Rasterization, or "What is `glBegin(GL_LINES)` really doing?"

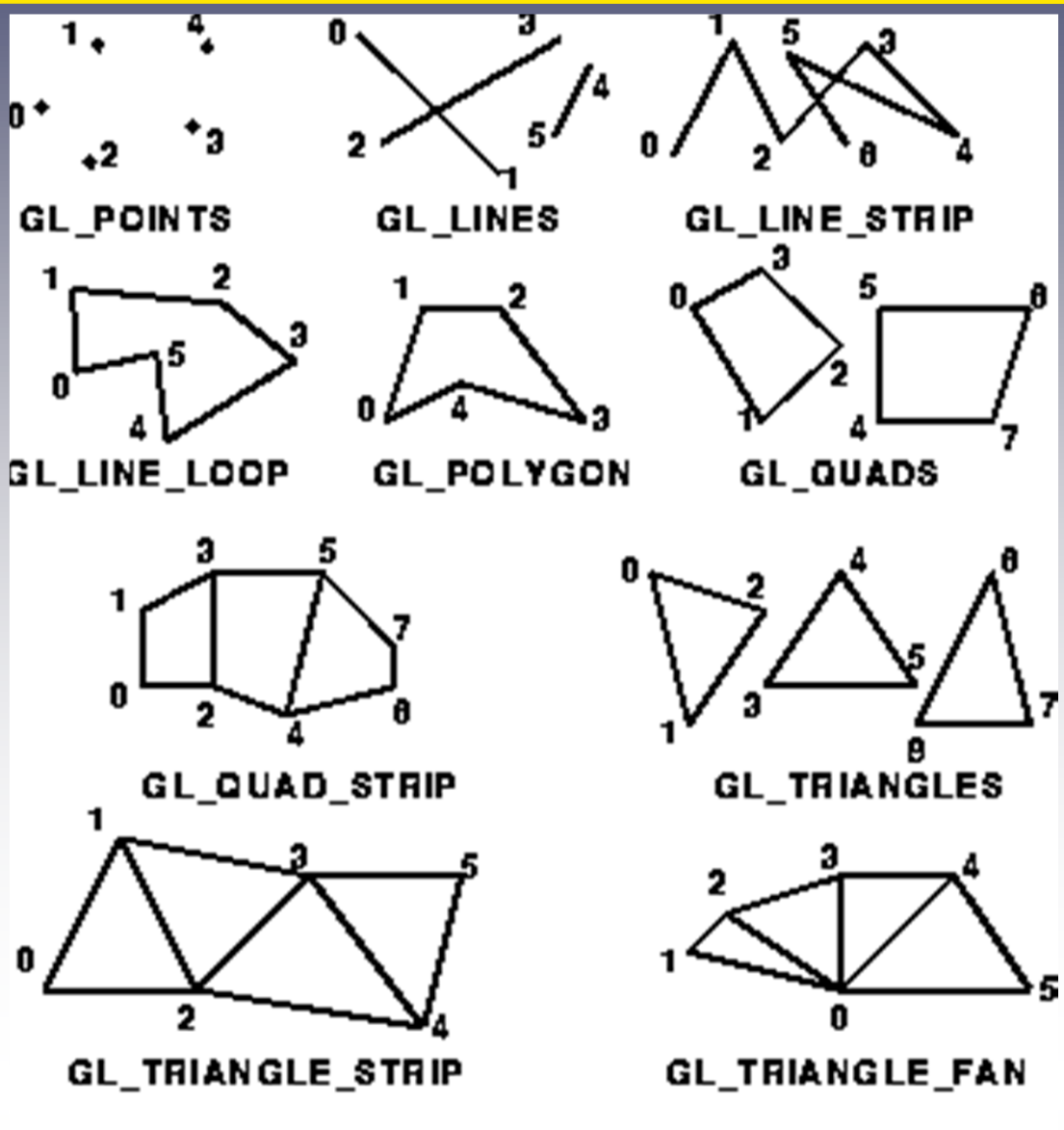
Prof. Christopher Rasmussen

for Prof. Yu's CISC 440/640

Outline

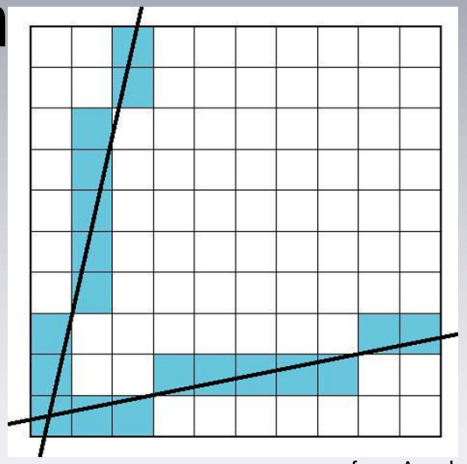
- Rasterizing lines
  - DDA/parametric algorithm
  - Midpoint/Bresenham's algorithm
- Rasterizing polygons/triangles

OpenGL Geometric Primitives



Rasterization: What is it?

- How to go from **floating-point coords** of geometric primitives' vertices to **integer coordinates** of pixels on screen
- Geometric primitives
  - Points: Round vertex location in screen coordinates



from Angel

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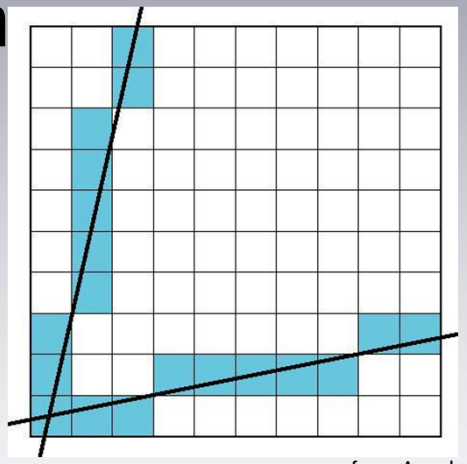

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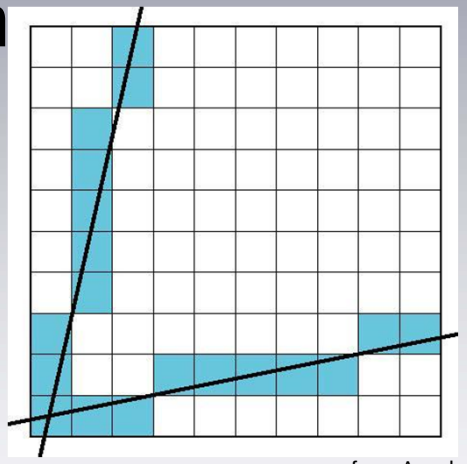
  - Polygons: How to fill area bounded by edges?



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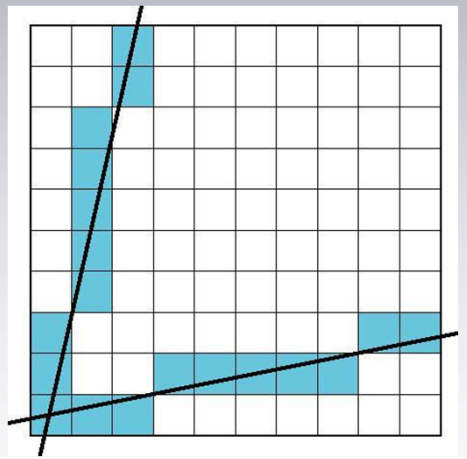
Rasterizing Lines: Goals

- Draw pixels as close to the ideal line as possible



Rasterizing Lines: Goals

- Draw pixels as close to the ideal line as possible
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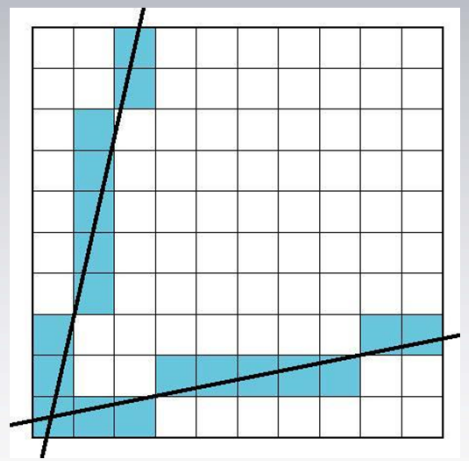

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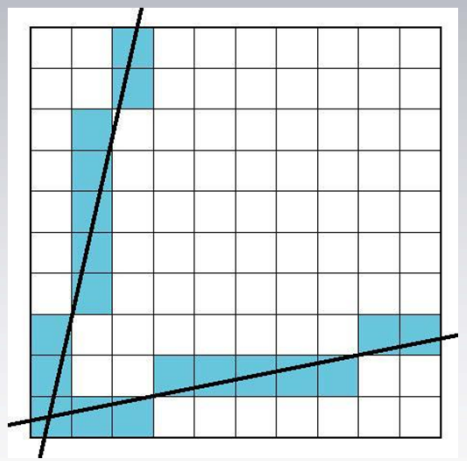
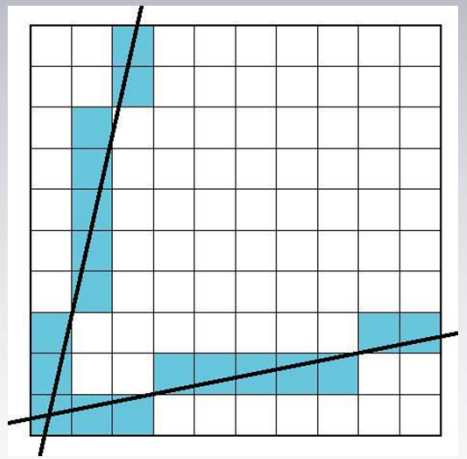


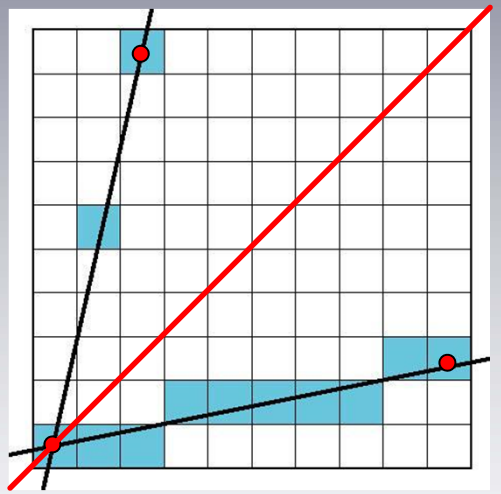
Rasterizing Lines: Goals

- Draw pixels as close to the ideal line as possible
- Use the minimum number of pixels without leaving any gaps
- Do it efficiently
- Extras
  - Handle different line styles
    - Width
    - Stippling (dotted and dashed lines)
    - Join styles for connected lines
  - Minimize aliasing (“jaggies”)



DDA/Parametric Line Drawing

- DDA stands for Digital Differential Analyzer, the name of a class of old machines used for plotting functions
- Slope-intercept form of a line: **`y = mx + b`**
  - **`m = dy/dx`**
  - **`b`** is where the line intersects the Y axis



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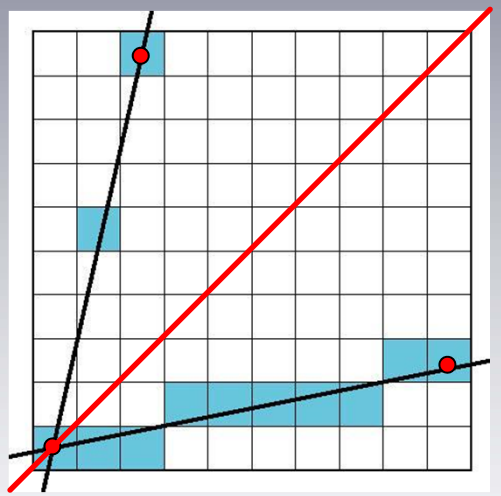
- DDA's basic idea: If we increment the **`x`** coordinate by 1 pixel at each step, the slope of the line tells us how much to increment **`y`** per step
  - I.e., **`m = dy/dx`**, so for **`dx = 1`**, **`dy = m`**



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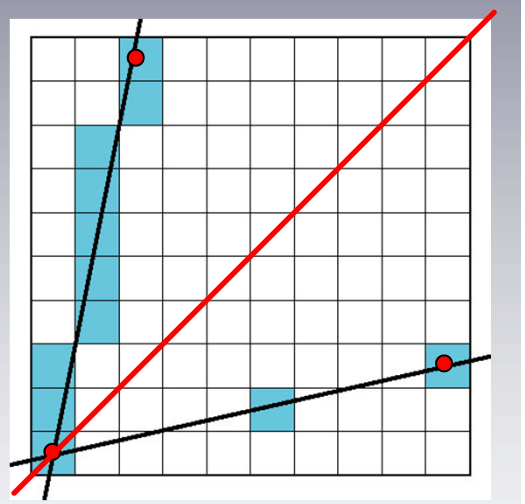
  - I.e., **`m = dy/dx`**, so for **`dx = 1`**, **`dy = m`**
  - This only works if **`m <= 1`**—otherwise there are gaps
    - Solution: Reverse axes and step in Y direction. Since now **`dy = 1`**, we get **`dx = 1/m`**



from Angel

DDA: Algorithm

1. Given endpoints $(x_0, y_0), (x_1, y_1)$
   - Integer coordinates: Round if endpoints were originally real-valued
   - Assume (x_0, y_0) is to the left of (x_1, y_1): Swap if they aren't

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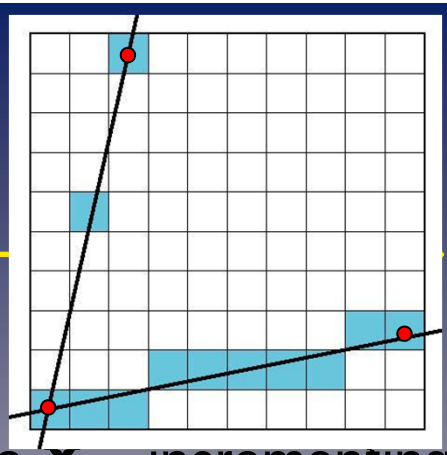


3. Iterate
   - If **`|m| <= 1`**: Iterate integer **`x`** from x_0 to x_1, incrementing by 1 each step
     - Initialize real **`y = y0`**
     - At each step, **`y += m`**, and plot point **`(x, round(y))`**
   - Else **`|m| > 1`**: Iterate integer **`y`** from y_0 to y_1, incrementing (or decrementing) by 1
     - Initialize real **`x = x0`**
     - At each step, **`x += 1/m`**, and plot **`(round(x), y)`**

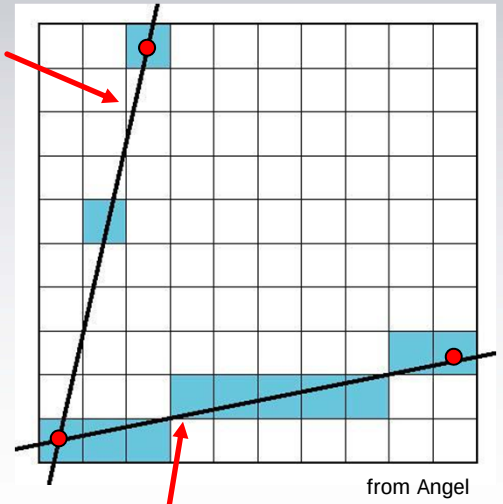
Midpoint/Bresenham's line drawing

- DDA is somewhat slow
  - Floating-point calculations, rounding are relatively expensive
- Big idea: Avoid rounding, do everything with integer arithmetic for speed
- Assume slope between 0 and 1
  - Again, handle lines with other slopes by using symmetry

Midpoint line drawing: Line equation

- Recall that the slope-intercept form of the line is
  y = (dy/dx)x + b
- Multiplying through by **dx**, we can rearrange this in **implicit form**:

F(x, y) = dy x - dx y + dx b = 0



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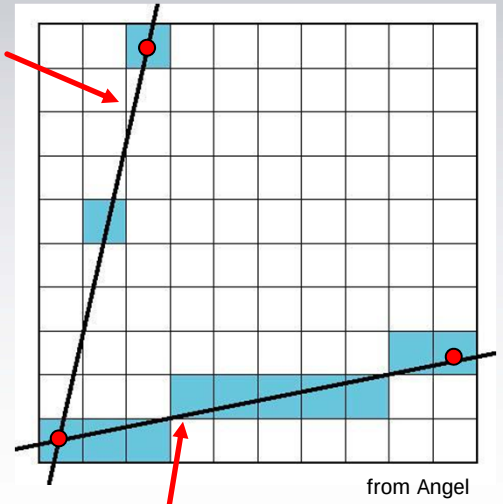
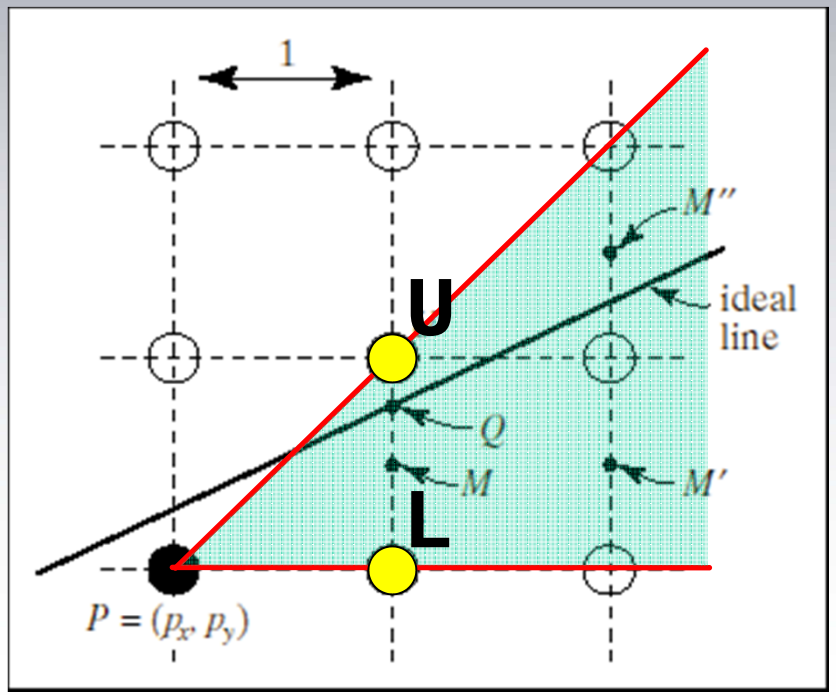
- **F** is:
  - Zero for points on the line
  - Positive for points **below** the line (**right** if slope > 1)
  - Negative for points **above** the line (**left** if slope > 1)
    - Examples: (0, 1), (1, 0), etc.



Midpoint line drawing: The Decision

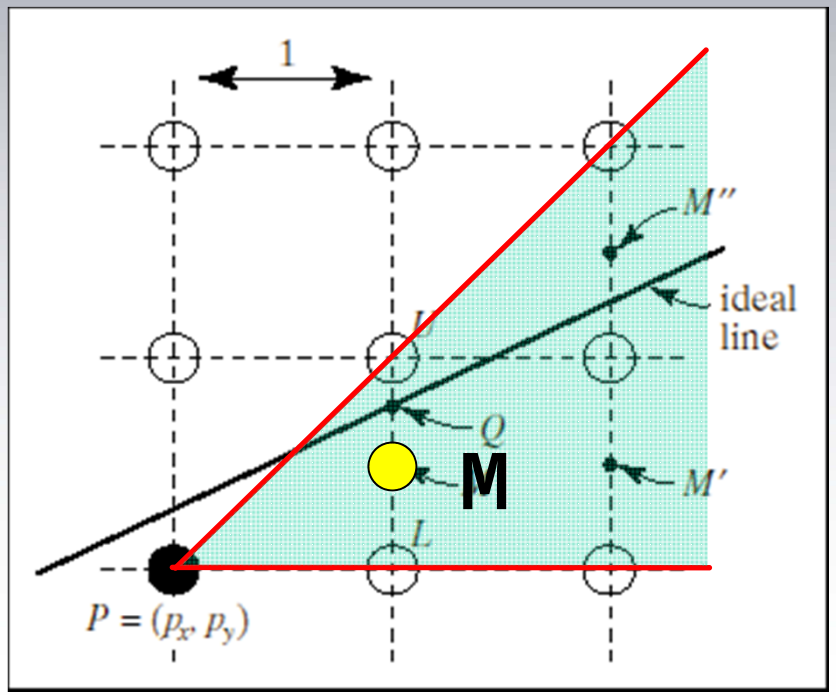
- Given our assumptions about the slope, after drawing **`(x, y)`** the only choice for the next pixel is between the upper pixel **`U = (x+1, y+1)`** and the lower one **`L = (x+1, y)`**
- We want to draw the one closest to the "ideal" line



from Hill

Midpoint line drawing: Midpoint decision

- After drawing **`(x, y)`**, in order to choose the next pixel to draw we consider the **midpoint** **`M = (x+1, y+0.5)`**
  - If **`M`** is **on** the line, then **`U`** and **`L`** are equidistant from the line



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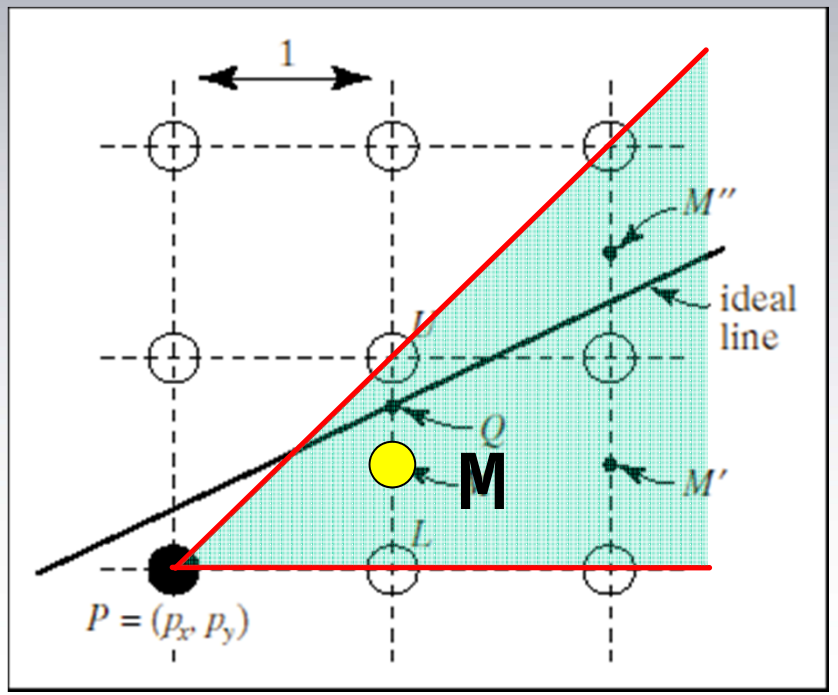

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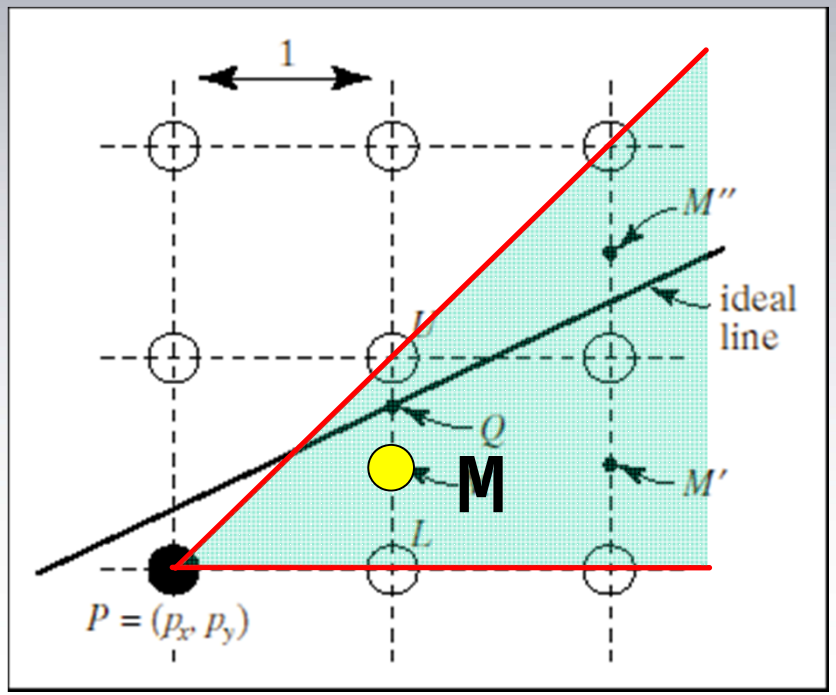
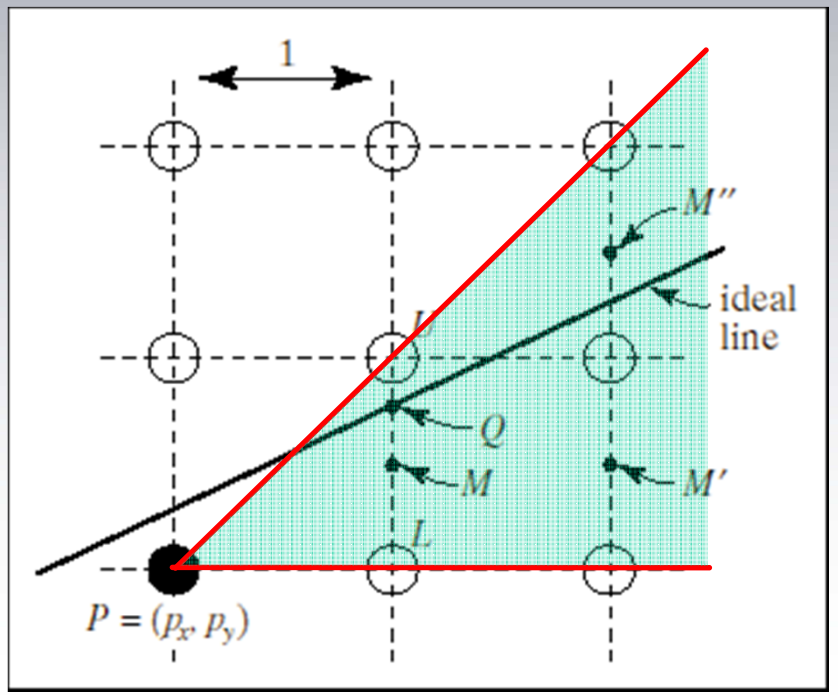


from Hill

Midpoint line drawing: Midpoint decision

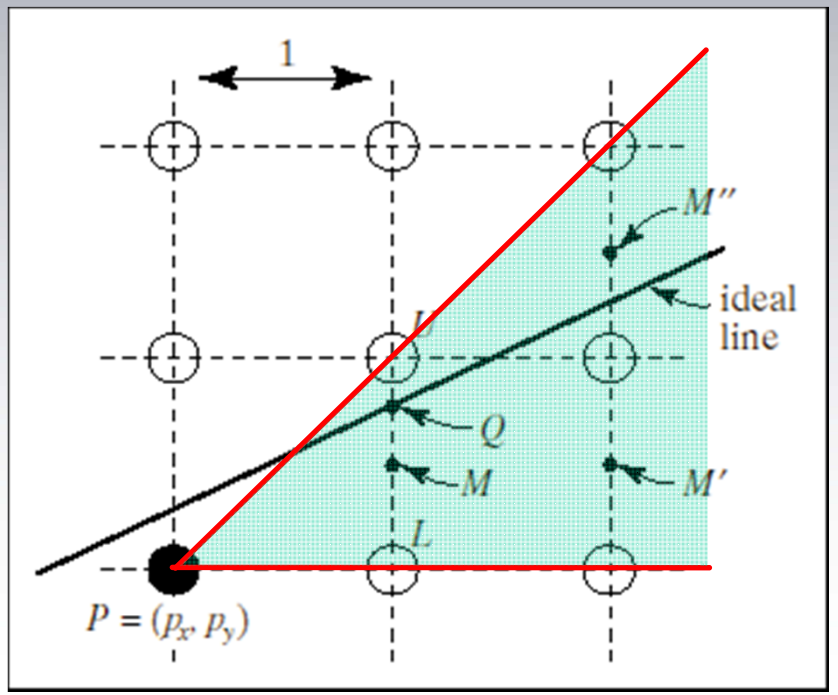
- So **F** is a **decision function** about which pixel to draw:
  - If **`F(M) = F(x+1, y+0.5) > 0`** (**M** below the line), pick **U**
  - If **`F(M) = F(x+1, y+0.5) <= 0`** (**M** above or on line), pick **L**



from Hill

Midpoint line drawing: Implementation

- Key efficiency insight: **F** does not have to be fully evaluated every step
- Suppose we do the full evaluation once and get **`F(x+1, y+0.5)`**



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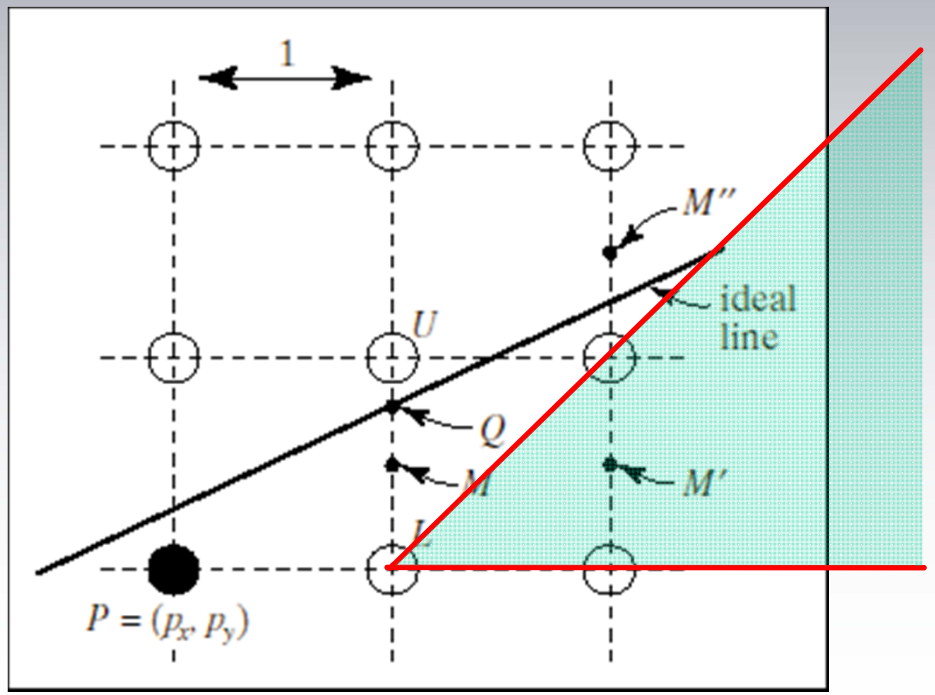
- Suppose we do the full evaluation once and get **`F(x+1, y+0.5)`**
  - If we choose **L**, next midpoint to evaluate **`M'`** is at **`F(x+2, y+0.5)`**



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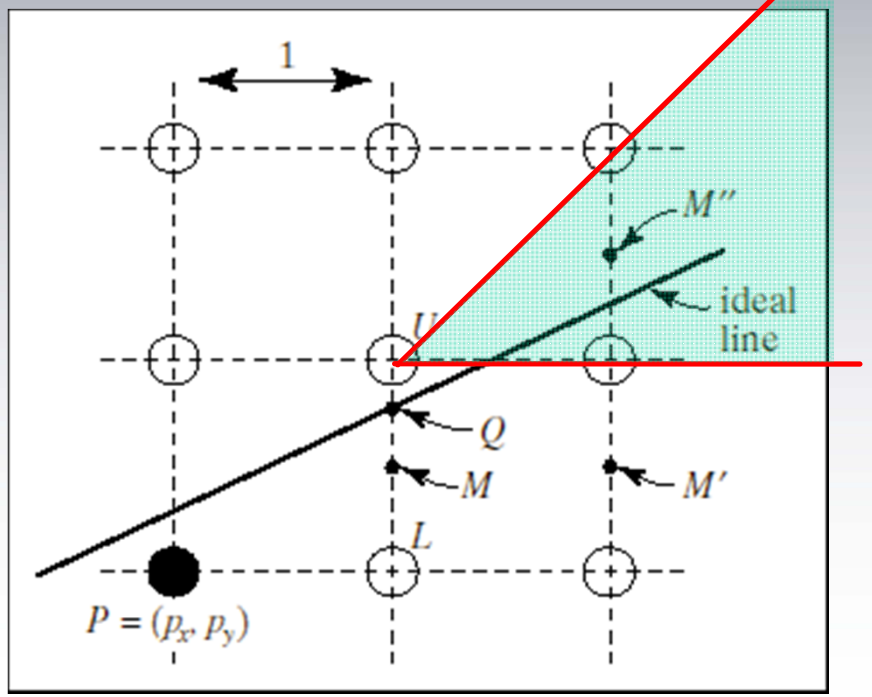
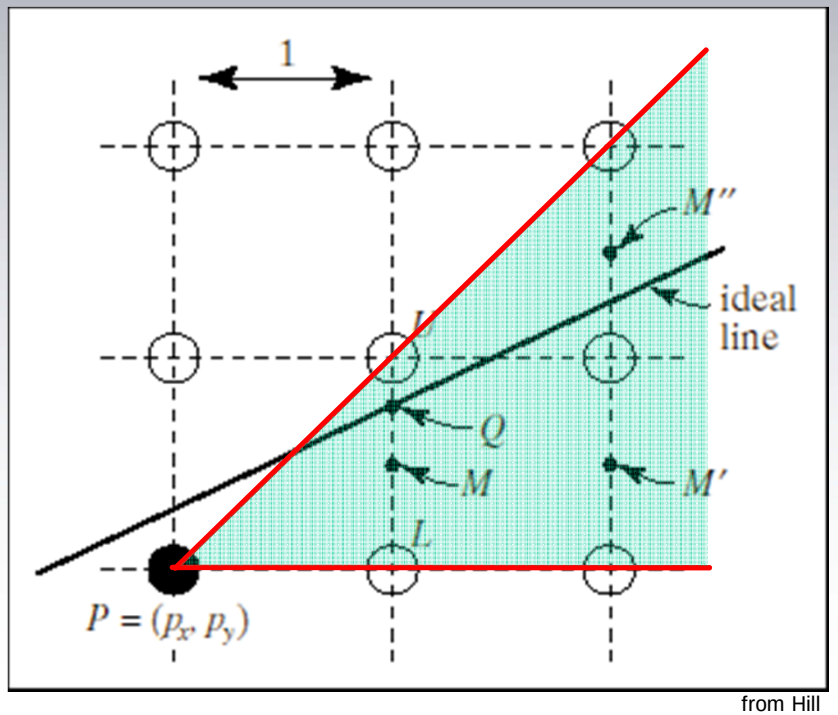
  - If we choose **L**, next midpoint to evaluate **M'** is at **F(x+2, y+0.5)**
  - If we choose **U**, next midpoint **M''** would be at **F(x+2, y+1.5)**



from Hill

Midpoint line drawing: Implementation

- Expanding these out using $F(x, y) = dy\ x - dx\ y + dx\ b$:
  - $F_M = F(x + 1, y + 0.5) = dy(x + 1) - dx(y + 0.5) + dx\ b$
  - $F_{M'} = F(x + 2, y + 0.5) = dy(x + 2) - dx(y + 0.5) + dx\ b$
  - $F_{M''} = F(x + 2, y + 1.5) = dy(x + 2) - dx(y + 1.5) + dx\ b$
- Note that $F_{M'} = F_M + dy$ and $F_{M''} = F_M + dy - dx$



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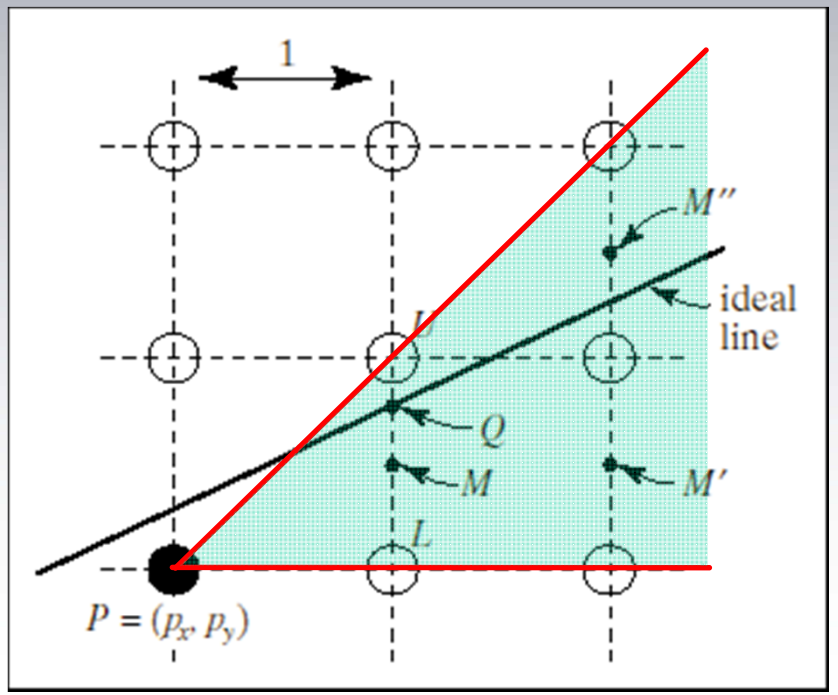
  - $F_{M''} = F(x + 2, y + 1.5) = dy(x + 2) - dx(y + 1.5) + dx\ b$
- Note that $F_{M'} = F_M + dy$ and $F_{M''} = F_M + dy - dx$
- So depending on whether we choose **L** or **U**, we just have to add **dy** or **dy - dx**, respectively, to the old value of **F** in order to get the new value



from Hill

Midpoint line drawing: Algorithm

- To initialize, we do a full calculation of **F** at the first midpoint next to the left line endpoint:

$$F(x_0 + 1,\ y_0 + 0.5) = dy(x_0 + 1) - dx(y_0 + 0.5) + dx\ b$$
$$= dy\ x_0 - dx\ y_0 + dx\ b + dy - 0.5\ dx$$
$$= F(x_0,\ y_0) + dy - 0.5\ dx$$

  - But $F(x_0, y_0) = 0$ since it's on the line, so our first $F = dy - 0.5\ dx$
  - Only the sign matters for the decision, so to make it an **integer value** we multiply by 2 to get $2F = 2\ dy - dx$
- To update, keep current values for **x** and **y** and a running total for **F**:
  - When **L** is chosen: **F += 2dy** and **x++**
  - When **U** is chosen: **F += 2(dy - dx)** and **x++** , **y++**

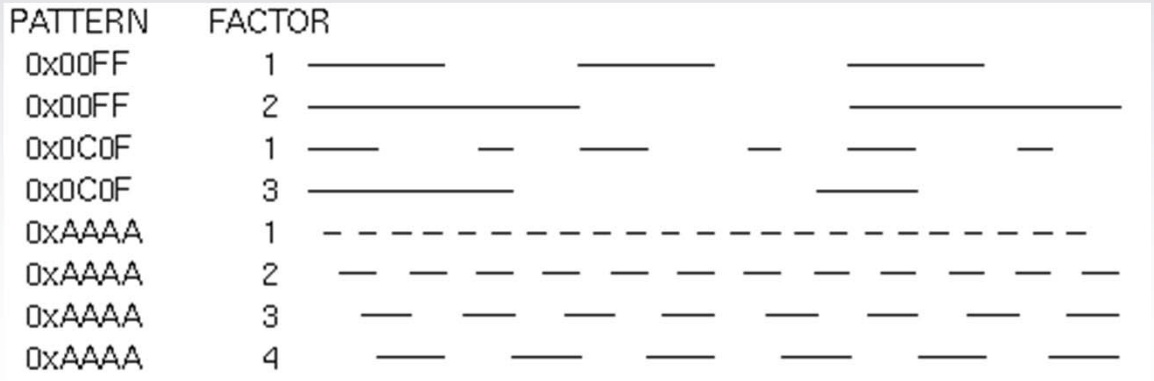
Line drawing speed

- 100,000 random lines in 500 x 500 window (average of 5 runs)

- DDA: 6.8 seconds
- Midpoint: 2.5 seconds
- OpenGL using **`GL_LINES`** (in software): 1.6 seconds

Extensions

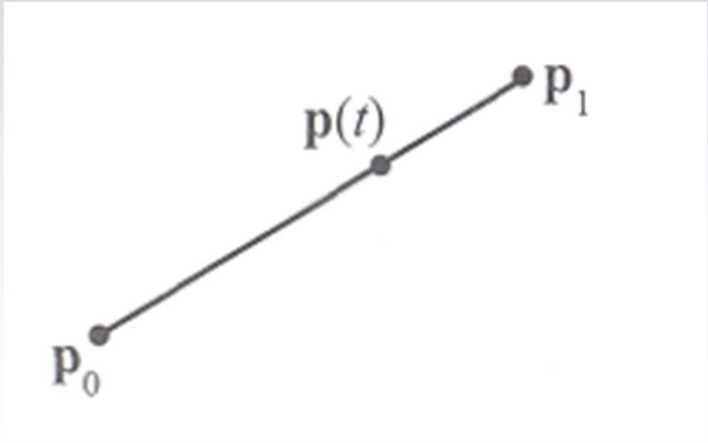
- How to draw thick (>1 pixel wide) lines?
  - Nested Bresenham's (perpendicular to main line at each step, or series of parallel lines)
- Stippled/dashed lines
  - Add state (pen up/down) inside loop
  - OpenGL: `glLineStipple(factor, pattern)`, `glEnable(GL_LINE_STIPPLE)`



Blending via Linear Interpolation

- Different colors at endpoints can be blended as we rasterize: vary color with distance fraction
- Parametric definition of a line segment:

$$\mathbf{p}(t) = \mathbf{p}_0 + t(\mathbf{p}_1 - \mathbf{p}_0), \text{ where } t \in [0, 1]$$
$$= \mathbf{p}_0 - t\,\mathbf{p}_0 + t\,\mathbf{p}_1$$
$$= (1 - t)\mathbf{p}_0 + t\,\mathbf{p}_1$$

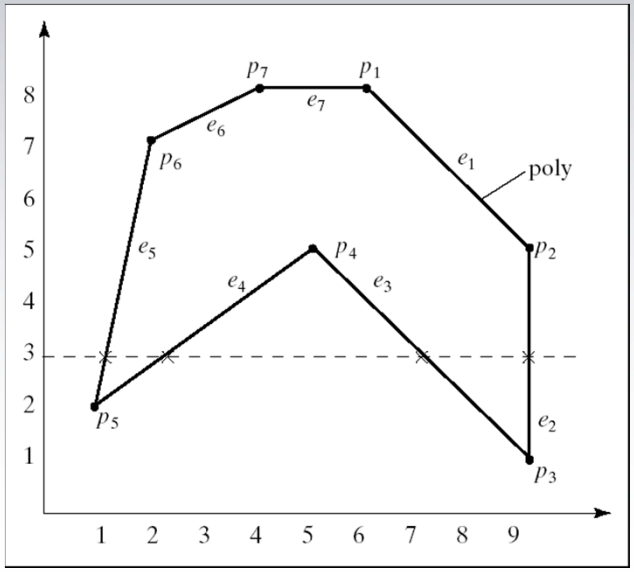


from Akenine-Möller & Haines

like a “blend” of the two endpoints

Polygon Rasterization

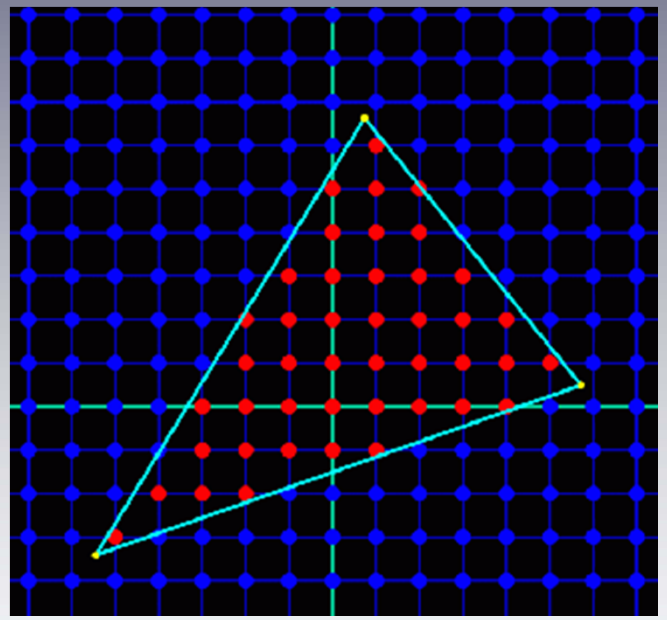
- Given a set of vertices, want to fill the interior
- Basic procedure:
  - Iterate over **scan lines** between top and bottom vertex
  - For each scan line, find all intersections with polygon edges
  - Sort intersections by x-value and fill pixel runs between even-odd pairs of intersections



from Hill

Rasterizing triangles

- Special case of polygon rasterization
  - Exactly two **active** edges at all times
- One method:
  - Fill scanline table between top and bottom vertex with leftmost and rightmost side by using DDA or midpoint algorithm to follow edges
  - Traverse table scanline by scanline, fill run from left to right



General polygons → triangles

- Can convert arbitrary polygon to set of triangles via **tesselation** (e.g., **gluTess*()** functions)