Vector OFDM Systems

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Outline

- Background
  - Current Standards
- OFDM Overview
- Vector OFDM
- VOFDM with Matrix Modulation
- Linear Receivers
- CDD VOFDM for Multiple Antennas
- Conclusion and Future Research
Communications

wireless

transmission through EM wave propagation

wired
Signal:

\[ A(t) \cos(p(t)) \] carry information

Channel:

a media that wave carrying information propagates through

\[ \rightarrow \text{Approximately a linear system} \]

system frequency response

\[ f \text{ (Hz)} \]
signal \( \cos(p(t)) \)

System/channel frequency response

(add)approximately flat

none flat

additive white Gaussian (AWGN)

intersymbol interference (ISI)
In broadband systems

channel frequency response

f (Hz)

this is wide

It is not possible to be flat → ISI occurs

Wireless: Multipath

Wired: None flat ISI channel
None ideal
Wired (modem): Channel is fixed and has high SNR

- < 9.6 kbs/s  equalization (Lucky 60s)
- 9.6 kbs/s  TCM + equalization (DFE)
- 14.4 kbs/s
- 28.8 kbs/s  TCM + equalization
- 56 kbs/s

Asymmetric Digital Subscriber Line (ADSL)

- 6 Mbs/s  orthogonal frequency division multiplexing (OFDM)
  or called discrete multi-tone (DMT)

<table>
<thead>
<tr>
<th>Data Rate</th>
<th>Wire Size</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5 or 2 Mbps</td>
<td>0.5 mm</td>
<td>5.5 Km</td>
</tr>
<tr>
<td>1.5 or 2 Mbps</td>
<td>0.4 mm</td>
<td>4.6 Km</td>
</tr>
<tr>
<td>6.1 Mbps</td>
<td>0.5 mm</td>
<td>3.7 Km</td>
</tr>
<tr>
<td>6.1 Mbps</td>
<td>0.4 mm</td>
<td>2.7 Km</td>
</tr>
</tbody>
</table>

Squeeze more bits to a symbol

Use more bandwidth

The physical layer was mainly on dealing with ISI
Wireless Systems: Channel varies/fades and not high SNR

outdoor

multipaths

indoor

multiple reflections
Multipath

symbol duration

narrowband case

T 2T 3T 4T

1/T bandwidth

time spread

multipath

No intersymbol interferences
When the bandwidth is too wide or $T$ is too small, the time spread may be across over multiple symbols. In this case, intersymbol interference (ISI) occurs.

In practice, $h(n)$ may have Doppler spread, i.e., time-varying

$x(t)$: transmitted signal; $r(t)$: received signal

$$r(t) = \sum_{n=0}^{L} h(n)x(t - nT) + W(t)$$

In practice, $h(n)$ may have Doppler spread, i.e., time-varying
<table>
<thead>
<tr>
<th>Modulation Method</th>
<th>Bandwidth</th>
<th>Multipath/Modulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2G (IS-95)</td>
<td>1.23 MHz</td>
<td>Almost optimal for single path (or equivalent)</td>
</tr>
<tr>
<td>3G (WCDMA CDMA2000)</td>
<td>&lt; 11 MHz</td>
<td>6--8 multipaths (or equivalent) almost the break point to use CDMA</td>
</tr>
<tr>
<td>IEEE 802.11b (LAN)</td>
<td>similar to 3G</td>
<td></td>
</tr>
<tr>
<td>IEEE 802.11a (LAN)</td>
<td>20 MHz</td>
<td>16 multipaths (or equivalent) OFDM</td>
</tr>
<tr>
<td>IEEE 802.11n (LAN)</td>
<td>20 &amp; 40MHz</td>
<td>40MHz doubles everything in 20MHz OFDM</td>
</tr>
<tr>
<td>4G LTE</td>
<td>20 MHz</td>
<td>16 multipaths (or equivalent) OFDM and SC-FDE</td>
</tr>
<tr>
<td>5G</td>
<td>100 MHz or more</td>
<td>OFDM</td>
</tr>
<tr>
<td>6G</td>
<td>????</td>
<td>????</td>
</tr>
</tbody>
</table>
Digital Wireless Standards vs. Bandwidth (#of Multipaths)

- A standard is determined by a bandwidth (so far)
- 2G: 1.23MHz, almost the highest for non-ISI (or highest for TDMA in cellular systems)
  - Both TDMA and CDMA (DS spread spectrum) work well
- 3G: ~10 MHz, a few multipaths, highest for CDMA
  - Due to the ISI and wireless varying channels, time domain equalization may not work well, TDMA is not used, but CDMA (DS spread spectrum) is used in all standards since it is good to resist a few chip level time delays (RAKE receiver)
- 4G: 20 MHz, more multipaths
  - Even CDMA RAKE receiver may not work well
  - OFDM is adopted (down link)
    - Due to wireless channel varying, the number of subcarriers, \( N=64 \), is used, 25% data overhead for the cyclic prefix (CP) to deal with the multipaths
- 5G: 100 MHz, OFDM
Some Comments on These Standards

The modulation schemes for all these standards are determined by the way to deal with ISI.

- In my opinion, multi-access or multi-cells is NOT the problem to determine which basic modulation is used.
- Adding more antennas or not is the hardware choice and may not determine a basic modulation (?)
- A basic modulation has to be simple.

Dealing with ISI is the key!
6G: Bandwidth $\gg 20$ MHz (?)

- Can OFDM Still Work?
  - Much more multipaths exist
    - much large CP length to deal with multipaths
    - much large number $N$ of subcarriers/IFFT_size
    - may lead to break down OFDM??
      - High PAPR (?)
      - Time varying channels (?)

- Is multiband OFDM bandwidth efficient?
  - Five 20 MHz bandwidth OFDM systems to form 100 MHz band

- What bandwidth will be the breakpoint for OFDM in cellular systems? How large will a bandwidth go?
  - Can we make it work with a fixed $N$ while it still can deal with the increased # of multipaths?

- We next think about single antenna VOFDM [Xia, TCOM, August, 2001, also ICC 2000]
OFDM and VOFDM

OFDM: orthogonal frequency division multiplexing

VOFDM: vector OFDM

- It is **NOT** Cisco’s for multiple antennas, where every antenna employs OFDM
  - Cisco’s is MIMO-OFDM that is a trivial concept
- It is not trivial for single transmit antenna systems

Today’s focus
Why OFDM? ------- Rough Idea

- Non-flat channel

Narrow subchannels using multiple subcarriers

- These subchannels may have overlapped spectrums. So, OFDM is bandwidth efficient
  - The analog signals in these subchannels are not orthogonal each other.
  - Their discrete/sampled signals are orthogonal each other.

Each subchannel is narrow and therefore more flat

- It does not have ISI.
OFDM

$x(n)$ → S/P → N-point IFFT → Cyclic Prefix insertion → P/S

block based modulation

P/S → N-point FFT → S/P → Cyclic Prefix removal

+$\rightarrow$ ISI $H(z)$
a block of $N$ information symbols

$$x_{N-1} \ldots x_0 \rightarrow \text{IFFT} \rightarrow X_0 \rightarrow \text{cyclic prefix adding} \rightarrow X_0 \rightarrow \cdots \rightarrow X_{\Gamma-1} \rightarrow X_{N-1} \rightarrow \cdots \rightarrow X_0 \rightarrow \text{transmit}$$

$$H(z)$$

$$h \ast X$$

$$h \bigcirc \ast X$$

two blocks

cyclic prefix removal

$$h \bigcirc \ast X$$

$$h \bigcirc \ast X$$

$$Y_k = H_k \cdot x_k, \ k = 0, 1, \ldots, N-1$$

$$Y_k = h \bigcirc \ast X$$

$$H_k = H(e^{j2\pi \frac{k}{N}}) = \sum_{n=0}^{L} h(n)e^{-j2\pi nk}$$

cyclic convolution
ISI channel

\[ y(k) = \sum_{n=0}^{L} h(n)x(k-n) + \eta(k) \]

adding cyclic prefix as an additional data rate overhead

N ISI-free subchannels

\[ Y_k = H_k \cdot x_k + \eta_k, \quad k = 0, 1, \ldots, N - 1 \]

Each subchannel corresponds to a DFT component \( H_k \) of the ISI channel. If the frequency component \( H_k \) is small (bad), then this subchannel is bad.

For 20 MHz Channel, OFDM

\[ L \leq 16 \]
\[ N=64 \]
\[ \Gamma=L=16, \quad 25\% \text{ data rate overhead} \]
What Really Matters for OFDM??

- When the DFT size $N$ is large, the CP is negligible. Although we may not have the ISI free identities $y_k = H_k x_k$, we may get the approximate ISI free equations: $y_k \approx H_k x_k$
- CP insertion is not the most essential part of the OFDM
- OFDM converts less finite contellation, such as BPSK, QPSK, to more level amplitudes and phases

- High peak-to-average power ratio (PAPR): $\text{PAPR} = N$
- When all signals $x_k$ are the same, the power is $N^2$
Single Antenna Vector OFDM System

OFDM, when data rate overhead $\Gamma \geq L$

Receiver:

$$Y_k = H_k x_k + \eta_k$$

$N$ many scalar channels/equations

---

VOFDM, when data rate overhead $\tilde{\Gamma} \geq \tilde{L} \approx \frac{L}{M}$

Receiver:

$$Y_k = H_k x_k + \eta_k$$

$N$ many $M$ by 1 vector channels/equations

---

This may be a simple way to see the transmission of OTFS.
Comments on VOFDM and OTFS

- An analog pulse $g(t)$ is skipped here but can be always added to VOFDM signals in real transmission similar to OFDM.

- The CP length does not have to be exactly a multiple of the vector size $M$: $\tilde{M}$.
  - The CP part can be truncated to any length that is not less than the channel length $L$ to avoid the inter-block-interference.

- The transmission of VOFDM is exactly the same as that of OTFS.
VOFDM: Vectorized Channel

- The ISI channel $H(z)$ is converted into $N$ vector channels

$$Y_k = H_k \times_k + \eta_k, \quad k=0,1,\ldots,N-1, \quad (1)$$

where $H_k$ is the $M$ by $M$ blocked version of the original frequency responses of the ISI $H(z)$:

$$H_k = H(e^{j2\pi k/N}), \quad H(z) = \begin{bmatrix} H_0(z) & z^{-1}H_{M-1}(z) & \cdots & z^{-1}H_1(z) \\ H_1(z) & H_0(z) & \cdots & z^{-1}H_2(z) \\ \vdots & \vdots & \ddots & \vdots \\ H_{M-1}(z) & H_{M-2}(z) & \cdots & H_0(z) \end{bmatrix}$$

$$H_m(z) = \sum_{l=0}^{L'} h(Ml + m)z^{-l}, \quad 0 \leq m \leq M - 1.$$ $$\tilde{L}' = \left[ \frac{L}{M} \right]$$

$m$th polyphase component of $H(z)$
Vectorized Channel Example

If \( H(z) = 1 + 0.9z^{-1} - 0.8z^{-2} + 0.6z^{-3} + 0.5z^{-4} - 0.4z^{-5} \), vector size \( M=2 \), then, its polyphase components are

\[
H_0(z) = 1 - 0.8z^{-1} + 0.5z^{-2}, \quad H_1(z) = 0.9 + 0.6z^{-1} - 0.4z^{-2}
\]

and the vector channel coefficient matrices are

\[
\begin{align*}
\underline{H}(z) &= \begin{bmatrix} H_0(z) & z^{-1}H_1(z) \\ H_1(z) & H_0(z) \end{bmatrix} \\
L &= 5 \\
\tilde{L} &= \left\lfloor \frac{L}{M} \right\rfloor = \left\lfloor \frac{5}{2} \right\rfloor = 3
\end{align*}
\]

\[
= \begin{bmatrix} 1 & 0 \\ 0.9 & 1 \end{bmatrix} + \begin{bmatrix} -0.8 & 0.9 \\ 0.6 & -0.8 \end{bmatrix}z^{-1} + \begin{bmatrix} 0.5 & 0.6 \\ -0.4 & 0.5 \end{bmatrix}z^{-2} + \begin{bmatrix} 0 & -0.4 \\ 0 & 0 \end{bmatrix}z^{-3}
\]
Why VOFDM Is Good for Channels with Doppler Spread

- The vectorized channel matrix $H(z)$ is pseudo-circulant and can be diagonalized by $M$-point DFT matrix $W_M$ with a diagonal phase shift matrix $\Lambda(z) = \text{diag}(1, z^{-1}, \ldots, z^{-M+1})$ as follows:

$$H(z^M) = (W_M^* \Lambda(z))^{-1} \text{diag}(H(z), H(zW_M), \ldots, H(zW_M^{M-1}))W_M^* \Lambda(z)$$

where $W_M = e^{-j\frac{2\pi}{M}}$. Thus, matrices $H_k = H(W_N^{-k})$ can be diagonalized by $W_M^* \Lambda(W_N^{-k})$, $k=0,1,\ldots,N-1$.

- The receiver equation (1) becomes

$$\tilde{Y}_k = \text{diag}(H(W_N^{-k}), H(W_N^{-k}W_M), \ldots, H(W_N^{-k}W_M^{M-1})) W_M^* \text{diag}(1, W_{MN}^k, \ldots, W_{MN}^{k(M-1)}) x_k + \tilde{n}_k$$

This frequency domain part is similar to the channel in time domain for single antenna systems. Or diagonal space-time coded MIMO systems.

This part is similar to the precoding to achieve signal space diversity for time-varying channel. Or diagonal space-time block coding to achieve spatial diversity.

When channel varies with Doppler spread, it can collect multipath diversity and/or signal space diversity. This can be seen later even with the MMSE receiver.
VOFDM vs OTFS

- The VOFDM receiver equation to demodulate
  \[ Y_k = H_k x_k + \eta_k \]
  - It coincides with that of OTFS when the channel is stationary/quasi-static at both Tx and Rx.


- In fact, the transmission of OTFS is the same as that of VOFDM, no matter the channel is stationary or not.
  - The transmitted signals of OTFS and VOFDM are the same in either discrete-time sequence or continuous-time waveform.
  - VOFDM has also been studied over time-varying channels in my book *Modulated Coding for Intersymbol Interference Channels*, New York, Marcel Dekker, 2000: Section 7.4

- Some other names proposed later in the literature:
  - OSDM, Quadrature OFDMA (or A-OFDM)
**VOFDM, OFDM, SC-FDE**

- When $M=1$, VOFDM=OFDM
- When $M>L$ and the FFT size $N$ is 1, VOFDM=SC-FDE:
  - at the transmitter, no IFFT is implemented (so the PAPR is not changed) but just CP of the information symbols is inserted; low PAPR.
  - at the receiver, both FFT and IFFT, and frequency domain equalizer are implemented.
- VOFDM is a bridge between OFDM and SC-FDE.
  - Its ML receiver complexity is also in the middle.
Time domain single carrier vs. equalization

Maximum # symbols in ISI

VOFDM
No, or 2, or 3, ..., or Maximum # (you choose) symbols in ISI

Frequency domain OFDM
No ISI

Single antenna VOFDM is in the middle between single carrier and OFDM in terms of dealing with ISI
VOFDM: Some Other Advantages

- Cyclic prefix data rate overhead reduction when the FFT/IFFT size is fixed
  - For OFDM, it is $\frac{L}{N}$
  - For VOFDM, it is $\frac{L}{MN}$

For fixed cyclic data rate overhead, the FFT/IFFT size can be reduced by M times
  - The IFFT size reduction reduces the peak-to-average power ratio (PAPR), which is important in cellular communications.
VOFDM: Some Other Advantages

- VOFDM can be combined with matrix modulation: at the receiver $Y_k = H_k x_k + W_k$ where $x_k$ are vectors of information bits or symbols. By grouping two vectors of size 2 together considering BPSK for each information symbol, these vectors become 16 matrices

\[
\begin{bmatrix}
    x_{11} & x_{12} \\
    x_{21} & x_{22}
\end{bmatrix}: x_{ij} \in \{1,-1\}
\]

- These 16 matrices are not good in terms of matrix modulation due to the channel matrices have random components (fading).
  - These 16 matrices can be replaced by the ones with the best known diversity product.
16 best known 2 by 2 unitary matrices in the literature (Liang-Xia, IEEE Trans. Information Theory, Aug. 2002) with the best known diversity product (the minimum absolute value of all the determinants of difference matrices of any two distinct matrices):

\[
\left\{ \begin{array}{cc}
e^{j\pi/8} & 0 \\
0 & e^{j3\pi/8}
\end{array} \right\}
\left[ \begin{array}{cc}
\cos \frac{l\pi}{2} & \sin \frac{l\pi}{2} \\
-\sin \frac{l\pi}{2} & \cos \frac{l\pi}{2}
\end{array} \right]
\left[ \begin{array}{cc}
e^{j\pi/4} & 0 \\
0 & e^{-j\pi/4}
\end{array} \right] : 0 \leq l \leq 15
\]
Simulations

DVB

CP data rate overhead is the same for the two curves, matrix modulation is not used.
ML receivers

comparison between OFDM with BPSK and VOFDM with 2x2 unitary matrix modulations

- OFDM + coherent BPSK
- VOFDM + coherent 2x2 unitary matrix modulation
- OFDM + differential BPSK
- VOFDM + differential 2x2 unitary matrix modulation

Differential no channel information

Coherent with channel information

- Zero-Forcing (ZF) receiver
- Minimum mean square error (MMSE) receiver
Detection SNR Gap Between ZF and MMSE Receivers

**Theorem 1:** Denote the $m$-th column of $\mathbf{H}_l$ as $\mathbf{h}_{l,m}$ and the matrix of $\mathbf{H}_l$ after deleting the $m$-th column as $\mathbf{H}_{l,m}$, which is an $M \times (M - 1)$ matrix. When $\rho \to \infty$, the gap between the detection SNRs of the ZF-V-OFDM and the MMSE-V-OFDM can be written as

$$\lim_{\rho \to \infty} \left( \rho_{i}^{\text{MMSE}} - \rho_{i}^{ZF} \right) = \left\| \mathbf{h}_l^H \mathbf{H}_{l,m} \left( \mathbf{H}_{l,m}^H \mathbf{H}_{l,m} \right)^{-1} \right\|^2,$$

(12) which is independent of $m$. As $\rho \to \infty$, the ratio between $\rho_{i}^{\text{MMSE}}$ and $\rho_{i}^{ZF}$ approaches 1, i.e.,

$$\lim_{\rho \to \infty} \frac{\rho_{i}^{\text{MMSE}}}{\rho_{i}^{ZF}} = 1.$$
Detection SNR Gap Between ZF and MMSE Receivers

- For V-OFDM, the SNR gap between ZF and MMSE detections doesn’t approach zero as SNR approaches infinity.

- On average, the performance gap increases with the vector block (VB) block size $M$ and the maximum delay $L$ (or $D$ used as below) of the channel.
Detection SNR Gap Between ZF and MMSE Receivers

\(D = L\)
The Performance Independence of Vector Block Index

- **Theorem 2:** For ZF-VOFDM and MMSE-VOFDM, after averaging over all the channel, the $NM$ transmitted symbols have the same error rate performance.
  - For VOFDM with ML receiver (i.e., ML-VOFDM), different VBs may have different performances (See Han *et al.* 2010 and Cheng *et al.* 2011).
  - However, for VOFDM with ZF and MMSE receivers, all the VBs have the same performance.
The Performance Independence of Vector Block Index

![Graph: MMSE-V-OFDM SER Performance of Different VBs](image_url)
Diversity Order of MMSE and ZF Receivers

- **Definition of the diversity order**

\[
d(R, M, D, N) = - \lim_{{\rho \to \infty}} \frac{\log P_{\text{ser}}(R, M, D, N)}{\log \rho}.
\]

- \( R \) is the spectrum efficiency defined as bits/sec/Hz

- **Theorem 5**: For MMSE-V-OFDM, the diversity order \( d^{\text{MMSE}}(R, M, D, N) \) equals

\[
d^{\text{MMSE}}(R, M, D, N) = \min \left\{ \lfloor M2^{-R} \rfloor, D \right\} + 1.
\]

- **Theorem 6**: For ZF-V-OFDM, the diversity order \( d^{\text{ZF}}(R, M, D, N) = 1. \)
Diversity Order of MMSE and ZF Receivers

- Both ZF and MMSE detections are scalar detections, they have the similar complexities.
- However, the MMSE detection can exploit the diversity inside the VOFDM, while ZF detection cannot.
- The only required extra information for MMSE detection is the channel SNR, which can be obtained at the receiver.
Diversity Order of MMSE Receiver

\[
diversity\ order = \min\left\{ \left\lfloor M 2^{-R} \right\rfloor, D \right\} + 1
\]

Same diversity order
Diversity Order of MMSE Receiver

\[
\text{diversity order} = \min\left\{ M 2^{-R}, D \right\} + 1
\]

![Graph showing MMSE-V-OFDM Outage Rate at Different D with M=16 and R=4]

- Diversity order = 1
- Diversity order = 2

\( D = L \)
Diversity Order of MMSE Receiver

\[
\text{diversity order} = \min \left\{ M \frac{2^{-R}}{D} \right\} + 1
\]

\( D = L \)

Diversity order = 1

\( M = 8 \)

Diversity order = 3

\( M = 16 \)
Performances for ML and MMSE Receivers

D=32, R=2

SER of ML-V-OFDM vs. MMSE-V-OFDM at Different M with D=32, R=2

- M=1
- M=2
- M=4
- M=16

MMSE

D=L
Diversity Order of ZF Receiver

ZF-V-OFDM has the same performance of the conventional OFDM at high SNR.

MMSE-V-OFDM can exploit the diversity in V-OFDM and has better performance.

\[ D = L \]
Multiple Antenna VOFDM Using Cyclic Delay Diversity (CDD)

- CDD can be used to collect both spatial and multipath diversities in a MIMO-OFDM systems

\[
\begin{align*}
    h_{11}, h_{12}, \ldots, h_{1L} \\
    \vdots \\
    h_{n_t1}, h_{n_t2}, \ldots, h_{n_tL}
\end{align*}
\]

After CDD

\[
\begin{align*}
    h_{11}, \ldots, h_{1L}, \ldots, h_{n_t1}, \ldots, h_{n_tL}
\end{align*}
\]

It is equivalent to

if \( N \geq n_t L \)

When the bandwidth is larger, the number \( L \) of multipaths will be larger too. Then, CDD in this case may not be able to collect full spatial and multipath diversities anymore.
Multiple Antenna VOFDM Using Cyclic Delay Diversity (CDD)

- CDD VOFDM can collect both spatial and multipath diversities despite of a large bandwidth

\[ H_{11}, H_{12}, \ldots, H_{\frac{L}{M}} \]

\[ H_{n_t 1}, H_{n_t 2}, \ldots, H_{\frac{L}{M}} \]

After CDD

It is equivalent to

\[ H_{11}, \ldots, H_{\frac{L}{M}}, \ldots, H_{n_t 1}, \ldots, H_{\frac{L}{M}} \]

if \( N \geq n_t \frac{L}{M} \)

The number of multipaths is equivalently reduced by \( M \) times for VOFDM with a vector size \( M \)
Conclusion and Future Research

- VOFDM can be used either to reduce the PAPR by reducing the IFFT size while at a fixed the CP data rate overhead; or reduce the CP data rate overhead while at a fixed the IFFT size.
- VOFDM provides a tradeoff between the receiver complexity, performance, PAPR, CP overhead for an ISI channel.
- VOFDM is in the middle between single carrier and OFDM systems in terms of dealing with ISI.
- The transmitted signals of OTFS and VOFDM are identical.
  - Good for channels with both time and Doppler spreads.
  - VOFDM was also studied over time-varying channels in Section 7.4: X.-G. Xia, *Modulated Coding for Intersymbol Interference Channels*, New York, Marcel Dekker, 2000.
- CDD VOFDM for multi-antennas can collect both spatial and multipath diversities, where CDD OFDM is not be able to do so in a large bandwidth system.
Recall Physical Layer Communications Developments in Recent Decades for Both Wireless and Wired Systems

- It has been always on dealing with ISI

Time domain single carrier vs. equalization

- Maximum # symbols in ISI

VOFDM

- No, or 2, or 3, ..., or Maximum # (you choose) symbols in ISI

Frequency domain OFDM

- No ISI

Is this VOFDM something to think about after OFDM?

Or what’s next???
References


Thank you!