An Orthogonal Space–Time Coded CPM System With Fast Decoding for Two Transmit Antennas

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***Abstract*—Trellis-coded space–time (TC-ST) coding for continuous-phase modulation (TC-ST-CPM) was recently proposed by Zhang and Fitz. In this paper, we propose an orthogonal space–time coding for CPM (OST-CPM) systems and two transmit antennas. In the proposed OST-CPM, signals from two transmit antennas at any time t are orthogonal while both of them have continuous phases. Similar to Alamouti's OST coding for phase-shift keying (PSK) and quadrature amplitude modulation (QAM) systems, the newly proposed OST-CPM has a fast decoding algorithm.**

***Index Terms*—Alamouti's scheme, continuous-phase modulation, orthogonality, space–time coding.**

I. Introduction

SPACE–TIME coding for multiple transmit antennas has attracted considerable attention due to its potential capacity increase, see, for example, [1]–[8]. Due to a large number of codewords for a reasonable rate space–time code, its decoding complexity may be prohibitively high. Alamouti [5] recently proposed an *orthogonal* space–time (OST) code design for two transmit antennas such that the decoding is fast, i.e., symbol-by-symbol decoding, and has the full diversity. This idea has been extended to a general number of transmit antennas by Tarokh, Jafarkani, and Calderbank [6], and further generalized in [8]. The key reason for the fast decoding of OST codes is the orthogonality that enables maximum-likelihood (ML) decoding of multiple symbols to be reduced into ML decoding of individual symbols.

Note that the above mentioned space–time coding schemes are for phase-shift keying (PSK) and quadrature amplitude modulation (QAM) modulation systems. Continuous-phase modulation (CPM), on the other hand, has also been widely used due to its spectral efficiency and wireless fading resistance [8], such as in the Global System for Mobile Communications (GSM) standard. Zhang and Fitz in [10] recently proposed a trellis-coded space–time CPM (TC-ST-CPM) system. The goal of this paper is to design an orthogonal space–time coding for CPM system similar to the OST for PSK and QAM systems. The difficulty for the OST code design for CPM systems arises due to the constraint of the continuous phase of a transmitted signal.

In this paper, by modifying Alamouti's scheme, an OST code design for two transmit antennas and both full and partial response CPM systems (OST-CPM) is given. For the newly proposed full response OST-CPM, we develop a fast decoding algorithm that is *not* simply the one for Alamouti's scheme, which is briefly explained as follows. Because of the memory in the CPM, the symbols sent by different transmit antennas cannot be separated independently at the receiver, which is different from Alamouti's scheme for QAM systems. These symbols, however, can be separated into several independent subsets of independent symbols on each branch of a CPM trellis and these subsets depend on the modulation indexes used in the CPM system. Then, the joint ML decoding of multiple symbols becomes a subset index searching and a symbol-by-symbol searching on each branch at a state. Furthermore, the number of states is the number of subsets, i.e., the coset size, which is at most the same as the one in a single-antenna CPM system as we shall see later in details. Because the coset size only depends on the CPM indexes and does not depend on the size of CPM symbols, it is usually small compared to the CPM symbol constellation size. For example, the coset size is two for the CPM with index $h = 1/2$. Therefore, the demodulation complexity can be significantly reduced.

The paper is organized as follows. In Section II, we describe the system model. In Section III, we present the OST-CPM design for a full-response CPM system. In Section IV, we propose a fast demodulation scheme. In Section V, we study the performance. In Section VI, we generalize the OST-CPM design for full-response CPM systems obtained in Section III to partial-response CPM systems. In Section VII, we present some simulation results.

II. System Model

We adopt some notations from [10]. In this paper, we consider a mobile communication system with two transmit antennas $L_t = 2$ and L_r receive antennas, which is shown in Fig. 1. Let $\boldsymbol{d}_m = (d_{m,0}, d_{m,1}, \ldots)$ denote the information symbol sequence for the mth transmit antenna (after the channel coding if there is any). The signal $y_n(t, \boldsymbol{d})$ received by the nth receive antenna can be written as [9], [10]

$$y_n(t, \boldsymbol{d}) = \sum_{m=1}^{2} \alpha_{m,n}(t) s_m(t, \boldsymbol{d}_m) + W_n(t) \tag{1}$$

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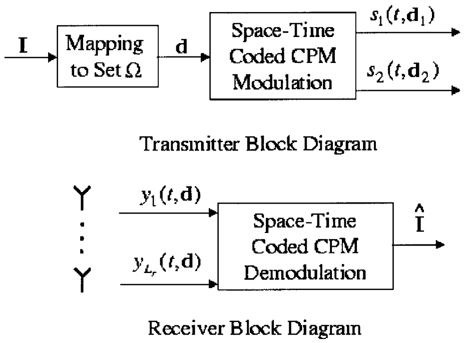

Fig. 1. Space–time CPM diagram.

where $W_n(t)$ is the additive noise, $\alpha_{m,n}(t)$ is the channel gain from the mth transmit antenna to the nth receive antenna, and

$$s_m(t, \boldsymbol{d}_m) = \sqrt{\frac{1}{T}} \exp\left\{j2\pi\left[\phi_0 + \Phi_m(t, \boldsymbol{d}_m(l))\right]\right\} \tag{2}$$

and

$$\Phi_m(t, \boldsymbol{d}_m(l)) = h_m \sum_{i=0}^{l} d_{m,i}\, q(t-(i-1)T), \qquad (l-1)T \le t \le lT \tag{3}$$

and

$$\boldsymbol{d}_m(l) = (d_{m,0}, d_{m,1}, \ldots, d_{m,l})$$

is the lth modulation symbol sequence of the mth transmit antenna and $d_{m,i}$ comes from the signal constellation set

$$\Omega \triangleq \{-M+1, -M+3, \ldots, -1, 1, \ldots, M-1\} \tag{4}$$

M is an even number, h_m is the modulation index of the CPM, T is the symbol time duration, and $q(t)$ is the phase smoothing response function.

When $h_m = \frac{2m_0}{p}$, where m_0 and p are relatively prime integers, the phase $\Phi_m(t, \boldsymbol{d}_m(l))$ can be expressed as [9], [10], for $(l-1)T \le t \le lT$

$$\Phi_m(t, \boldsymbol{d}_m(l)) = \theta_m(l-\gamma) + h_m \sum_{i=l-\gamma+1}^{l} d_{m,i}\, q(t-(i-1)T) \tag{5}$$

where γ is the modulation memory size and

$$\theta_m(l-\gamma) = \frac{h_m}{2} \sum_{i \le l-\gamma} d_{m,i} \tag{6}$$

which belongs to the set Ω_θ defined as (after modulo 1)

$$\Omega_\theta \triangleq \left\{0, \frac{1}{p}, \frac{2}{p}, \ldots, \frac{p-1}{p}\right\}. \tag{7}$$

When $\gamma = 1$, this system is called a full response system. In this case, for $(l-1)T \le t \le lT$ the phase $\Phi_m(t, \boldsymbol{d}_m(l))$ is

$$\Phi_m(t, \boldsymbol{d}_m(l)) = \theta_m(l-1) + h_m d_{m,l}\, q(t-(l-1)T). \tag{8}$$

Thus, $\Phi_m(t, \boldsymbol{d}_m(l))$ has a trellis structure with states in Ω_θ. For the space–time coded CPM, the phase $(\Phi_1(t, \boldsymbol{d}_1(l)), \Phi_2(t, \boldsymbol{d}_2(l)))$ has a trellis structure with states in the product set $\Omega_\theta \times \Omega_\theta$, i.e., the number of states increases exponentially with the number of transmit antennas.

The ML demodulation of the information sequences $(\boldsymbol{d}_1', \boldsymbol{d}_2')$ of length N_c is [9], [10]

$$\left(\hat{\boldsymbol{d}}_1, \hat{\boldsymbol{d}}_2\right) = \arg\min_{(\boldsymbol{d}_1, \boldsymbol{d}_2)} \left\{ \sum_{n=1}^{L_r} \int_0^{N_c T} \left| y_n(t, \boldsymbol{d}') - \sum_{m=1}^{2} \alpha_{m,n}(t) s_m(t, \boldsymbol{d}_m) \right|^2 dt \right\}. \tag{9}$$

One can see that, in the above ML demodulation, for both sequences $\boldsymbol{d}_1'$ and $\boldsymbol{d}_2'$ there are M^2 branches leaving and coming to each state in the trellis structure, which is large with large M. In addition, as we explained earlier, the number of states increases exponentially with the number of transmit antennas. We next propose an OST-CPM scheme so that the symbols coming from different transmit antennas can be separated at the receivers and therefore the ML demodulation complexity can be reduced. For convenience, we first study the full response CPM systems with simpler notations and then generalize it to partial-response CPM systems with more complex notations.

III. OST–Encoded CPM Design for Full-Response CPM Systems

The information sequence is first mapped into the sequence

$$\boldsymbol{d} = \{d_{1,0}, d_{2,0}, d_{1,1}, d_{2,1}, \ldots, d_{1,l}, d_{2,l}, \ldots, d_{1,L}, d_{2,L}\}$$

of symbols $d_{m,l} \in \Omega$. The sequence $\boldsymbol{d}$ is then modulated with the CPM to generate two CPM-modulated signal waveforms $s_m(t, \boldsymbol{d})$, $m = 1, 2$. These two CPM-modulated signals are transmitted by the two transmit antennas simultaneously. The main goal of this section is to design the CPM waveforms $s_m(t, \boldsymbol{d})$, $m = 1, 2$, such that the rows of the matrix

$$\begin{bmatrix} s_1(t, \boldsymbol{d}) & s_1(t+T, \boldsymbol{d}) \\ s_2(t, \boldsymbol{d}) & s_2(t+T, \boldsymbol{d}) \end{bmatrix} \tag{10}$$

are orthogonal for each t for the fast demodulation to be studied in the next section. As a remark, the above orthogonality is between two waveforms from two transmit antennas and different from the orthogonality in the minimum shift-keying (MSK) modulation, where two waveforms corresponding to two different information symbols are orthogonal. Also note that, in Alamouti's OST coding [5], the OST code is

$$\begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}$$

and the first antenna transmits x_1 and x_2 while the second antenna transmits $-x_2^*$ and x_1^*. Clearly, the signals between the two transmit antennas are orthogonal.

To do so, we use two smoothing phase response functions $q(t)$ and $q_0(t)$ with $q(0) = q_0(0) = 0$ and $q(T) = q_0(T) = 1/2$. The symbols $d_{1,l}$ and $d_{2,l}$ are jointly encoded. Assume the modulation index $h = \frac{2m_0}{p}$, where m_0 and p are relatively prime integers. At the time slot between $2(l-1)T$ and $2lT$, the following signals are sent through the first transmit antenna:

$$s_1(t,\boldsymbol{d}) = \exp\{j2\pi\Phi_1(t,\boldsymbol{d})\}, \qquad 2(l-1)T \le t \le 2lT \tag{11}$$

where

$$\Phi_1(t,\boldsymbol{d}) = \theta_1(2(l-1)) + hd_{1,l}q(t-2(l-1)T), \quad \text{for } 2(l-1)T \le t \le (2l-1)T \tag{12}$$

$$\Phi_1(t,\boldsymbol{d}) = \theta_1(2l-1) + hd_{2,l}q(t-(2l-1)T), \quad \text{for } (2l-1)T \le t \le 2lT \tag{13}$$

and

$$\theta_1(2(l-1)) = \Phi_1(2(l-1)T,\boldsymbol{d}) \tag{14}$$

$$\theta_1(2l-1) = \Phi_1((2l-1)T,\boldsymbol{d}). \tag{15}$$

At the time slot between $2(l-1)T$ and $2lT$, the following signals are sent through the second transmit antenna:

$$s_2(t,\boldsymbol{d}) = \exp\{j2\pi\Phi_2(t,\boldsymbol{d})\}, \qquad 2(l-1)T \le t \le 2lT \tag{16}$$

where

$$\Phi_2(t,\boldsymbol{d}) = \theta_2(2(l-1)) - hd_{2,l}q(t-2(l-1)T) + c_l q_0(t-2(l-1)T), \quad \text{for } 2(l-1)T \le t \le (2l-1)T \tag{17}$$

$$\Phi_2(t,\boldsymbol{d}) = \theta_2(2l-1) - hd_{1,l}q(t-(2l-1)T) + c_l q_0(t-(2l-1)T), \quad \text{for } (2l-1)T \le t \le 2lT \tag{18}$$

and

$$\theta_2(2(l-1)) = \Phi_2(2(l-1)T,\boldsymbol{d}) \tag{19}$$

$$\theta_2(2l-1) = \Phi_2((2l-1)T,\boldsymbol{d}) \tag{20}$$

and

$$c_l = 1 + \frac{4m_0}{p} + 2\,\mathrm{mod}\left(\frac{(m_{1,l}+m_{2,l})2m_0}{p}, 1\right) \tag{21}$$

where $\mathrm{mod}\,(x,y)$ is the modulo operation of x with base y, $m_{1,l}$ and $m_{2,l}$ are the integers such that

$$d_{1,l} = 2m_{1,l} + 1 \tag{22}$$

$$d_{2,l} = 2m_{2,l} + 1. \tag{23}$$

Thus, using (21)–(23), the parameters c_l, $d_{1,l}$, and $d_{2,l}$ satisfy the following relationship:

$$c_l - (d_{1,l} + d_{2,l})\,h = 2k+1 \tag{24}$$

where k is an integer. By noticing $h = 2m_0/p$, from (21) we find that c_l has only p_0 possible values for all different symbol values of $d_{1,l}$ and $d_{2,l}$, where

$$p_0 = \begin{cases} p, & \text{if } p \text{ is odd} \\ \frac{p}{2}, & \text{if } p \text{ is even.} \end{cases} \tag{25}$$

We now want to check that the row vectors of the transmission signal matrix defined in (10) are orthogonal for each t. From (13) and (18), for any t, $2(l-1)T \le t \le (2l-1)T$, we have

$$\Phi_1(t+T,\boldsymbol{d}) - \Phi_2(t+T,\boldsymbol{d}) = \theta_1(2l-1) - \theta_2(2l-1) + h(d_{2,l}+d_{1,l})q(t-(2l-2)T) - c_l q_0(t-(2l-2)T).$$

Since

$$\begin{aligned}\theta_1(2l-1) &= \theta_1(2l-2) + d_{1,l}hq(T) = \theta_1(2l-2) + d_{1,l}\frac{h}{2}\\ \theta_2(2l-1) &= \theta_2(2l-2) - d_{2,l}hq(T) + c_l q_0(T)\\ &= \theta_2(2l-2) - d_{2,l}\frac{h}{2} + \frac{c_l}{2}\end{aligned}$$

we have

$$\begin{aligned}&\Phi_1(t+T,\boldsymbol{d}) - \Phi_2(t+T,\boldsymbol{d})\\ &= \theta_1(2l-2) - \theta_2(2l-2) + \frac{(d_{1,l}+d_{2,l})h}{2} - \frac{c_l}{2}\\ &\qquad + h(d_{2,l}+d_{1,l})q(t-(2l-2)T) - c_l q_0(t-(2l-2)T)\\ &\overset{\text{(i)}}{=} -k - \frac{1}{2} + \theta_1(2l-2) - \theta_2(2l-2)\\ &\qquad + (d_{2,l}+d_{1,l})hq(t-(2l-2)T) - c_l q_0(t-(2l-2)T)\\ &= \Phi_1(t,\boldsymbol{d}) - \Phi_2(t,\boldsymbol{d}) - k - \frac{1}{2}\end{aligned} \tag{26}$$

where step (i) follows from (24). Therefore, we have

$$s_1(t,\boldsymbol{d})s_2^*(t,\boldsymbol{d}) = -s_1(t+T,\boldsymbol{d})s_2^*(t+T,\boldsymbol{d}) \tag{27}$$

i.e., the rows of the matrix in (10) are orthogonal.

We next describe the detailed relationship between $s_1(t,\boldsymbol{d})$ and $s_2(t,\boldsymbol{d})$, which will be used in the next section for the demodulation.

We now decompose the set Ω into p_0 disjoint subsets $\Omega_1, \Omega_2, \ldots, \Omega_{p_0}$ as follows: for $g = 1, 2, \ldots, p_0$

$$\Omega_g \triangleq \{-M+2g-1, -M+2g-1+2p_0, -M+2g-1+4p_0, \ldots\} \cap \Omega.$$

By doing so, it is not hard to see that the value of c_l only depends on the indexes $g_1(l)$ and $g_2(l)$ of the subsets $\Omega_{g_1(l)}$ and $\Omega_{g_2(l)}$ to which the information symbols $d_{1,l}$ and $d_{2,l}$ belong, respectively. Therefore, c_l can be written as $c_l = c_{g_1(l),g_2(l)}$. Let

$$G \triangleq \{1, 2, \ldots, p_0\} \tag{28}$$

denote the set of all subset indexes g in representing Ω_g.

Because $q(t) = q_0(t) = 1/2$ for $t \ge T$ (for full response CPM systems), it is easy to check that

$$2\pi\theta_1(2l) = 2\pi\theta_2(2l) \bmod 2\pi, \qquad l = 1, 2, \ldots$$

if the same initial states $\theta_2(0)$ and $\theta_1(0)$ are used, for example, $\theta_2(0) = \theta_1(0) = 0$. In this case, using (22) and (23)

$$\begin{aligned}\theta_1(2l) = \theta_2(2l) &= \sum_{i=1}^{l}(d_{1,i}+d_{2,i})\frac{h}{2}\\ &= \sum_{i=1}^{l}(m_{1,i}+m_{2,i}+1)h = \frac{2m_0M}{p}\end{aligned} \tag{29}$$

for some integer M. Thus, for simplicity we assume $\theta_1(2l) = \theta_2(2l), l = 1, 2, \ldots, N_c$. Similar to the discussion on c_l in (21), there are only p_0 possible values of $\theta_1(2l) = \theta_2(2l)$ in the modulo 1 sense, where p_0 is defined in (25). As we will see later, these possible values of $\theta_1(2l) = \theta_2(2l)$ are the states in the ML demodulation trellis of the above proposed OST-CPM and, therefore, the number of states for the ML demodulation is p_0.

For convenience, for $m = 1, 2$, let

$$s_{m,l}(t) \triangleq \exp\left\{j2\pi d_{m,l} h q(t - 2(l-1)T)\right\}, \quad 2(l-1)T \le t \le (2l-1)T \tag{30}$$

$$\Theta(l) \triangleq \exp\{j2\pi\theta_m(2(l-1))\} \tag{31}$$

and

$$s_m(t, 2l-1) \triangleq s_m(t, \boldsymbol{d}), \qquad 2(l-1)T \le t \le (2l-1)T$$
$$s_m(t, 2l) \triangleq s_m(t, \boldsymbol{d}), \qquad (2l-1)T \le t \le 2lT. \tag{32}$$

Since $d_{1,l}$ and $d_{2,l}$ are independent of each other, the above $s_{1,l}(t)$ and $s_{2,l}(t)$ are also independent of each other.

By the assumption that $\theta_1(2l) = \theta_2(2l)$, we have

$$s_1(t, 2l-1) = \Theta(l) s_{1,l}(t) \tag{33}$$

$$s_1(t, 2l) = \Theta(l) \exp\left\{j2\pi \frac{d_{1,l} h}{2}\right\} s_{2,l}(t-T) \tag{34}$$

where the value of the term

$$\exp\left\{j2\pi \frac{d_{1,l} h}{2}\right\}$$

in (34) only depends on the index number $g_1(l)$ of the subset $\Omega_{g_1(l)}$ to which the information symbol $d_{1,l}$ belongs. So, (34) can be rewritten as

$$s_1(t, 2l) = \Theta(l) A_{g_1(l)} s_{2,l}(t) \tag{35}$$

where $A_{g_1(l)}$ is a constant and $d_{1,l} \in \Omega_{g_1(l)}$.

Going back to (16), we have

$$\begin{aligned} s_2(t, 2l-1) &= \Theta(l) \exp\left\{j2\pi c_l q_0(t)\right\} s_{2,l}^*(t) \\ &= \Theta(l) \exp\left\{j2\pi c_{g_1(l),g_2(l)} q_0(t)\right\} s_{2,l}^*(t) \\ &= \Theta(l) C_{g_1(l),g_2(l)}(t) s_{2,l}^*(t) \end{aligned} \tag{36}$$

where

$$C_{g_1(l),g_2(l)}(t) = \exp\left\{j2\pi c_{g_1(l),g_2(l)} q_0(t)\right\}$$

and $c_{g_1(l),g_2(l)}$ depends only on the indexes $g_1(l)$, $g_2(l)$ of subsets $\Omega_{g_1(l)}$ and $\Omega_{g_2(l)}$ to which the information symbols $d_{1,l}$ and $d_{2,l}$ belong, as we explained earlier. Also,

$$\begin{aligned} s_2(t, 2l) &= \Theta(l) \exp\left\{-j2\pi \frac{d_{2,l} h}{2}\right\} \exp\left\{j2\pi \frac{c_l}{2}\right\} \\ &\quad \cdot \exp\left\{j2\pi c_l q_0(t)\right\} s_{1,l}^*(t) \\ &= \Theta(l) A_{g_2(l)} B_{g_1(l),g_2(l)} C_{g_1(l),g_2(l)}(t) s_{1,l}^*(t) \end{aligned} \tag{37}$$

where

$$B_{g_1(l),g_2(l)} = \exp\left\{j2\pi \frac{c_{g_1(l),g_2(l)}}{2}\right\} = \exp\left\{j2\pi \frac{c_l}{2}\right\}.$$

IV. A Fast Demodulation Algorithm

Consider the OST-CPM with even number N_c proposed in Section III. Let $\hat{\boldsymbol{d}}$ be the output of the ML demodulator (9)

$$\hat{\boldsymbol{d}} = \arg\min_{\boldsymbol{d}} \left\{ \sum_{n=1}^{L_r} \int_0^{N_c T} \left| y_n(t, \boldsymbol{d}') - \sum_{m=1}^{2} \alpha_{m,n}(t) s_m(t, \boldsymbol{d}) \right|^2 dt \right\}. \tag{38}$$

By the trellis structure of the CPM, the sequence detection in (38) can be implemented using Viterbi algorithm. In Viterbi algorithm, one needs to start from a state $\theta_m(2l)$ and select the survivor path from the M^2 incoming branches. There are p_0 previous states $\theta_m(2(l-1))_k, k = 1, \ldots, p_0$, at the last observation time $t = 2(l-1)T$. Among these M^2 incoming branches, there are $\frac{M^2}{p_0}$ branches coming from state $\theta_m(2(l-1))_k$ for each k. In the following, (39)–(41) are used to provide a fast algorithm to find the best path P_k among these $\frac{M^2}{p_0}$ branches which come through state $\theta_m(2(l-1)l)_k$. Then, P_k is compared with the other $p_0 - 1$ paths $P_{k'}$ that are from the preceding states and arrive at the state $\theta_m(2l)$ to find the survivor path, where $P_{k'}$ is obtained in the same way as P_k, i.e., it is the best path that comes from state $\theta_m(2(l-1))_{k'}, k' \neq k, k' = 1, \ldots, p_0$, arrives at state $\theta_m(2l)$. Next, we give the detailed algorithm for searching the best path P_k.

In order to search the best path P_k, the input $(d_{1,l}, d_{2,l})$ and the distance from previous state $\theta_m(2(l-1))_k$ to the current state $\theta_m(2l)$ need to be obtained, where the input $(d_{1,l}, d_{2,l})$ causes the state transfer from $\theta_m(2(l-1))_k$ to $\theta_m(2l)$. Thus, we need to search all the branch metrics at the stage l as follows:

$$\left(\hat{d}_{1,l}, \hat{d}_{2,l}\right) = \arg\min_{(d_{1,l}, d_{2,l}) \in \Omega \times \Omega} \left\{ \int_{2(l-1)T}^{2lT} \left| y_n(t, \boldsymbol{d}') - \sum_{m=1}^{2} \alpha_{m,n}(t) s_m(t, \boldsymbol{d}) \right|^2 dt \right\}. \tag{39}$$

We next want to simplify the above branch searching by taking advantage of the orthogonality of the space–time coded CPM design obtained in Section III.

Assume that the channel $\alpha_{m,n}(t)$ is known at the receiver, i.e., coherent detection, and constant during a space–time coding block $[2(l-1)T, 2lT]$. So, for convenience, $\alpha_{m,n}(t)$ is rewritten as $\alpha_{m,n}$. Also for convenience, the received signal $y_n(t, \boldsymbol{d}')$ is simply written as $y(t)$ by dropping the receive antenna index n and the transmitted symbol sequence $\boldsymbol{d}'$ in the following derivations. From the orthogonality of the signals $s_1(t, \boldsymbol{d})$ and $s_2(t, \boldsymbol{d})$ and the notations from (30)–(37), (39) can be rewritten as (40) at the bottom of the following page. Because $s_{1,l}(t)$ and $s_{2,l}(t)$ are independent of each other, $A_{g_1(l)}$, $A_{g_2(l)}$, $B_{g_1(l),g_2(l)}$, and $C_{g_1(l),g_2(l)}(t)$ only depend on the index of subsets $\Omega_{g_1(l)}$ and $\Omega_{g_2(l)}$, (40) can be decomposed as (41) at the bottom of the following page, where

$$\begin{aligned} F_1(d_{1,l}, g_1, g_2) &= \left|y(t) - \alpha_{1,n}\Theta(l) s_{1,l}(t)\right|^2 \\ &+ \left|y(t+T) - \alpha_{2,n}\Theta(l) A_{g_2(l)} B_{g_1(l),g_2(l)} C_{g_1(l),g_2(l)}(t) s_{1,l}^*(t)\right|^2 \end{aligned} \tag{42}$$

$$\begin{aligned} F_2(d_{2,l}, g_1, g_2) &= \left|y(t) - \alpha_{2,n}\Theta(l) C_{g_1(l),g_2(l)}(t) s_{2,l}^*(t)\right|^2 \\ &+ \left|y(t+T) - \alpha_{1,n}\Theta(l) A_{g_1(l)} s_{2,l}(t)\right|^2. \end{aligned} \tag{43}$$

The number of comparisons in the original branch searching (39) is $\frac{M^2}{p_0}$ while the one in (41) from one of the previous states $\theta_m(2(l-1))_k, k = 1, \ldots, p_0$, to the current state $\theta_m(2l)$ is $2p_0M$. Since p_0 defined in (25) only depends on the CPM modulation index h and does not depend on the signal constellation size M, it is usually much smaller than M. As an example, when $h = 1/2$ is used, $p_0 = 2$. In this case, the number of branch searching (from one of the previous states to the current state) times is $4M$ while the original one is $M^2/2$. Furthermore, all memories in the decoding are from $\Theta(l)$ as we can see from the above derivations. Thus, from (31), we know that the states are the possible values of $\theta_1(2(l-1)) = \theta_2(2(l-1))$ and, therefore, there are only p_0 states in the trellis as we explained before. From (25), one can see $p_0 \leq p$, which is smaller than p^2, the number of states of a general space–time block coded full response CPM system. From (28), (29), and (31), it is not hard to see that, for each fixed pair $(g_1, g_2) \in G \times G$, all pairs $(d_{1,l}, d_{2,l}) \in \Omega_{g_1} \times \Omega_{g_2}$ correspond to a single state $\Theta(l+1)$. From a state $\Theta(l)$ to a state $\Theta(l+1)$, there are M^2/p_0 multiple parallel paths. The searching in (41) tells us that, using the orthogonality, the M^2/p_0 parallel path searchings from a state to a state in Viterbi algorithm are reduced to $2Mp_0$ parallel path searchings as shown in Fig. 2.

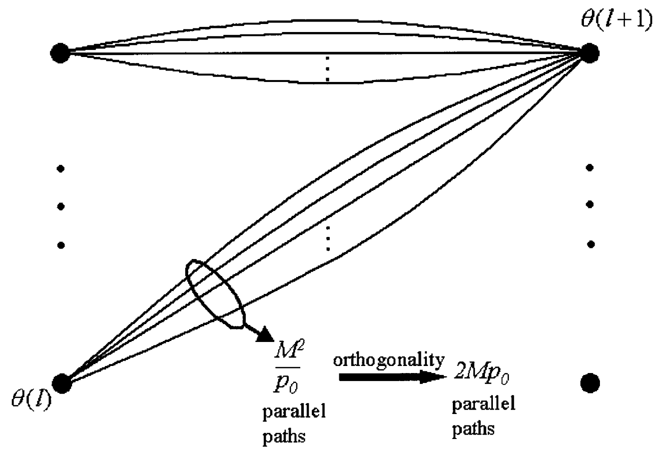

Fig. 2. Parallel paths between two states.

TABLE I
COMPLEXITY COMPARISON.

Full Response CPM Schemes	Complexities for 2 Symbols
Delay diversity [10]	$2pM^2$
OST-CPM	$2pp_0^2M$
Single-antenna CPM	$2pM$

The complexities of single-antenna CPM, the existing delay diversity CPM mentioned in [10], and our proposed OST-CPM, are listed in Table I.

Since p or p_0 depends on the CPM index h and does not depend on the CPM symbol size M, from Table I, one can see that, when M is large, OST-CPM has a lower complexity than the delay diversity does. Consider the case when $h = \frac{1}{2}$ that is used often. In this case, $p_0 = 2$ and the complexity of the OST-CPM is less than that of the delay diversity scheme when $M > 4$. Another scheme (we call it the mapping scheme) is mentioned in [10] for two transmit antennas and full-response CPM systems. In the mapping scheme, one information symbol is mapped to two different waveforms and are then transmitted at the same time from two antennas. This scheme has a lower complexity than the delay scheme and has the same complexity as the OST-CPM in the case of full responses. For partial-response CPM systems, the complexity of the mapping scheme is higher than that of the OST-CPM [11].

V. PERFORMANCE ANALYSIS

For the performance analysis, the basic idea is the same as those in [2]–[6]. We use $S(t, \boldsymbol{d})$ to denote the 2×2 transmitted

$$\begin{aligned}\left(\hat{d}_{1,l}, \hat{d}_{2,l}\right) &= \underset{(d_{1,l},d_{2,l})\in\Omega\times\Omega}{\arg\min} \left\{ \int_{2(l-1)T}^{(2l-1)T} \left[\left| y(t) - \sum_{m=1}^{2} \alpha_{m,n} s_m(t, 2l-1) \right|^2 + \left| y(t+T) - \sum_{m=1}^{2} \alpha_{m,n} s_m(t, 2l) \right|^2 \right] dt \right\} \\ &= \underset{(d_{1,l},d_{2,l})\in\Omega\times\Omega}{\arg\min} \left\{ \int_{2(l-1)T}^{(2l-1)T} \left[|y(t) - \alpha_{1,n} s_1(t, 2l-1)|^2 + |y(t) - \alpha_{2,n} s_2(t, 2l-1)|^2 \right.\right. \\ &\quad \left.\left. + |y(t+T) - \alpha_{1,n} s_1(t, 2l)|^2 + |y(t+T) - \alpha_{2,n} s_2(t, 2l)|^2 - |y(t)|^2 - |y(t+T)|^2 \right] dt \right\} \\ &= \underset{(d_{1,l},d_{2,l})\in\Omega\times\Omega}{\arg\min} \left\{ \int_{2(l-1)T}^{(2l-1)T} \left[|y(t) - \alpha_{1,n}\Theta(l) s_{1,l}(t)|^2 + \left| y(t) - \alpha_{2,n}\Theta(l) C_{g_1(l),g_2(l)}(t) s_{2,l}^*(t) \right|^2 \right.\right. \\ &\quad + \left| y(t+T) - \alpha_{2,n}\Theta(l) A_{g_2(l)} B_{g_1(l),g_2(l)} C_{g_1(l),g_2(l)}(t) s_{1,l}^*(t) \right|^2 \\ &\quad \left.\left. + \left| y(t+T) - \alpha_{1,n}\Theta(l) A_{g_1(l)} s_{2,l}(t) \right|^2 \right] dt - \int_{2(l-1)T}^{(2l-1)T} \left(|y(t)|^2 + |y(t+T)|^2 \right) dt \right\}. \end{aligned} \tag{40}$$

$$\begin{aligned}\left(\hat{d}_{1,l}, \hat{d}_{2,l}\right) = \underset{(g_1,g_2)\in G\times G}{\arg\min} &\left\{ \underset{d_{1,l}\in\Omega_{g_1}}{\arg\min} \left[\int_{2(l-1)T}^{(2l-1)T} F_1(d_{1,l}, g_1, g_2) dt \right] + \underset{d_{2,l}\in\Omega_{g_2}}{\arg\min} \left[\int_{2(l-1)T}^{(2l-1)T} F_2(d_{2,l}, g_1, g_2) dt \right] \right. \\ &\left. - \int_{2(l-1)T}^{(2l-1)T} \left[|y(t)|^2 + |y(t+T)|^2 \right] dt \right\} \end{aligned} \tag{41}$$

signal matrix in (10) with information symbol sequence $\boldsymbol{d}$ and $Y(t,\boldsymbol{d}')$ to denote the $L_r \times 2$ received signal matrix as

$$Y(t,\boldsymbol{d}') = \begin{bmatrix} y_1(t,\boldsymbol{d}') & y_1(t+T,\boldsymbol{d}') \\ \vdots & \vdots \\ y_{L_r}(t,\boldsymbol{d}') & y_{L_r}(t+T,\boldsymbol{d}') \end{bmatrix}$$

where $\boldsymbol{d}'$ is a transmitted symbol sequence. Then, the objective function in the ML demodulation (38)–(40) can be reformulated as

$$\int_{2(l-1)T}^{(2l-1)T} \left\|Y(t,\boldsymbol{d}') - AS(t,\boldsymbol{d})\right\|_F^2 dt = \int_{2(l-1)T}^{(2l-1)T} \left\|A[S(t,\boldsymbol{d}') - S(t,\boldsymbol{d})] + W(t)\right\|_F^2 dt \quad (44)$$

where $\|\cdot\|_F$ denotes the Frobenious norm, i.e., the sum of all the magnitudes squared of the matrix, $A = (\alpha_{nm})$ is the channel coefficient matrix, and $W(t)$ is the additive white Gaussian noise of the channel.

To analyze the pairwise error probability from $\boldsymbol{d}'$ to $\hat{\boldsymbol{d}}$, let us first see the difference matrix $S(t,\boldsymbol{d}') - S(t,\hat{\boldsymbol{d}})$ and consider $t \in [0,T]$ for convenience. From (11)–(21), we have (45) at the bottom of the page, where

$$a_{11} = \exp(j2\pi\theta_1(2(l-1))), \qquad a_{12} = \exp(j2\pi\theta_1(2l-1))$$
$$a_{21} = \exp(j2\pi\theta_2(2(l-1))), \qquad a_{22} = \exp(j2\pi\theta_2(2l-1)).$$

Note that the smoothing response functions $q(t)$ and $q_0(t)$ are continuous and take all values between 0 and $1/2$. Therefore, when $d'_{m,l} \neq \hat{d}_{m,l}$ for some m, there exists a time interval I of t in $[0,T]$ such that the set of values of $s_{mi}(t,\boldsymbol{d}')$ and the set of values of $s_{mi}(t,\hat{\boldsymbol{d}})$ are disjoint for $t \in I$ and some m and i. This means that there exist $t_0 \in (0,T)$ and $\delta > 0$ such that, the difference matrix $S(t_0,\boldsymbol{d}') - S(t_0,\hat{\boldsymbol{d}})$ has full rank and its all singular values $\lambda_m(t_0) > 0$, $m = 1,2$, and furthermore

$$\left\|A[S(t,\boldsymbol{d}') - S(t,\hat{\boldsymbol{d}})]\right\|_F^2 \geq \frac{1}{2}\left\|A[S(t_0,\boldsymbol{d}') - S(t_0,\hat{\boldsymbol{d}})]\right\|_F^2, \text{ for } t \in [t_0-\delta, t_0+\delta].$$

Therefore,

$$\begin{aligned}\int_{2(l-1)T}^{(2l-1)T} \left\|A[S(t,\boldsymbol{d}') - S(t,\hat{\boldsymbol{d}})]\right\|_F^2 dt &\geq \int_{t_0-\delta}^{t_0+\delta} \left\|A[S(t,\boldsymbol{d}') - S(t,\hat{\boldsymbol{d}})]\right\|_F^2 dt \\ &\geq \delta\left\|A[S(t_0,\boldsymbol{d}') - S(t_0,\hat{\boldsymbol{d}})]\right\|_F^2. \end{aligned} \quad (46)$$

Then, the pair error probability $P(\boldsymbol{d}' \rightarrow \hat{\boldsymbol{d}})$ can be upper-bounded in a similar way to that developed in the literature for PAM/PSK/QAM systems as follows:

$$P(\boldsymbol{d}' \rightarrow \hat{\boldsymbol{d}}) < \eta\left(\prod_{m=1}^{\nu}\lambda_m(t_0)\right)^{-2L_r} (\text{SNR})^{-\nu L_r} \quad (47)$$

where $\nu = 2$ in this case, i.e., the full-rank diversity, and $\eta > 0$ is a constant. From the preceding derivations, one can see that the full-rank criterion still holds for the space–time coded CPM performance.

What we want to mention here is that, similar to Alamouti's scheme for a PSK or QAM signal, the diversity product (or coding gain/advantage) of our design is not small. It is not less than $d_{\min}^2$ where $d_{\min}$ is the free distance of one antenna CPM system. Another remark is that, in order to have a fast decoding algorithm as developed previously, the orthogonality at each time t is not necessary and it only needs the waveform orthogonality in the L^2 sense, i.e., the inner product of the two waveforms transmitted by two transmit antennas is zero. The question, then, becomes whether it is possible to design higher rate space–time coded system with this relaxed orthogonality condition. So far, this question is still open. As a final remark, in our OST-CPM designs, the orthogonality constraint forces the spectrum of the transmitted signals to be extended but it is not significant for a high data rate system.

VI. OST-Encoded CPM Design for Partial Response CPM Systems

In this section, we want to generalize the OST-CPM design from full-response CPM systems obtained in Section III to partial-response CPM systems.

Let $q(t)$ and $q_0(t)$ be two smoothing phase response functions with modulation memory sizes γ and $\gamma_0 \leq \gamma$, respectively, where $q(0) = q_0(0) = 0$, $q(t) = 1/2$ for $t \geq \gamma T$, and $q_0(t) = 1/2$ for $t \geq \gamma_0 T$. We next want to generalize (11)–(21) to the above $q(t)$ and $q_0(t)$.

Let $\boldsymbol{d} = (d_i)$ be an independent and identically distributed (i.i.d.) information symbol sequence. In this section, for notational convenience, we use the notation d_i rather than $d_{m,i}$ as in Section III for the full-response CPM. At the time slot between $2(l-1)T$ and $2lT$, the following signals are sent through the first transmit antenna:

$$s_1(t,\boldsymbol{d}) = \exp\{j2\pi\Phi_1(t,\boldsymbol{d})\}, \quad 2(l-1)T \leq t \leq 2lT \quad (48)$$

where

$$\Phi_1(t,\boldsymbol{d}) = \theta_1(2l-1-\gamma) + h\sum_{i=2l-\gamma}^{2l-1} d_i q(t-(i-1)T), \quad \text{for } 2(l-1)T \leq t \leq (2l-1)T \quad (49)$$

$$\Phi_1(t,\boldsymbol{d}) = \theta_1(2l-\gamma) + h\sum_{i=2l-\gamma+1}^{2l} d_i q(t-(i-1)T), \quad \text{for } (2l-1)T \leq t \leq 2lT. \quad (50)$$

At the time slot between $2(l-1)T$ and $2lT$, the following signals are sent through the second transmit antenna:

$$s_2(t,\boldsymbol{d}) = \exp\{j2\pi\Phi_2(t,\boldsymbol{d})\}, \quad 2(l-1)T \leq t \leq 2lT \quad (51)$$

$$\begin{aligned} S(t,\boldsymbol{d}) &= \begin{bmatrix} a_{11}\exp(j2\pi h d_{1,l}q(t)) & a_{12}\exp(j2\pi h d_{2,l}q(t)) \\ a_{21}\exp(j2\pi(-hd_{2,l}q(t)+c_l q_0(t))) & a_{22}\exp(j2\pi(-hd_{1,l}q(t)+c_l q_0(t))) \end{bmatrix} \\ &\triangleq \begin{bmatrix} s_{11}(t,\boldsymbol{d}) & s_{12}(t,\boldsymbol{d}) \\ s_{21}(t,\boldsymbol{d}) & s_{22}(t,\boldsymbol{d}) \end{bmatrix} \end{aligned} \quad (45)$$

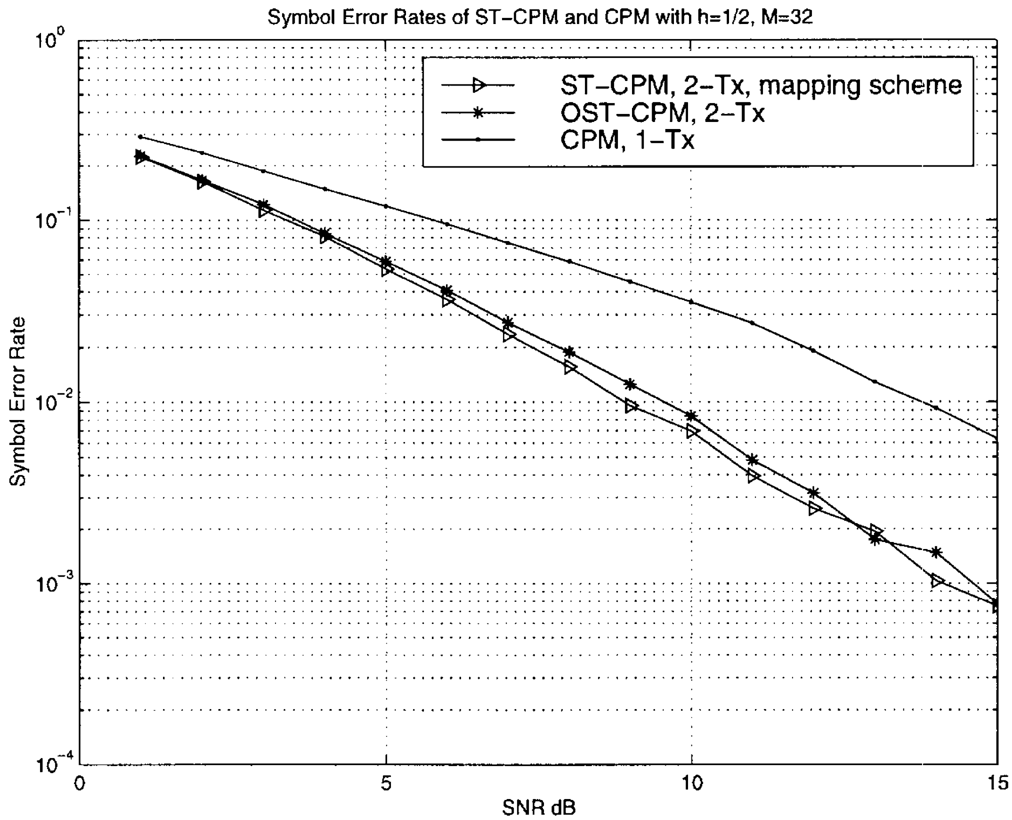

Fig. 3. Performance comparison of CPM and OST-CPM with one receive antenna.

where

$$\Phi_2(t,\boldsymbol{d}) = \theta_2(2l-1-\gamma) - h\sum_{i=2l-\gamma}^{2l-1} d_{i+1}q(t-(i-1)T) + c_l \sum_{i=2l-\gamma_0}^{2l-1} q_0(t-(i-1)T), \quad \text{for } 2(l-1)T \le t \le (2l-1)T \tag{52}$$

$$\Phi_2(t,\boldsymbol{d}) = \theta_2(2l-\gamma) - h\sum_{i=2l-\gamma+1}^{2l} d_{i-1}q(t-(i-1)T) + c_l \sum_{i=2l-\gamma_0+1}^{2l} q_0(t-(i-1)T), \quad \text{for } (2l-1)T \le t \le 2lT \tag{53}$$

where c_l is defined by

$$c_l = 2k+1+hd_{2l-\gamma} + 2h\sum_{i=2l-\gamma}^{2l-1} d_{i+1}q((2l-i)T) - 2h\sum_{i=2l-\gamma+1}^{2l} d_{i-1}q((2l-i)T) \tag{54}$$

where k is an integer such that $|c_l|$ is the smallest for a given sequence d_i. Therefore, c_l depends on d_i. Unlike the case in (21) when $\gamma = 1$ studied in Section III, the above c_l may not necessarily have only p_0 possible values.

We now want to check the orthogonality between vectors $[s_1(t,\boldsymbol{d}), s_1(t+T,\boldsymbol{d})]$ and $[s_2(t,\boldsymbol{d}), s_2(t+T,\boldsymbol{d})]$ for each $t \in [(2l-2)T, (2l-1)T]$. It is not difficult to check that the phases $\Phi_m(t,\boldsymbol{d})$ are continuous in terms of t. Similar to (6), we have

$$\theta_1(2l-1-\gamma) = \Phi_1((2l-1-\gamma)T,\boldsymbol{d}) = \frac{h}{2}\sum_{i\le 2l-1-\gamma} d_i \tag{55}$$

$$\theta_1(2l-\gamma) = \Phi_1((2l-\gamma)T,\boldsymbol{d}) = \frac{h}{2}\sum_{i\le 2l-\gamma} d_i = \theta_1(2l-1-\gamma) + \frac{h}{2}d_{2l-\gamma}. \tag{56}$$

Furthermore, by evaluating the continuity of $\Phi_2(t,\boldsymbol{d})$ at $t = (2l-1)T$ in its definition in (52) and (53), we have

$$\theta_2(2l-\gamma) = \theta_2(2l-1-\gamma) + h\sum_{i=2l-\gamma+1}^{2l} d_{i-1}q((2l-i)T) - h\sum_{i=2l-\gamma}^{2l-1} d_{i+1}q((2l-i)T) + \frac{c_l}{2}. \tag{57}$$

Then, similar to (26), the following equality can be verified after some algebra:

$$\Phi_1(t+T,\boldsymbol{d}) - \Phi_2(t+T,\boldsymbol{d}) = \Phi_1(t,\boldsymbol{d}) - \Phi_2(t,\boldsymbol{d}) - k - \frac{1}{2}.$$

Therefore, we have shown the orthogonality

$$s_1(t,\boldsymbol{d})s_2^*(t,\boldsymbol{d}) = -s_1(t+T,\boldsymbol{d})s_2^*(t+T,\boldsymbol{d}).$$

The partial-response CPM presented above is different from the full-response CPM in the sense that c_l takes much more possible values and it is hard to develop a fast decoding algorithm as in the full response case. However, another orthogonal space–time code design for partial-response CPM with a fast algorithm is obtained in our current work [11] where an additional differential encoding is adopted.

VII. Simulation Results

In this section, some simulation results of CPM, space–time CPM with mapping scheme mentioned in [10] and OST-CPM for two transmit and one receive antennas over fading channels are given. The fading channel is quasi-static and flat, i.e., constant in the CPM or the OST-CPM symbol duration but fading in different symbols. In the simulations shown in Fig. 3, we use full-response CPM modulation with modulation index $h = 1/2$, smoothing phase function $q(t) = \frac{1}{2T}t$ when $t \in [0, T]$, $q(t) = 0$ when $t \leq 0$, $q(t) = 1/2$ when $t > T$; smoothing phase function $q_0(t) = t/2 - T/(2\pi)\sin(2\pi t/T)$ when $t \in [0, T]$, $q(t) = 0$ when $t \leq 0$, $q(t) = 1/2$ when $t > T$. The signal constellation size is $M = 32$. From Fig. 3, we can see that the performances of OST-CPM and ST-CPM with mapping scheme [10] are similar and much better than that of a single-antenna CPM. Since space–time CPM with delay diversity has almost the same performance as that of OST-CPM but has a much higher decoding complexity, the simulation results of space–time CPM with delay diversity is not shown here.

VIII. Conclusion

In this paper, we proposed an OST-CPM for two transmit antennas, in which the signals from two transmit antennas are orthogonal at any time t while both of them have continuous phases. With our proposed OST-CPM, we derived a fast ML demodulation algorithm.

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References

[1] G. J. Foschini, "Layered space-time architecture for wireless communication in fading environment when using multi-elements antennas," *Bell Labs. Tech. J.*, vol. 1, no. 2, pp. 41–59, 1996.

[2] T. L. Marzetta and B. M. Hochwald, "Capacity of a mobile multiple-antenna communication link in Rayleigh flat fading," *IEEE Trans. Inform. Theory*, vol. 45, pp. 139–157, Jan. 1999.

[3] J.-C. Guey, M. P. Fitz, M. R. Bell, and W.-Y. Kuo, "Signal design for transmitter diversity wireless communication systems over Rayleigh fading channels," *IEEE Trans. Commun.*, vol. 47, pp. 527–537, Apr. 1999.

[4] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, pp. 744–764, Mar. 1998.

[5] S. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, pp. 1451–1458, Aug. 1998.

[6] V. Tarokh, H. Jafarkani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1456–1567, July 1999.

[7] T. Tarokh, V. A. Naguib, N. Seshadri, and A. R. Calderbank, "Combined array processing and space–time coding," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1121–1128, May 1999.

[8] B. Hassibi and B. Hochwald, "High-rate codes that are linear in space and time," preprint, 2000.

[9] J. B. Anderson, T. Aulin, and C. Sunberg, *Digital Phase Modulation*. New York: Plenum, 1986.

[10] X. Zhang and M. P. Fitz, "Space-time coding for Rayleigh fading channels in CPM system," in *Proc. 38th Annu. Allerton Conf. Communication, Control, and Computing*, Monticello, IL, Oct. 2000.

[11] D. Wang, G. Wang, and X.-G. Xia, "An orthogonal space-time coded partial response CPM system with fast decoding for two transmit antennas," preprint, Aug. 2003.