# New Precoding for Intersymbol Interference Cancellation Using Nonmaximally Decimated Multirate Filterbanks with Ideal FIR Equalizers

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***Abstract*—In this paper, we propose a new precoding method for intersymbol interference (ISI) cancellation by using nonmaximally decimated multirate filterbanks. Unlike the existing precoding methods, such as the TH and trellis precodings, the new precoding**

**i) may be independent of the ISI channel;**
**ii) is linear and does not have to implement any modulo operation;**
**iii) gives the ideal FIR equalization at the receiver for any FIR ISI channel including spectral-null channels;**
**iv) expands the transmission bandwidth in a minimum amount.**

**The precoding is built on nonmaximally decimated multirate filterbanks. Based on multirate filterbank theory, we present a necessary and sufficient condition on an FIR ISI transfer function in terms of its zero set such that there is a linear FIR $N \times K$ precoder so that an ideal FIR equalizer exists, where the integers $K$ and $N$ are arbitrarily fixed. The condition is easy to check. As a consequence of the condition, for any given FIR ISI transfer function (not identically 0), there always exist such linear FIR precoders. Moreover, for almost all given FIR ISI transfer functions, there exist linear FIR precoders with size $N \times (N-1)$, i.e., the bandwidth is expanded by $1/N$. In addition to the conditions on the ISI transfer functions, a method for the design of the linear FIR precoders and the ideal FIR equalizers is also given. Numerical examples are presented to illustrate the theory.**

## I. Introduction

INTERSYMBOL interference (ISI) is a common problem in telecommunication systems, such as terrestrial television broadcasting, digital data communication systems, and cellular mobile communication systems. The main reasons for the ISI are because of high-speed transmission and multipath fading. There have been considerable studies for these problems, such as [1]–[29] and [33]–[40]. These studies can be primarily split into three categories:

i) post equalization, such as least-mean-squared (LMS) equalizer and decision feedback equalization (DFE), for example, [1]–[3], [18]–[29], and [36]–[39];
ii) multicarrier modulation to increase transmission symbol length, for example, [4]–[6];
iii) precoding techniques, such as Tomlinson–Harashima (TH) precoding [7], [8], trellis precoding by Eyuboglu and Forney [9], [10], matched spectral null precoding in partial response channels [12], and other precoding schemes, for example, [13]–[17] and [40].

The basic idea for DFE is that once an information symbol has been detected, the ISI that it causes on future symbols may be estimated and subtracted out prior to symbol detection. DFE usually consists of a feedforward filter and feedback filter. The feedback filter is driven by decisions of the output of the detector, and its coefficients are adjusted to cancel the ISI on the current symbol that results from past detected symbols. The coefficient adjustment may be done via a linear equalizer with LMS algorithms. The convergence of these iterative algorithms are dependent of the channel characteristics. When a channel is spectral null or frequency selective fading, these algorithms are very slow and, therefore, become computationally expensive. The performance of the existing linear equalizers significantly degrades over frequency selective fading channels. Although DFE has better performance than the existing linear equalizers when the frequency fading is in the middle of a passband, it does not offer much improvement in other fading cases. For more details, see, for example, [3] and [35]. In post equalization techniques, there are many research results (see, for example, [18]–[29] and [36]–[39] on blind equalizations where channel characteristics are assumed unknown. In blind equalization techniques, there are approximately three groups of results:

i) high-order statistics techniques;
ii) second-order cyclostationary statistics techniques with oversampling;
iii) antenna array (smart antenna) multireceiver techniques;

where there is a considerable amount of overlaps between ii) and iii).

A block diagram for TH precoding is shown in Fig. 1, where the basic idea is to equalize the signal before transmission. With TH precoding there are two drawbacks: i) The transmitter needs to know the channel characteristics, and ii) the precoding is not reliable when the ISI channel $H(z)$ has spectral null or frequency selective fading characteristics, which is because the pre-equalizer mod$[1/H(z), M]$ oscillates in a dramatic way when $H(z)$ is close to zero. The trellis precoding scheme proposed by Eyuboglu and Forney [9] whitens the noise at the equalizer output. This scheme combines precoding and trellis

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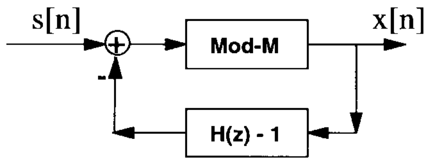



Fig. 1. TH precoder.

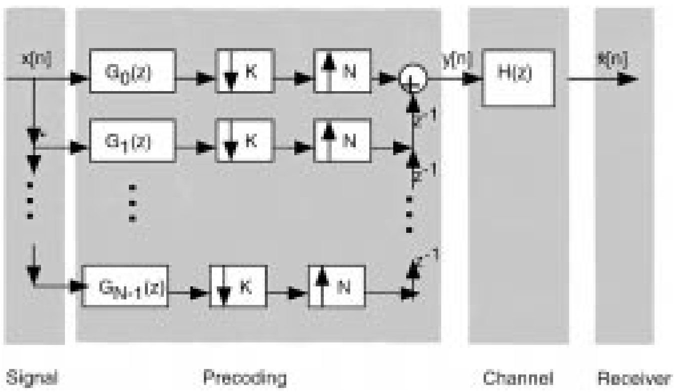



Fig. 2. Nonmaximally decimated multirate filterbank in a communication channel with ISI.

shaping. There are also similar drawbacks about this approach.

i) The transmitter also needs to know the ISI channel characteristics.
ii) The trellis shaping depends on the ISI channel.
iii) The trellis precoding technique may not be suitable for spectral-null channels either.

In the matched spectral null precoding scheme [12] in partial response channels, certain error control codes are chosen to match the spectral nulls of partial response channels in order to lose less signal information through the channel. This approach is mainly for magnetic recording systems.

We now propose a multirate filterbank as a precoder before transmission (shown in Fig. 2), where $\downarrow K$ indicates downsampling by $K$, and $\uparrow N$ indicates upsampling by $N$, i.e., inserting $N-1$ zeros between two adjacent samples, and $H(z)$ is the ISI transfer function. Later, we will see a multirate filterbank decoder for the receiver to eliminate the ISI. If input signal $x[n]$ in Fig. 2 can be completely recovered from the received signal $\hat{x}[n]$ through an FIR linear system, we call that the system in Fig. 2 has perfect reconstruction (PR) or an FIR ideal linear equalizer. In what follows, we use "precoder" and "multirate filterbank" interchangeably.

With the precoder proposed in Fig. 2, there are three questions to be answered:

i) What is the condition on $H(z)$ such that there exists a multirate filterbank with $N$ channels and decimation by $K$ in Fig. 2 so that $x[n]$ can be recovered from $\hat{x}[n]$ through an FIR linear system?
ii) If the condition on $H(z)$ in the first question is satisfied, how does one design a multirate filterbank in Fig. 2 to eliminate the ISI?
iii) If both of these two problems are solved, how does the receiver recover the input signal $x[n]$ from the received $\hat{x}[n]$?

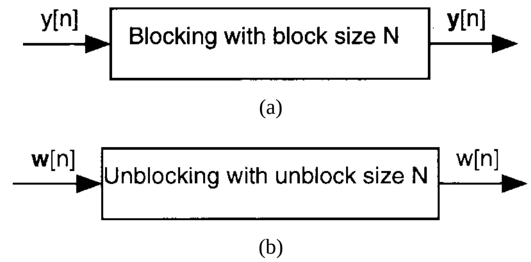



Fig. 3. Blocking and unblocking.

Next, we want to find brief solutions for these questions. When $K = 1, G_0(z) = 1, G_1(z) = \cdots = G_{N-1}(z) = 0$, the precoding scheme in Fig. 2 is equivalent to the fractionally spaced equalizer studied, for example, [36]–[39], where the receiver needs to sample a signal $N$ times faster than the baud sampling. When $K = 1$, the precoding concept has appeared in [39] by Tsatsanis and Giannakis, where the precoder $G_l(z) = c_l, l = 0, 1, \cdots, N-1$ for $N$ constants $c_l$ was used. As we can see, the case of $K = 1$ is a very special case in our precoding scheme, and moreover, our new precoding scheme in Fig. 2 provides other potential precoders $G_l(z), l = 0, 1, \cdots, N-1$ rather than only constants $c_l$, which allows one to search the optimal one with respect to an individual channel.

When $K \geq N$ and there are $N$ interference channels instead of a single channel $H(z)$ in Fig. 2, a detailed analysis was given by Nguyen [31]. When $K > N$, as mentioned in [31], PR is impossible, but partial alias cancellation filterbanks were proposed in [31]. The applications discussed in [31] are in wide-band radio communications, where only part of the signal frequencies is of interest to the user. In this paper, we are interested in applications in the ISI channels with PR systems in Fig. 2 and, therefore, the case of $K < N$. This also implies that unlike the existing precoding techniques, the new precoding expands the transmission bandwidth, which is what we lose for the new precoding method, and fortunately, we will show that the bandwidth expansion can be as small as possible in theory.

An intuitive way to reduce the ISI generated from a lowpass $H(z)$ is to smoothly interpolate $x[n]$ with a large enough number of interpolations between samples of $x[n]$ so that the interpolated one has the lowpass property. However, two drawbacks about this approach may occur. One is that it usually requires a large amount of increasing of data rate (number of interpolations between samples). The other is that a good frequency band structure for a nonlowpass, such as bandpass, filter $H(z)$ is required for PR. In this paper, we want to solve the above three problems systematically. Given two integers $0 < K < N$, we present a necessary and sufficient condition (see Theorem 1) on an FIR filter $H(z)$ such that there exists an FIR nonmaximally decimated multirate filterbank with $N$ channels and decimation by $K$ so that $x[n]$ can be recovered from $\hat{x}[n]$ in Fig. 2 with an FIR synthesis bank. The condition we found is basically very weak. In fact, it can be proved that for any given FIR filter $H(z)$ not identically 0, there always exists an FIR nonmaximally decimated multirate filterbank in Fig. 2 for recovering $x[n]$

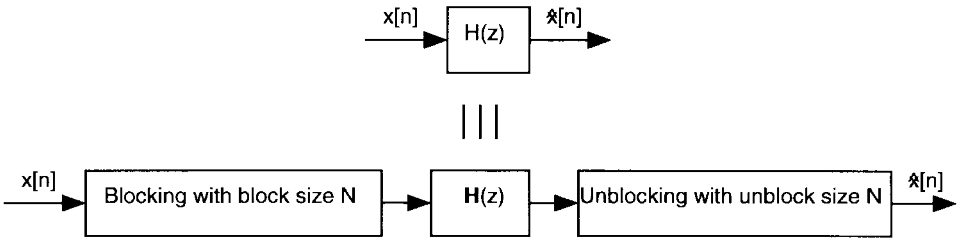



Fig. 4. Equivalence of an LTI system and its blocked version.

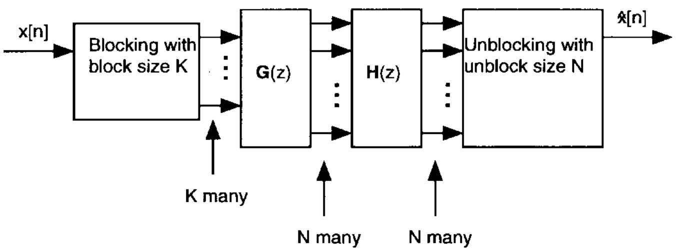
from $\hat{x}[n]$. A nonmaximally decimated filterbank precoder requires a higher transmission rate with the increasing amount proportional to the difference $N-K$. It is clear that the smallest $N-K$ is 1. In other words, a multirate filterbank with $N$ channels and decimation by $N-1$ has the smallest increasing of a transmission rate, and therefore, it is usually desired. We show that a multirate filterbank with $N$ channels and decimation by $N-1$ exists in Fig. 2 for PR if and only if any two sets of zeros of the polynomials $H(zW_N^l)$ of $z^{-1}$ for $l=0,1,\cdots,N-1$ do not intersect, where $W_N=e^{-2\pi\sqrt{-1}/N}$. This condition is true almost surely. Various examples are presented. With the above conditions, we also derive some results on the submatrices of a pseudo-circulant polynomial matrices [32]. Constructions of FIR nonmaximally decimated multirate filterbanks and their FIR syntheses for the reconstruction for a given $H(z)$ in Fig. 2 are provided. Numerical examples are presented to illustrate the theory, which also indicates that the technique we developed for eliminating the ISI is robust.

This paper is organized as follows. In Section II, we present necessary and sufficient conditions on $H(z)$. We also discuss the construction of nonmaximally decimated multirate filterbanks for eliminating ISI. In Section III, we present examples and the reconstruction method. In Section IV, we consider applications of the ISI cancellation.

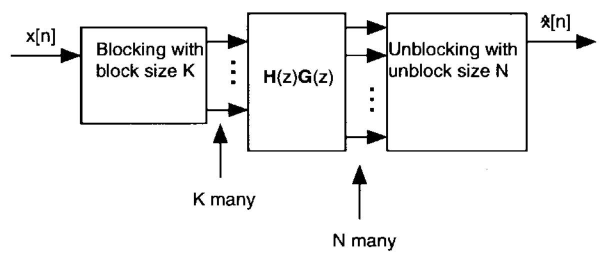
## II. A Necessary and Sufficient Condition

In this section, we study necessary and sufficient conditions on the ISI transfer functions $H(z)$ in Fig. 2 such that there exists a nonmaximally decimated multirate filterbank with $N$ channels and decimation by $K$ and such that an ideal FIR linear equalizer exists. We also present a design method for an FIR nonmaximally decimated multirate filterbank for eliminating the ISI. Throughout this paper, boldface lowercase letters denote vector-valued sequences, capital letters denote transfer functions, and boldface capital letters denote function matrices (or polynomial matrices). We first consider the case when $K$ and $N$ ($0<K<N$) are two arbitrarily fixed integers.

Before we go to the results, let us see some fundamentals on blocking and linear time invariant (LTI) systems. We then convert the system in Fig. 2 into a single multirate system. The output $\mathbf{y}[n]$ shown in Fig. 3(a) of the blocked $y[n]$ with block size $N$ is the vector-valued signal $\mathbf{y}[n]=(y[Nn], y[Nn-1], \cdots, y[Nn-N+1])^T$, where $^T$ indicates transpose. Conversely, the output $w[n]$ shown in Fig. 3(b) of the unblocked vector-valued signal $\mathbf{w}[n]=[w_0[n], w_1[n], \cdots, w_{N-1}[n])^T$




(a)




(b)

Fig. 5. Equivalent systems of the system in Fig. 2.

with unblock size $N$ is $w[n]=w_k[l]$ when $n=Nl-k$ for $k=0,1,\cdots,N-1$. In particular, when $w[n]=(y[Nn], y[Nn-1], \cdots, y[Nn-N+1])^T$, then $w[n]=y[n]$.

Let $H(z)=\sum_n h[n]z^{-n}$ and $H_j(z)$ be its $j$th forward polyphase component with $N$ channels, i.e., $H_j(z)=\sum_n h[Nn+j]z^{-n}, 0\le j\le N-1$. With $H_j(z), 0\le j\le N-1$, we form the following $N\times N$ pseudo-circulant matrix $\mathbf{H}(z)$ (see [30], [32])

$$\mathbf{H}(z)=\begin{bmatrix} H_0(z) & z^{-1}H_{N-1}(z) & \cdots & z^{-1}H_1(z) \\ H_1(z) & H_0(z) & \cdots & z^{-1}H_2(z) \\ \vdots & \vdots & \cdots & \vdots \\ H_{N-2}(z) & H_{N-3}(z) & \cdots & z^{-1}H_{N-1}(z) \\ H_{N-1}(z) & H_{N-2}(z) & \cdots & H_0(z) \end{bmatrix}. \tag{2.1}$$

Then, we have the equivalence for an LTI system and blocking process shown in Fig. 4, where $\mathbf{H}(z)$ is from (2.1) and is called the *blocked version* of $H(z)$; see [30] and [32].

For $0\le l\le N-1$, let $G_{l,j}(z)$ be the $j$th forward polyphase component of the $l$th filter $G_l(z)$ in Fig. 2 with $K$ channels, i.e., $G_{l,j}(z)=\sum_n g_l[Kn+j]z^{-n}$, when $G_l(z)=\sum_n g_l[n]z^{-n}$, for $0\le j\le K-1$. Let $\mathbf{G}(z)$ be the polyphase matrix of the filterbank $G_0(z), G_1(z), \cdots, G_{N-1}(z)$ in Fig. 2: $\mathbf{G}(z)=[G_{l,j}(z)]_{N\times K}$. Then, the system in Fig. 2 is equivalent to the one in Fig. 5(a), which is also equivalent to the one in Fig. 5(b).

Therefore, the PR of $x[n]$ from $\hat{x}[n]$ in Fig. 2 is equivalent to the one of the linear multirate system $\mathbf{H}(z)\mathbf{G}(z)$ in Fig. 4(b). Notice that $\mathbf{H}(z)\mathbf{G}(z)$ is an $N \times K$ function matrix of $z^{-1}$. To analyze it, we need a property on the pseudo-circulant matrix $\mathbf{H}(z)$ in (2.1). In fact, $\mathbf{H}(z)$ can be diagonalized as follows.

Let $\mathbf{W}_N$ be the $N \times N$ DFT matrix, i.e., $\mathbf{W}_N \triangleq (W_N^{jk})_{0\le j,k\le N-1}$, where $W_N = e^{-2\pi\sqrt{-1}/N}$. Let $\mathbf{\Lambda}(z)$ be the diagonal matrix

$$\mathbf{\Lambda}(z) \triangleq \text{diag}\,(1, z^{-1}, \cdots, z^{-N+1}).$$

Notice that the transpose of the matrix $\mathbf{H}(z)$ is the forward polyphase matrix of the $N$ filters $H(z)$, $z^{-1}H(z)$, $\cdots$, $z^{-N+1}H(z)$ in $N$ channels

$$[H(z), z^{-1}H(z), \cdots, z^{-N+1}H(z)] = (1, z^{-1}, \cdots, z^{-N+1})\mathbf{H}(z^N).$$

Replacing $z$ by $zW_N^l$ for $l = 0, 1, \cdots, N-1$ in the above equality, we have the following $N \times N$ matrix multiplications

$$\hat{\mathbf{H}}(z) = \mathbf{W}_N^*\mathbf{\Lambda}(z)\mathbf{H}(z^N) \tag{2.2}$$

where we have (2.3), shown at the bottom of the page. Let

$$\mathbf{V}(z) \triangleq \text{diag}\,[H(z), H(zW_N), \cdots, H(zW_N^{N-1})]. \tag{2.4}$$

Then, the matrix $\hat{\mathbf{H}}(z)$ in (2.3) can be rewritten as

$$\hat{\mathbf{H}}(z) = \mathbf{V}(z)\mathbf{W}_N^*\mathbf{\Lambda}(z).$$

This completes the following diagonalization of $\mathbf{H}(z^N)$ by combining (2.2)

$$\mathbf{H}(z^N) = [\mathbf{W}_N^*\mathbf{\Lambda}(z)]^\dagger\mathbf{V}(z)\mathbf{W}_N^*\mathbf{\Lambda}(z) \tag{2.5}$$

where $^\dagger$ means the inverse.

From now on, we assume all filters in Fig. 2 are FIR, and the PR of the system in Fig. 2 means the overall system function $\mathbf{H}(z)\mathbf{G}(z)$ has an FIR inverse.

The PR of the multirate system $\mathbf{H}(z)\mathbf{G}(z)$ is equivalent to the one of the multirate system $\mathbf{H}(z^N)\mathbf{G}(z^N)$. In fact, if $\mathbf{H}(z^N)\mathbf{G}(z^N)$ has PR, then any input signal $X(z)$ can be reconstructed from $\mathbf{H}(z^N)\mathbf{G}(z^N)X(z)$. Thus, $X(z^N)$ can be reconstructed from $\mathbf{H}(z^N)\mathbf{G}(z^N)X(z^N)$ with an FIR synthesis filterbank. In other words, any $X(z)$ can be reconstructed from $\mathbf{H}(z)\mathbf{G}(z)X(z)$ with an FIR synthesis filterbank. This implies the PR of $\mathbf{H}(z)\mathbf{G}(z)$. Conversely, we assume the PR of $\mathbf{H}(z)\mathbf{G}(z)$, which is equivalent to that there exists an FIR inverse, i.e., there is an FIR $K \times N$ polynomial matrix $\mathbf{Q}(z)$ such that

$$\mathbf{Q}(z)\mathbf{H}(z)\hat{\mathbf{G}}(z) = I_K$$

where $I_K$ is the $K \times K$ identity matrix. Thus, we also have

$$\mathbf{Q}(z^N)\mathbf{H}(z^N)\hat{\mathbf{G}}(z^N) = I_K.$$

It implies that $\mathbf{H}(z^N)\hat{\mathbf{G}}(z^N)$ has an FIR inverse (or PR).

We, thus, consider the PR of $\mathbf{H}(z^N)\mathbf{G}(z^N)$. By (2.5)

$$\mathbf{H}(z^N)\mathbf{G}(z^N) = [\mathbf{W}_N^*\mathbf{\Lambda}(z)]^\dagger\mathbf{V}(z)\mathbf{W}_N^*\mathbf{\Lambda}(z)\mathbf{G}(z^N). \tag{2.6}$$

It is clear that $[\mathbf{W}_N^*\mathbf{\Lambda}(z)]^\dagger = \mathbf{\Lambda}(z^{-1})\mathbf{W}_N$, which is paraunitary. Let

$$\hat{\mathbf{G}}(z) \triangleq \mathbf{W}_N^*\mathbf{\Lambda}(z)\mathbf{G}(z^N).$$

Then, the PR of $\mathbf{H}(z)\mathbf{G}(z)$ is equivalent to the one of $\mathbf{V}(z)\hat{\mathbf{G}}(z)$. Notice that the size of the matrix $\mathbf{V}(z)\hat{\mathbf{G}}(z)$ is $N \times K$.

On the other hand, $\mathbf{V}(z)\hat{\mathbf{G}}(z)$ has an FIR inverse equivalent to that of the greatest common divisor (gcd) of all determinants of all $K \times K$ submatrices of the $N \times K$ matrix $\mathbf{V}(z)\hat{\mathbf{G}}(z)$ that is $cz^{-d}$ for a nonzero constant $c$ and an integer $d$; see, for example, [32]. Since $\mathbf{V}(z)$ is diagonal and of the form (2.4), the above condition for the PR can be simplified further as follows.

Without loss of the generality, we assume

$$H(z) = \sum_{k=0}^{P} h[k]z^{-k}$$

where $h[0] \neq 0$, $h[P] \neq 0$, and $P \geq 1$. Let $S$ denote the set of all zeros of the polynomial $H(z)$ of $z^{-1}$: $S \triangleq \{z_1, z_2, \cdots, z_P\}$ with $H(z_l) = 0$, where $z_l, 1 \le l \le P$ may not be necessarily distinct. For a constant $c$, let $cS \triangleq \{cz_1, cz_2, ..., cz_P\}$, which is a rotated version of $S$. We have the following result for the PR.

*Theorem 1:* There exists an FIR nonmaximally decimated multirate filterbank in Fig. 2 such that the system in Fig. 2 has an FIR ideal linear equalizer if and only if

$$\bigcap_{0\le l_1<l_2<\cdots<l_K\le N-1} (S_{l_1} \cup S_{l_2} \cup \cdots \cup S_{l_K}) = \phi \tag{2.7}$$

where $S_{l_k} = W_N^{l_k}S$, $k = 1, 2, \cdots, K$.

Theorem 1 tells us that there exists a multirate filterbank in Fig. 2 for the ideal linear equalization if and only if the intersection of the unions of any $K$ sets of all $N$ rotated zero sets of $S$ with angles $l2\pi/N, l = 0, 1, \cdots, N-1$ of the ISI transfer function $H(z)$ is empty. When $K = N$, the intersection in (2.7) contains at least $S$, which is not empty. This implies that when $K = N$, the system in Fig. 2 does not have PR in the sense of nonexistence of FIR inverses. This is not surprising because any maximally decimated multirate filterbank does not add any redundancy to the signal and, therefore, does not have any error correction capability.

$$\hat{\mathbf{H}}(z) \triangleq \begin{bmatrix} H(z) & z^{-1}H(z) & \cdots & z^{-N+1}H(z) \\ H(zW_N) & z^{-1}W_N^{-1}H(zW_N) & \cdots & z^{-N+1}W_N^{-N+1}H(zW_N) \\ \vdots & \vdots & \cdots & \vdots \\ H(zW_N^{N-1}) & z^{-1}W_N^{(-1)(N-1)}H(zW_N^{N-1}) & \cdots & z^{-N+1}W_N^{(-N+1)(N-1)}H(zW_N^{N-1}) \end{bmatrix}. \tag{2.3}$$

*Proof:* We first prove the "necessary part." Assume the set

$$\bigcap_{0\le l_1<l_2<\cdots<l_K\le N-1} (S_{l_1}\cup S_{l_2}\cup\cdots\cup S_{l_K})\neq\phi.$$

This implies that the polynomials $\prod_{k=1}^{K} H(W_N^{l_k} z)$ for all possible $0\le l_1<l_2<\cdots<l_K\le N-1$ has at least a common zero $z_0$. In other words, they have a common factor $z^{-1}-z_0^{-1}$. By the form of $\mathbf{V}(z)\hat{\mathbf{G}}(z)$ and the diagonality of $\mathbf{V}(z)$, the polynomial $\prod_{k=1}^{K} H(W_N^{l_k} z)$ is a factor of the determinant of the submatrix of $\mathbf{V}(z)\hat{\mathbf{G}}(z)$ at the rows $l_1, l_2, \cdots, l_K$. When $l_1, l_2, \cdots, l_K$ run over all possible $0\le l_1<l_2<\cdots<l_K\le N-1$, the corresponding submatrices run over all possible $K\times K$ submatrices of $\mathbf{V}(z)\hat{\mathbf{G}}(z)$. Therefore, all determinants of all $K\times K$ submatrices of $\mathbf{V}(z)\hat{\mathbf{G}}(z)$ have at least a common factor $z^{-1}-z_0^{-1}$, no matter what $\hat{\mathbf{G}}(z)$ is. This proves that $\mathbf{V}(z)\hat{\mathbf{G}}(z)$ does not have an FIR inverse.

Let us prove the "sufficient part." Assume (2.7) is true. We construct

$$\mathbf{G}(z)=\begin{bmatrix} I_K \\ \mathbf{0}_{(N-K)\times K}\end{bmatrix} \tag{2.8}$$

where $\mathbf{0}_{(N-K)\times K}$ is the all-zero $(N-K)\times K$ matrix. Then

$$\begin{aligned}\hat{\mathbf{G}}(z) &= \mathbf{W}_N^*\mathbf{\Lambda}(z)\mathbf{G}(z^N)=\mathbf{W}_N^*\mathrm{diag}\ (1, z^{-1}, \cdots, z^{-K+1}) \\ &= (z^{-k}W_N^{-jk})_{0\le j\le N-1,\, 0\le k\le K-1}.\end{aligned}$$

It is not hard to see that the determinant of the $l_1, l_2, \cdots, l_K$ row submatrix of $\mathbf{V}(z)\hat{\mathbf{G}}(z)$ is

$$c_{l_1 l_2\cdots l_K}\prod_{j=1}^{K} H(zW_N^{l_j})z^{-(1+2+\cdots+K-1)} \tag{2.9}$$

where $0\le l_1<l_2<\cdots<l_K\le N-1, c_{l_1 l_2\cdots l_K}$ is the Vandermonde's determinant of a $K\times K$ submatrix of the following $N\times K$ matrix

$$\left(W_N^{-jk}\right)_{0\le j\le N-1,\, 0\le k\le K-1}$$

which is a nonzero constant. By (2.7), the gcd of all polynomials in (2.9) is $cz^{-d}$ for a nonzero constant $c$ and an integer $d$. This proves that the matrix $\mathbf{V}(z)\hat{\mathbf{G}}(z)$ has an FIR inverse and, therefore, completes the proof. □

By the fact that

$$\bigcap_{0\le l_1<l_2<\cdots<l_{K-1}\le N-1} (S_{l_1}\cup S_{l_2}\cup\cdots\cup S_{l_{K-1}})\subset \bigcap_{0\le l_1<l_2<\cdots<l_K\le N-1} (S_{l_1}\cup S_{l_2}\cup\cdots\cup S_{l_K})$$

we have the following immediate corollary.

*Corollary 1:* If there exists an FIR multirate filterbank with $N$ channels and decimation by $K$ in Fig. 2 so that the system in Fig. 2 has an ideal FIR linear equalizer, then there also exists an FIR multirate filterbank with $N$ channels and decimation by $K-1$ in Fig. 2 for the ideal linear equalization, where $K>1$.

Corollary 1 is not surprising. It is because that the decreasing of the decimation rate from $K$ to $K-1$ of a nonmaximally decimated multirate filterbank means the increase of the redundancy. If a multirate filterbank with less redundancy eliminates the ISI, then the multirate filterbank with much redundancy eliminates the ISI as well.

The proof of Theorem 1 also suggests a way to construct a nonmaximally decimated multirate filterbank in Fig. 2 to eliminate the ISI of $H(z)$. When $H(z)$ satisfies the condition in Theorem 1, to have the PR, it is good enough to set the polyphase matrix of the filterbank $G_0(z)$, ..., $G_{N-1}(z)$ in Fig. 2 to be the one in (2.8), i.e.

$$\mathbf{G}(z)=\begin{bmatrix} I_K \\ \mathbf{0}_{(N-K)\times K}\end{bmatrix}.$$

This precoder basically adds $N-K$ zeros for each $K$ symbols (or samples). It is certainly not necessary, as long as the $N\times K$ polynomial matrix $\mathbf{V}(z)\hat{\mathbf{G}}(z)$ has an FIR inverse. Put the above $\mathbf{G}(z)$ into the system in Fig. 5(b), and the overall system function becomes

$$\begin{aligned}\hat{\mathbf{H}}(z)\mathbf{G}(z) &= \mathbf{F}_K(z) \\ &\triangleq \begin{bmatrix} H_0(z) & z^{-1}H_{N-1}(z) & \cdots & z^{-1}H_{N-K+1}(z) \\ H_1(z) & H_0(z) & \cdots & z^{-1}H_{N-K+2}(z) \\ \vdots & \vdots & \cdots & \vdots \\ H_{K-1}(z) & H_{K-2}(z) & \cdots & H_0(z) \\ \vdots & \vdots & \cdots & \vdots \\ H_{N-1}(z) & H_{N-2}(z) & \cdots & H_{N-K}(z)\end{bmatrix}.\end{aligned} \tag{2.10}$$

By the proof of Theorem 1, when the condition (2.7) on $H(z)$ is satisfied, then $\mathbf{F}_k(z)$ in (2.10) has an FIR inverse. Conversely, if $\hat{\mathbf{H}}(z)\mathbf{G}(z)$ in (2.9) has an FIR inverse, then $x[n]$ can be recovered from $\hat{x}[n]$ in Fig. 5(b). Therefore, by Theorem 1, the condition (2.7) is satisfied. In addition, using Corollary 1, we have proved the following corollary.

*Corollary 2:* The $N\times K$ matrix $\mathbf{F}_K(z)$ in (2.10) with $0<K\le N$ has an FIR inverse if and only if the condition (2.7) is satisfied. The system in Fig. 2 has an ideal linear equalizer if, and only if, the matrix $\mathbf{F}_K(z)$ in (2.10) has an FIR inverse. If $\mathbf{F}_K(z)$ has an FIR inverse, then $\mathbf{F}_{K-1}(z)$ has an FIR inverse for $K>1$.

We now consider two special cases. The first case is when $K=1$. In this case, (2.7) becomes

$$\bigcap_{0\le l\le N-1} S_l=\phi. \tag{2.11}$$

By Theorem 1 and Corollary 2, we have the following result.

*Corollary 3:* There exists a multirate filterbank in Fig. 2 with $K=1$ for the ideal linear equalization if and only if

$$\gcd\ \{H(z), H(zW_N), \cdots, H(zW_N^{N-1})\}=c_1 z^{-d_1}$$

if and only if

$$\gcd\ \{H_0(z), H_1(z), \cdots, H_{N-1}(z)\}=c_2 z^{-d_2}$$

where $c_1$ and $c_2$ are two nonzero constants, and $d_1$ and $d_2$ are two integers.

The result in Corollary 3 coincides with the known result for fractionally spaced equalizers, i.e., there are no zeros of $H(z)$ equispaced on a circle with angle $2\pi/N$ separated one zero from another. From Corollary 3, we immediately have the following consequence.

*Corollary 4:* For any ISI transfer function $H(z)$ not identically zero, there always exists a nonmaximally decimated multirate filterbank in Fig. 2 for the ideal linear equalization of the system in Fig. 2.

The nonmaximally decimated multirate filterbank with $N$ channels and decimation by $K$ in Fig. 2 plays the coding role in eliminating the ISI generated from the ISI channel $H(z)$. We have already known that $K$ has to be less than $N$ for PR. In other words, the data rate has to be increased by $N-K>0$ for eliminating the ISI. In practice, the smallest data rate expansion is desired, which is $N-K=1$, or $K=N-1$. We next want to study this case.

*Theorem 2:* There exists an FIR multirate filterbank with $N$ channels and decimation by $N-1$ in Fig. 2 such that the system in Fig. 2 has an ideal FIR linear equalizer if and only if $S_l \cap S_k = \phi$, i.e., polynomials $H(zW_N^l)$ and $H(zW_N^k)$ are coprime for $0 \le l \neq k \le N \ -1$.

*Proof:* Theorem 2 can be proved by the following set equations.

$$\bigcap_{0\le l_1<l_2<\cdots<l_{N-1}\le N-1}\left(\bigcup_{k=1}^{N-1} S_{l_k}\right) = \bigcap_{l_0=0}^{N-1}\left(\bigcup_{l\neq l_0} S_l\right) = \bigcup_{l\neq k}(S_l \cap S_k).$$

□

Let us consider the case when the ISI transfer function $H(z) = a + z^{-1}$ with $|a| = 1$, i.e., the first-order case. In this case, the zero set $S = \{-1/a\}$. For a general $N, S_l = \{-W_N^l/a\}, l = 0, 1, \cdots, N-1$. Clearly, $S_l \cap S_k = \phi$ since $W_N^l \neq W_N^k$ when $0 \le l \neq k \le N-1$. By Theorem 2 and Corollary 1, we proved the following result.

*Corollary 5:* Assume the ISI transfer function $H(z) = a + z^{-1}$ in Fig. 2, where $|a| = 1$. Then, the system in Fig. 2 for the multirate filterbank $\mathbf{G}(z)$ in (2.8) always has PR for any integers $K$ and $N$ with $0 < K < N$.

Corollary 5 implies that any $\epsilon(>0)$ amount of data rate increasing in coding may eliminate the ISI generated from any first order ISI channel. This is because for any $\epsilon > 0$, there exists a positive integer $N$ such that $0 < 1-(N-1)/N < \epsilon$. We then use this $N$ as the number of channels and $N-1$ as the decimation ratio in the multirate filterbank in Fig. 2.

## III. Examples and Reconstruction

In this section, we study some examples and also the reconstruction of $x[n]$ from $\hat{x}[n]$ in Fig. 2, given $H(z)$ and an FIR nonmaximally decimated multirate filterbank in Fig. 2, where the system has perfect reconstruction. We first see some examples.

*Example 1:* $H(z) = 1 + z^{-1}$. By Corollary 5, one is able to recover $x[n]$ from $\hat{x}[n]$ when $\mathbf{G}(z)$ takes the form in (2.8) for any $0 < K < N$. Consider $K = 1$ and $N = 2$. In this

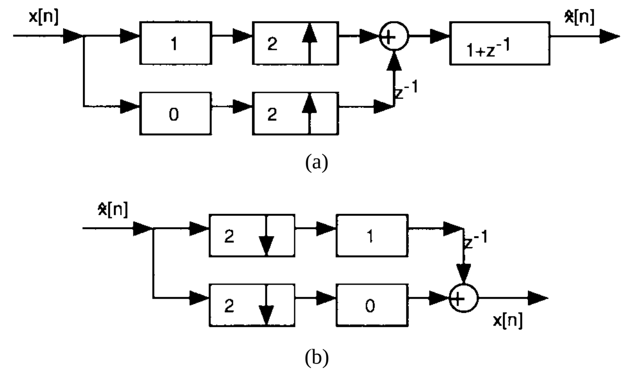



Fig. 6. (a) Transmission and channel parts. (b) Reconstruction.

case, the output $\hat{x}[n]$ in Fig. 2 is

$$\hat{x}[n]: \ \cdots, x[0], x[0], x[1], x[1], \cdots.$$

Clearly, Fig. 6(b) gives the reconstruction.

*Example 2:* $H(z) = (1+z^{-1})(1-z^{-1})$. In this case, the zero set $S = \{1, -1\}$. When $N$ is even, $S_0 = S_{N/2} = S$. By Theorem 2, it is impossible to recover $x[n]$ from $\hat{x}[n]$ in Fig. 2 for any FIR nonmaximally decimated multirate filterbank with two channels. However, for any odd $N > 2, S_l = \{W_N^l, -W_N^l\}, 0, \le l \le N-1$. Clearly, $S_l \cap S_k = \phi$ for $0 \le l \neq k \le N-1$. By Theorem 2, we proved that the system in Fig. 2 with the above $H(z)$ and the multirate filterbank $\mathbf{G}(z)$ in (2.8) with $0 < K < N > 2$ for odd $N$ always has an ideal FIR linear equalizer. This also implies that a little increasing of the data rate in coding may eliminate the ISI generated from the ISI channel.

*Example 3:* Consider a linear phase lowpass filter $H(z)$ of length 5 constructed from the Parks–McClellan algorithm of the optimal equiripple FIR filter design technique. The filter is

$$H(z) = \tfrac{1}{9}(1 + 2z^{-1} + 2.5z^{-2} + 2z^{-3} + z^{-4}). \qquad (3.1)$$

Its frequency and impulse responses are shown in Fig. 7. Its zeros and rotated zeros with angle $\pi$ are shown in Fig. 8(a). Its zeros and rotated zeros with angles $2\pi/3$, $4\pi/3$ are shown in Fig. 8(b). One can see that all of them are disjoint. By Theorem 2, the multirate filterbank $\mathbf{G}(z)$ with $N = 2$ or $N = 3$ gives the PR of the system in Fig. 2.

*Example 4:* Consider a linear-phase lowpass filter $H(z)$ of length 9 also constructed from the Parks–McClellan algorithm. Its frequency and impulse response are shown in Fig. 9, and zeros and rotated versions are shown in Fig. 10. One can see that with length 9, the lowpass property is much better than the one with length 5 in Example 3, and the rotated zeros are also disjoint. The lowpass property will be useful in applications in denoising.

After we have discussed the possibility to eliminate the ISI, the next problem is the reconstruction. Suppose an FIR nonmaximally decimated multirate filterbank is designed in Fig. 2, and it is able to eliminate the ISI generated from $H(z)$. We now want to construct another multirate filterbank for the receiver to reconstruct the original signal $x[n]$ from the received one $\hat{x}[n]$.

We consider a general nonmaximally decimated multirate filterbank $\mathbf{G}(z)$ in Fig. 5. By the above assumption,

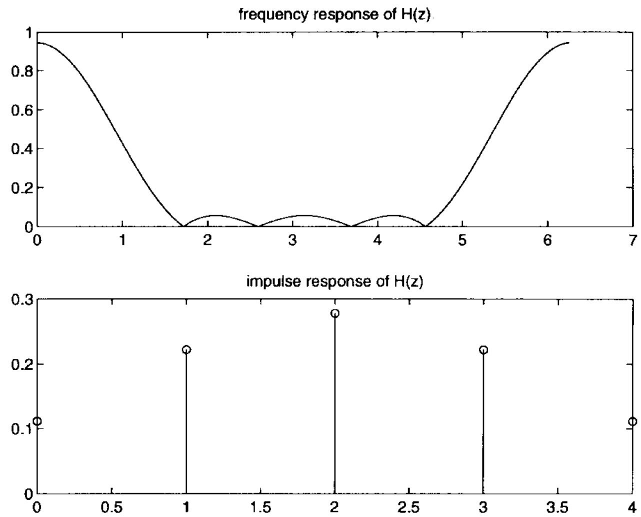



Fig. 7. Lowpass filter $H(z)$ with length 5.

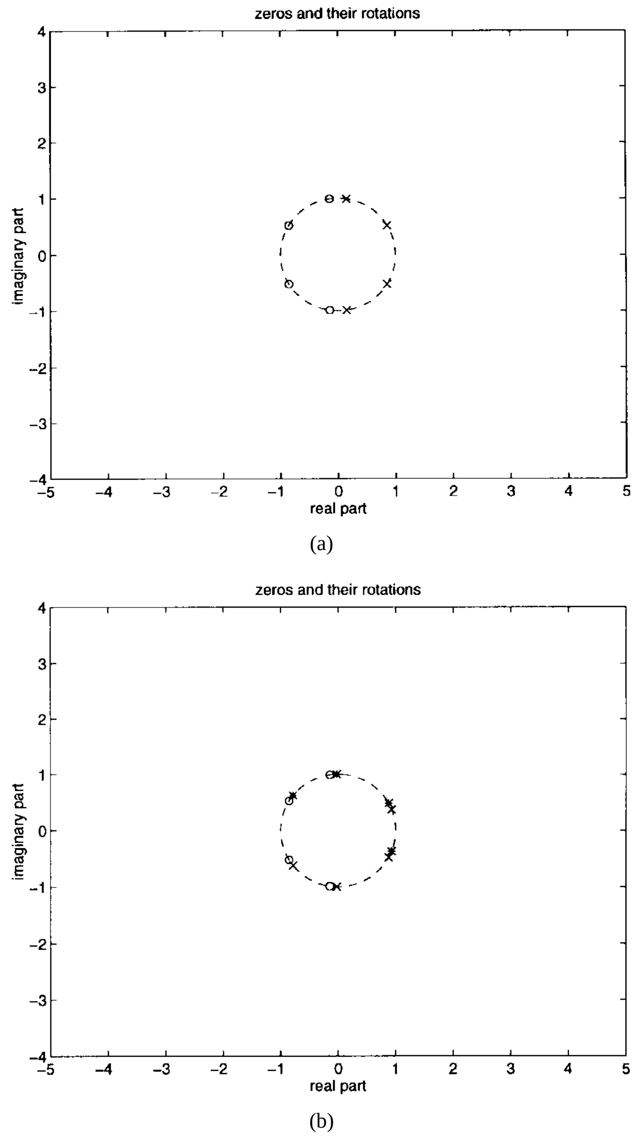



Fig. 8. Length 5 filter: (a) Zeros marked by "o," their rotations with angle $\pi$ marked by "x." (b) Zeros marked by "o," their rotations with angles $2\pi/3$ and $4\pi/3$ marked by "x" and "*," respectively. Dashed line: the unit circle.

we know that the overall $N \times K$ multirate system matrix $\mathbf{F}(z) \triangleq \mathbf{H}(z)\mathbf{G}(z)$ has an FIR inverse. The problem is then to find its inverse $\mathbf{F}^{-1}(z)$ in the sense that $\mathbf{F}^{-1}(z)\mathbf{F}(z) = I_K$. Then, $x[n]$ can be recovered by $\mathbf{F}^{-1}(z)\hat{x}[n]$. To find $\mathbf{F}^{-1}(z)$, we use the Smith form decomposition technique [32] as described below.

It is known [32] that any $N \times K$ polynomial matrix $\mathbf{F}(z)$, where all components are polynomials of $z^{-1}$, can be decomposed into a product of three polynomial matrices $\mathbf{U}(z)$, $\mathbf{\Lambda}(z)$, and $\mathbf{W}(z)$:

$$\mathbf{F}(z) = \mathbf{U}(z)\mathbf{\Lambda}(z)\mathbf{W}(z) \tag{3.2}$$

where $\mathbf{U}(z)$ and $\mathbf{W}(z)$ are $N \times N$ and $K \times K$ unimodular matrices, respectively, and $\mathbf{\Lambda}(z)$ is diagonal with the form

$$\mathbf{\Lambda}(z) = \begin{Bmatrix} \text{diag } [\lambda_1(z), \cdots, \lambda_\rho(z)] \\ 0_{(N-K)\times K} \end{Bmatrix}$$

where $\rho$ is the normal rank of $\mathbf{F}(z)$, $\lambda_l(z)$ divides $\lambda_{l+1}(z)$ for $l = 1, 2, \cdots, \rho - 1$, $\lambda_l(z) = \Delta_{l+1}(z)/\Delta_l(z)$ with $\Delta_1(z) = 1$, and $\Delta_l(z), l > 1$, which is the gcd of all the determinants of all the $(l-1) \times (l-1)$ submatrices of $\mathbf{F}(z)$. A square polynomial matrix is *unimodular* means that its determinant is a nonzero constant.

When $\mathbf{F}(z)$ has an inverse, we then have $\rho = K$ and $\Delta_{K+1}(z) = cz^{-d}$ for a nonzero constant $c$ and an integer $d$. Therefore, when $\mathbf{F}(z)$ has an inverse, the diagonal matrix $\mathbf{\Lambda}(z)$ in (3.2) has the form

$$\mathbf{\Lambda}(z) = \text{diag } (z^{-d_0}, z^{-d_1}, \cdots, z^{-d_{K-1}}) \tag{3.3}$$

for $K$ integers $d_0, d_1, \cdots, d_{K-1}$. Using the above analysis, we have that the overall system in Fig. 4 has the following decomposition

$$\mathbf{F}(z) = \mathbf{H}(z)\mathbf{G}(z) = \mathbf{U}(z)\mathbf{\Lambda}(z)\mathbf{W}(z) \tag{3.4}$$

where $\mathbf{U}(z)$ and $\mathbf{W}(z)$ are $N \times N$ and $K \times K$ unimodular matrices, respectively, and $\mathbf{\Lambda}(z)$ has the form in (3.3). With the form (3.4) of $\mathbf{F}(z)$ (see Fig. 10), its inverse is

$$\mathbf{F}^{-1}(z) = \mathbf{W}^{-1}(z)[\text{diag } (z^{d_0}, z^{d_1}, \cdots, z^{d_{K-1}}) \; \mathbf{0}_{K\times(N-K)}]\mathbf{U}^{-1}(z). \tag{3.5}$$

The reconstruction can be achieved by the diagram shown in Fig. 11.

Given a polynomial matrix, there is a systematic way to find its Smith form. For more details, see [32].

## IV. Applications in the ISI Cancellation

We now consider the application for the ISI cancellation. Example 3 in Section III is used as the ISI transfer function. For its frequency and impulse responses, see Fig. 7.

By the theory in Sections II and III, and the properties of its zero sets shown in Fig. 8(a), it is known that the nonmaximally decimated multirate filterbank with two channels and decimation 1 and its polyphase matrix in (2.8) is able to eliminate the ISI. This implies that when we insert 0 between each two samples $x[n], \cdots, x[0], 0, x[1], 0, \cdots$, which is the signal to

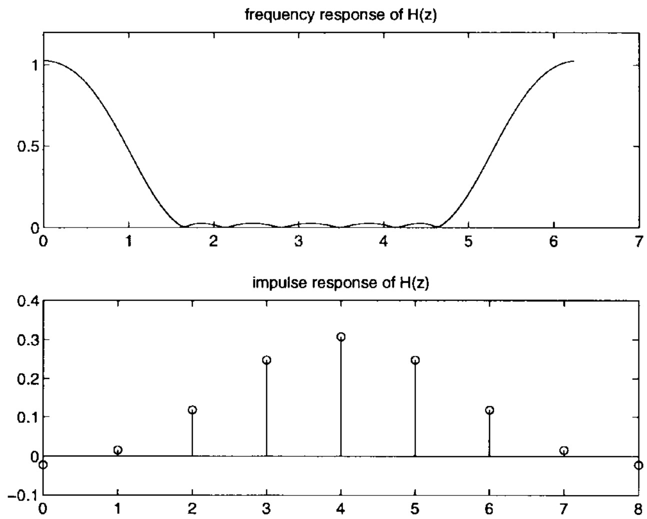



Fig. 9. Lowpass filter $H(z)$ with length 9.

be transmitted, we can reconstruct $x[n]$ from the output $\hat{x}[n]$ of the ISI transfer function $H(z)$. In this case, the overall system transfer matrix $\mathbf{F}(z)$ is

$$\mathbf{F}(z) = \frac{1}{9}\begin{bmatrix}1 + 2.5z^{-1} + z^{-2}\\ 2 + 2z^{-1}\end{bmatrix}. \tag{4.1}$$

Its inverse can be calculated as

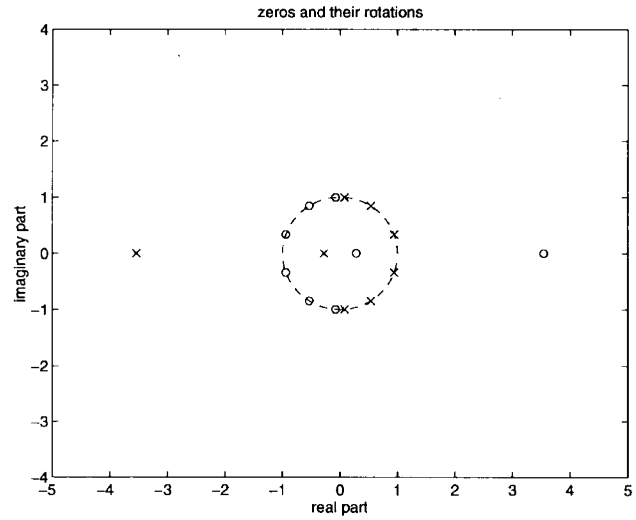
$$\mathbf{F}^{-1}(z) = 9\,(-2,\, 1.5 + z^{-1}). \tag{4.2}$$

With the above inverse, we apply the reconstruction scheme shown in Fig. 11 to the received signal. The simulation results are shown in Fig. 12 with the original signal $x[n]$, the ISI transfer function in the frequency domain, the received signal $\hat{x}[n]$ after the channel, and, finally, the reconstruction with mean square error $7.0704 \times 10^{-7}$. In addition to the ISI, if there is a random noise in transmission, the above reconstruction is robust. A numerical example is shown in Fig. 13, where the maximum magnitude of the additive channel white noise is 0.05, whereas the one for the original signal shown in Fig. 12 is 1. The mean square error for the reconstruction is 0.004.

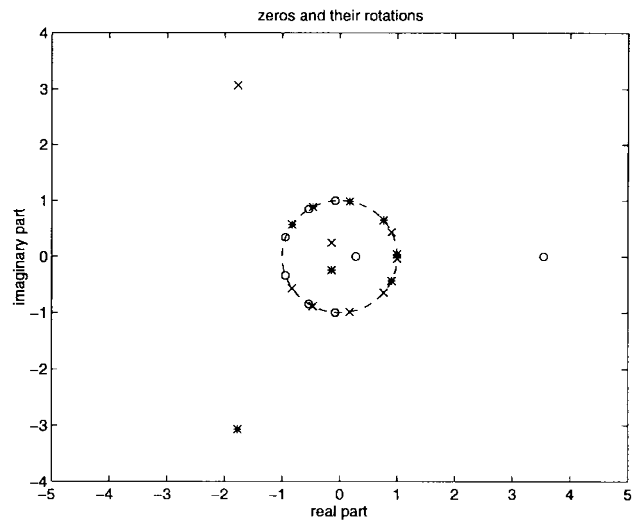
By the property of zeros and their rotations shown in Fig. 8(b), the above increasing of the transmission rate can be reduced by using the nonmaximally decimated multirate filterbank with three channels and decimation 2 in (2.8). In other words, the rate 1/2 can be reduced to 2/3. In this case, the overall system transfer matrix $\mathbf{F}(z)$ is

$$\mathbf{F}(z) = \frac{1}{9}\begin{bmatrix}1 + 2z^{-1} & 2.5z^{-1}\\ 2 + z^{-1} & 1 + 2z^{-1}\\ 2.5 & 2 + z^{-1}\end{bmatrix}. \tag{4.3}$$

Its inverse can be calculated as shown in (4.4), shown at the bottom of the page. Numerical simulations are given in Fig. 14 without random channel noise and, in Fig. 15, with additional channel additive white noise. One can see that the reconstruction is also robust.




(a)




(b)

Fig. 10. Length 9 filter: (a) Zeros marked by "o," their rotations with angle $\pi$ marked by "x." (b) Zeros marked by "o," their rotations with angle $2\pi/3$ and $4\pi/3$ marked by "x" and "*," respectively. Dashed line: the unit circle.

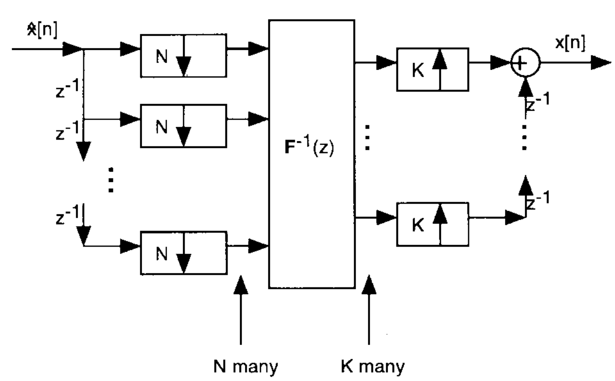



Fig. 11. Reconstruction.

$$\mathbf{F}^{-1}(z) = 9\begin{bmatrix}-\frac{16}{35} - \frac{56}{35}z^{-1} - \frac{24}{35}z^{-2} & \frac{68}{35} + \frac{130}{35}z^{-1} + \frac{48}{35}z^{-2} & -\frac{34}{35} - \frac{96}{35}z^{-1} - \frac{36}{35}z^{-2}\\ \frac{4}{7} + \frac{12}{7}z^{-1} & -\frac{17}{7} - \frac{24}{7}z^{-1} & \frac{12}{7} + \frac{18}{7}z^{-1}\end{bmatrix} \tag{4.4}$$

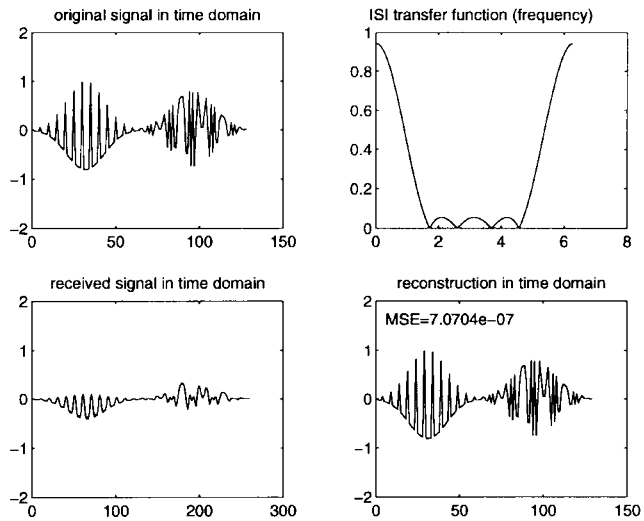



Fig. 12. Rate 1/2 multirate filterbank for the ISI cancellation without random noise in channel.

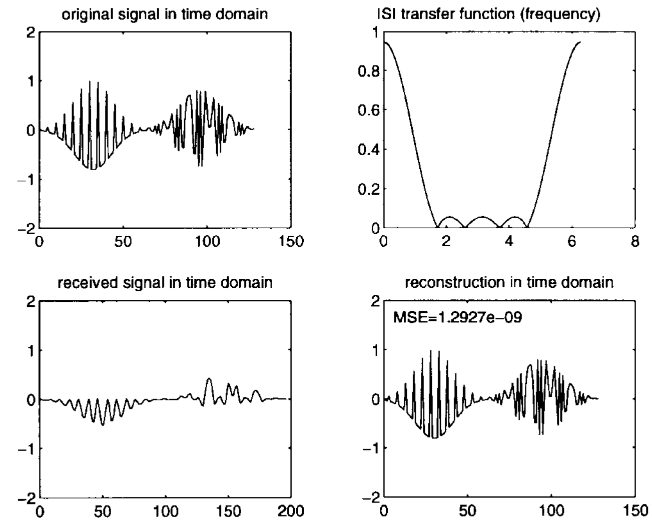



Fig. 14. Rate 2/3 multirate filterbank for the ISI cancellation without random noise in channel.

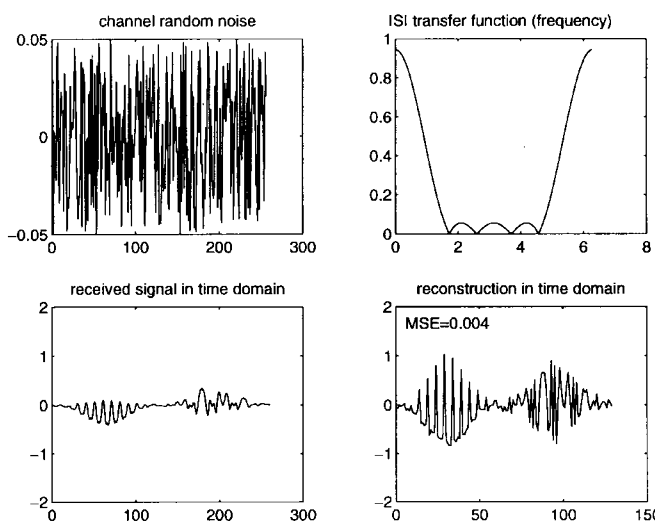



Fig. 13. Rate 1/2 multirate filterbank for the ISI cancellation with additional channel additive white noise.

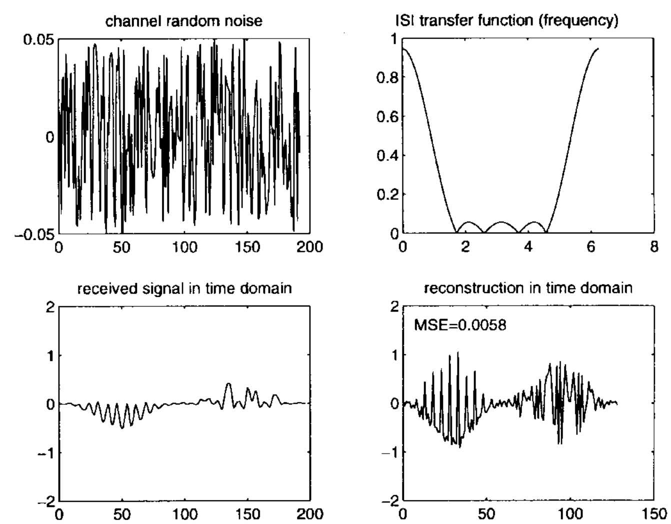



Fig. 15. Rate 2/3 multirate filterbank for the ISI cancellation with additional channel additive white noise.

*Remarks:* One can further reduce the data rate 2/3 by using a multirate filterbank in (2.8) with $N > 3$ and $K = N - 1$. For simplicity, we do not go to higher $N$'s here. Another point that should be noticed is that the above ISI cancellation technique is data independent. Although we use the Smith form decomposition technique for the equalization, it is certainly possible and might be better that some existing equalization techniques, such as [23]–[29], [36], [38], are applicable.

## V. Conclusions

In this paper, we have studied nonmaximally decimated multirate filterbanks as precoders for the ISI elimination, where each $K$ samples are expanded into $N$ samples. When $K = 1$, it is equivalent to the fractionally spaced equalizers, where the sampling rate is $N$ times faster than the baud rate in the receiver. We have found a necessary and sufficient condition on the ISI transfer function for the existence of an FIR ideal linear equalizer. The condition coincides with the known one for the fractionally spaced equalizers when $K = 1$. The condition is not difficult to check when the ISI transfer function is known. In particular, we obtained a simplified version of the condition for an FIR nonmaximally decimated multirate filterbank precoder with $N$ channels and the largest decimation, i.e., $K = N - 1$, which corresponds to the case of the smallest bandwidth expansion in the precoding. The condition can be stated as follows: All rotations of the zero set of the FIR transfer function $H(z)$ at angles $l2\pi/N$ for $l = 0, 1, \cdots, N - 1$ are disjoint from each other. These conditions are basically easy to satisfy. Thus, the approach in this paper suggests that the sampling rate that is $N/K$ times faster than the baud rate for the receiver may be good enough. Moreover, the approach in this paper also suggests the possibility of other precoders besides the trivial one in (2.8) or the constants in [39].

The new precoding method proposed in this paper differs from the existing precoding methods in the following aspects.

It is reliable for any FIR ISI channel, including spectral-null channels, may be independent of the ISI channel, does not implement any modulo operations and is linear; however, it expands the transmission bandwidth with a minimum amount as a sacrifice. This paper provides a framework on the ISI cancellation using multirate filterbanks as precoders. Many practical implementation issues still remain to be investigated in the future.

As a final remark, in this paper, the receiver needs to know the ISI channel characteristics. Most recently, we have studied precoding equalizations without knowing the ISI channel characteristics for the transmitter or the receiver in [41]–[46]. Particularly, ambiguity resistant precoders have been studied in [41]–[44] to combat the ISI.

## ACKNOWLEDGMENT

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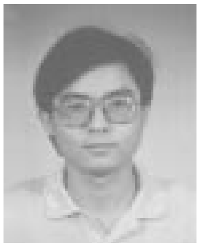

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