Correction to “A Simple Orthogonal Space-Time Coding Scheme for Asynchronous Cooperative Systems for Frequency Selective Fading Channels”

Zheng Li and Xiang-Gen Xia

In [1], equations (3) and (4) are not accurate. We need to perform the time reversal operation from the m-th to the (L_s+m-1)-th samples, where m is the maximum tap after sampling which is defined as $\max_{i,l}\{\tau_{l,SR_i}\}F_s$, F_s is the sampling rate and $L_s = N+\ell_{cp}$. We rewrite equations (3) and (4) as follows:

$$\overline{\mathbf{Y}}_{i1} = \sqrt{P_1}(\overline{\mathbf{X}}_1 * h_{SR_i})_{L_s} + \overline{\mathbf{n}}_{i1}, \quad (3)$$

$$\overline{\mathbf{Y}}_{i2} = \sqrt{P_1}(\overline{\mathbf{X}}_2 * h_{SR_i})_{L_s} + \overline{\mathbf{n}}_{i2} \quad (4)$$

where $(\overline{\mathbf{X}}_1 * h_{SR_i})_{L_s}$ denotes the L_s samples from the m-th to the (L_s+m-1)-th samples in the linear convolution $\overline{\mathbf{X}}_1 * h_{SR_i}$, and $(\overline{\mathbf{X}}_2 * h_{SR_i})_{L_s}$ is similarly defined. The reason for this change is because the signals in the above equations (3) and (4) are the signals the relay nodes can receive due to the time dispersion from the source to the relay nodes.

We also rewrite the appendix in the following.

APPENDIX

PROOF OF THE CLAIM

In the proof of the claim in the appendix on page 2223 in [1], the time reversal operator $\zeta(\cdot)$ should not directly act on the linear convolution $h * \overline{\mathbf{x}}$ and rather should act on its L_s samples from the m-th to the (L_s+m-1)-th samples, i.e., should act on $(h * \overline{\mathbf{x}})_{L_s}$. Thus, the correct proof of the claim is as follows.

Denote three vectors $h = [h_1, h_2, ..., h_m]^T$, $\mathbf{x} = [x_1, ..., x_N]^T$, and $\overline{\mathbf{x}} = [x_{N-(\ell_{cp}-1)}, ..., x_N, x_1, ..., x_N]^T$, $m < \ell_{cp} < N$. From the definition of linear convolution, we write $(h * \overline{\mathbf{x}})_{L_s}$ as follows:

$$\begin{bmatrix} x_{N-(\ell_{cp}-m)} & x_{N-(\ell_{cp}-(m-1))} & \cdots & x_{N-(\ell_{cp}-1)} \\ \vdots & \vdots & \ddots & \vdots \\ x_N & \vdots & \ddots & \vdots \\ x_1 & x_N & \ddots & \vdots \\ \vdots & x_1 & \ddots & x_N \\ x_N & \vdots & \ddots & x_1 \\ 0 & x_N & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & x_N \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_m \end{bmatrix}$$

where the matrix on the right hand side has size $L_s \times m$.

Then we can write $\mathbf{S} = \zeta((h * \overline{\mathbf{x}})_{L_s})$ as

$$\begin{bmatrix} x_{N-(\ell_{cp}-m)} & x_{N-(\ell_{cp}-(m-1))} & \cdots & x_{N-(\ell_{cp}-1)} \\ 0 & \vdots & \ddots & x_N \\ \vdots & 0 & \ddots & \vdots \\ 0 & x_N & \ddots & \vdots \\ x_N & \vdots & \ddots & x_1 \\ \vdots & x_1 & \ddots & x_N \\ x_1 & x_N & \ddots & \vdots \\ x_N & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ x_{N-(\ell_{cp}-(m+1))} & x_{N-(\ell_{cp}-m)} & \cdots & x_{N-(\ell_{cp}-2)} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_m \end{bmatrix}$$

In the following we will perform the two steps of CP removal on $\zeta((h * \overline{\mathbf{x}})_{L_s})$. The first step is equivalent to remove the first ℓ_{cp} rows of the $L_s \times m$ matrix above to construct an $N \times m$ sub-matrix $\mathbf{S}_{step1}$, which can be written as

$$\begin{bmatrix} x_{N-(\ell_{cp}-m)} & x_{N-(\ell_{cp}-(m-1))} & \cdots & x_{N-(\ell_{cp}-1)} \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & x_1 \\ \vdots & x_1 & \ddots & x_N \\ x_1 & x_N & \ddots & \vdots \\ x_N & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ x_{N-(\ell_{cp}-(m+1))} & x_{N-(\ell_{cp}-m)} & \cdots & x_{N-(\ell_{cp}-2)} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_m \end{bmatrix}$$

where the matrix above has size $N \times m$.

The second step is equivalent to shift the bottom $\ell_{cp} - (m-1)$ rows of the above $N \times m$ matrix to the top, $\mathbf{S}_{step2}$ can be

Zheng Li and Xiang-Gen Xia are with the Department of Electrical and Computer Engineering, University of Delaware, Newark, DE 19716. Email: {zhli, xxia}@ee.udel.edu.

written as

$$\begin{bmatrix} x_1 & x_N & \dots & x_{N-(m-2)} \\ x_N & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ x_{N-(\ell_{cp}-(m+1))} & x_{N-(\ell_{cp}-m)} & \ddots & x_1 \\ x_{N-(\ell_{cp}-m)} & x_{N-(\ell_{cp}-(m-1))} & \ddots & x_N \\ \vdots & \vdots & \ddots & \vdots \\ x_2 & x_1 & \dots & x_{N-(m-3)} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_m \end{bmatrix}.$$

Denote an $N \times 1$ vector $h' = [h_1, h_2, \dots, h_m, 0, \dots, 0]^T$. From the definition of circular convolution, $\mathbf{S}_{circ} = \zeta(h') \circledast \zeta(\mathbf{x})$ can be written as

$$\mathbf{S}_{circ} = \begin{bmatrix} x_1 & x_2 & \dots & x_N \\ x_N & x_1 & \dots & x_{N-1} \\ \vdots & x_N & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ x_2 & x_3 & \dots & x_1 \end{bmatrix} \begin{bmatrix} h_1 \\ 0 \\ \vdots \\ 0 \\ h_m \\ \vdots \\ h_2 \end{bmatrix}$$

It is not difficult to check that $\mathbf{S}_{circ} = \mathbf{S}_{step2}$, which implies the claim, i.e., $\zeta(h') \circledast \zeta(\mathbf{x})$ can be obtained after Step 2.

REFERENCES

[1] Zheng Li, Xiang-Gen Xia and Moon Ho Lee, "A simple orthogonal space-time coding scheme for asynchronous cooperative systems for frequency selective fading channels", *IEEE Trans. on Communications*, vol. 58, no. 8, pp. 2219-2224, Aug. 2010.