Vector OFDM Systems

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Outline

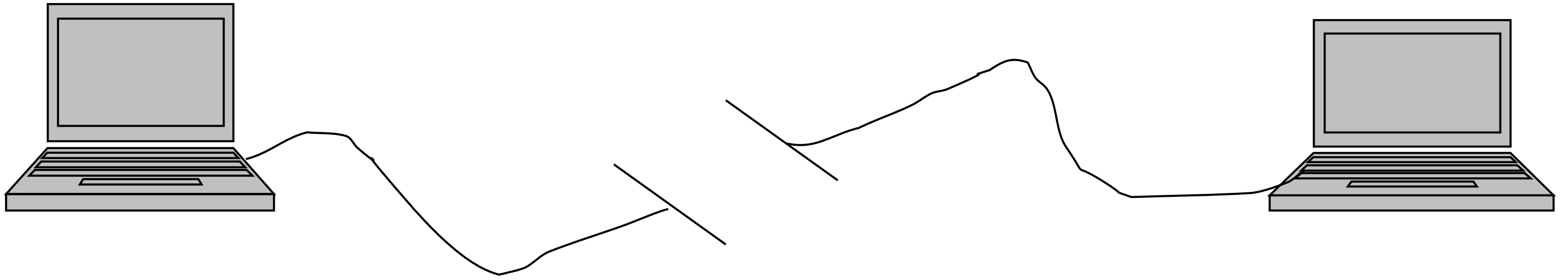
- Background
  - Current Standards
- OFDM Overview
- Vector OFDM
- VOFDM with Matrix Modulation
- Linear Receivers
- CDD VOFDM for Multiple Antennas
- Conclusion and Future Research

Communications

wireless

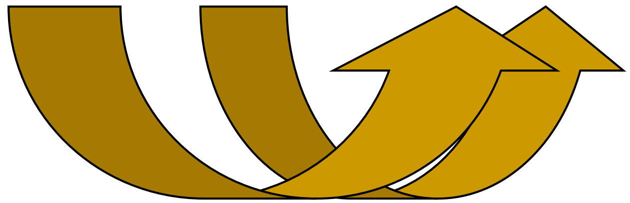
transmission through EM wave propagation

wired



Signal:

A(t) cos(p(t))   carry information

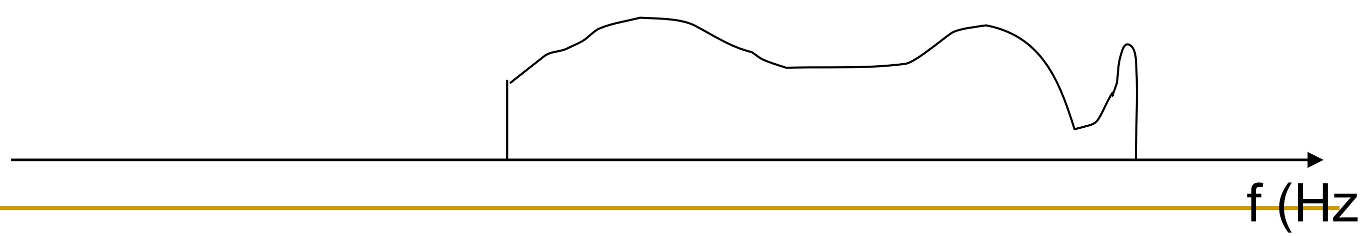


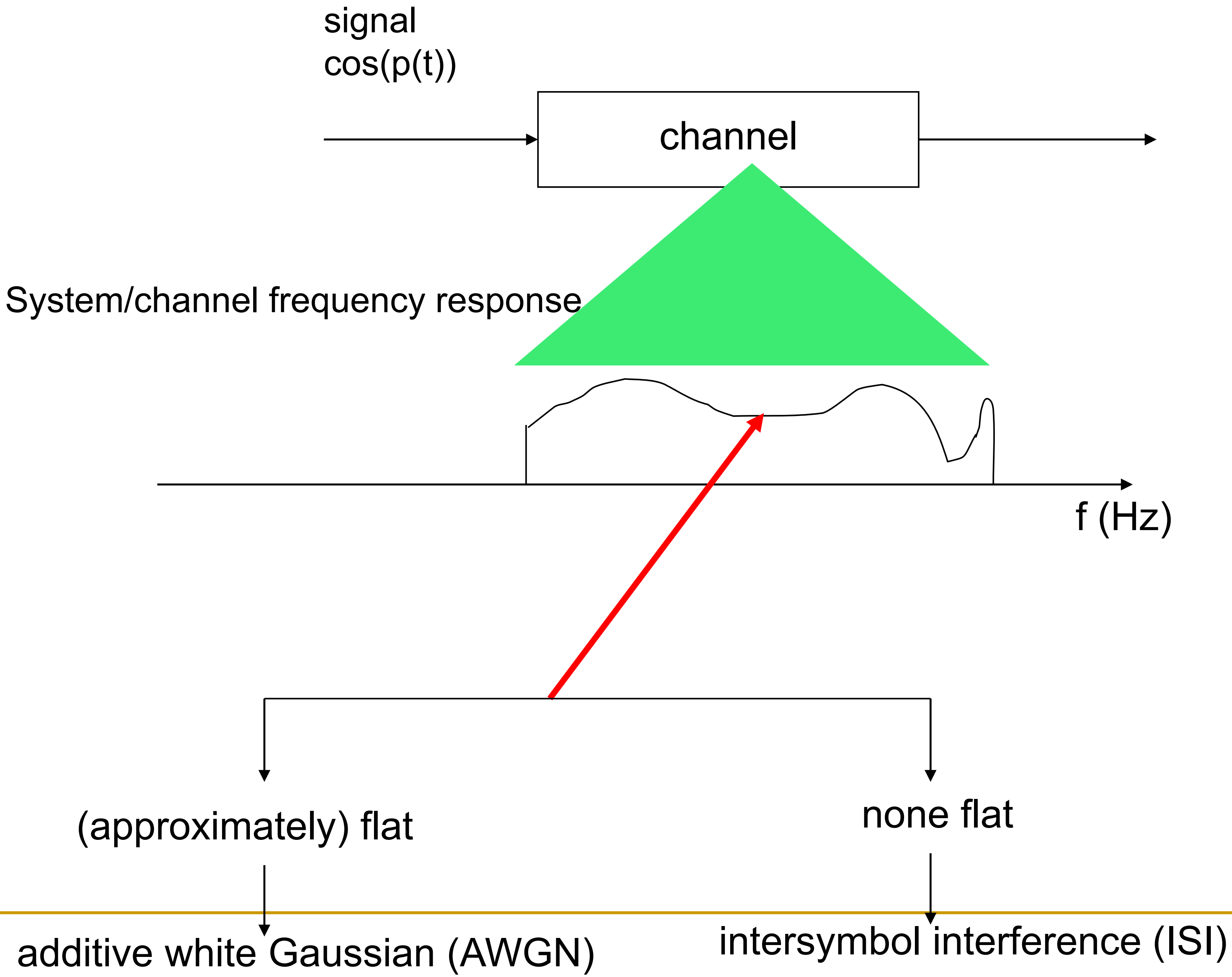
Channel:

a media that wave carrying information propagates through

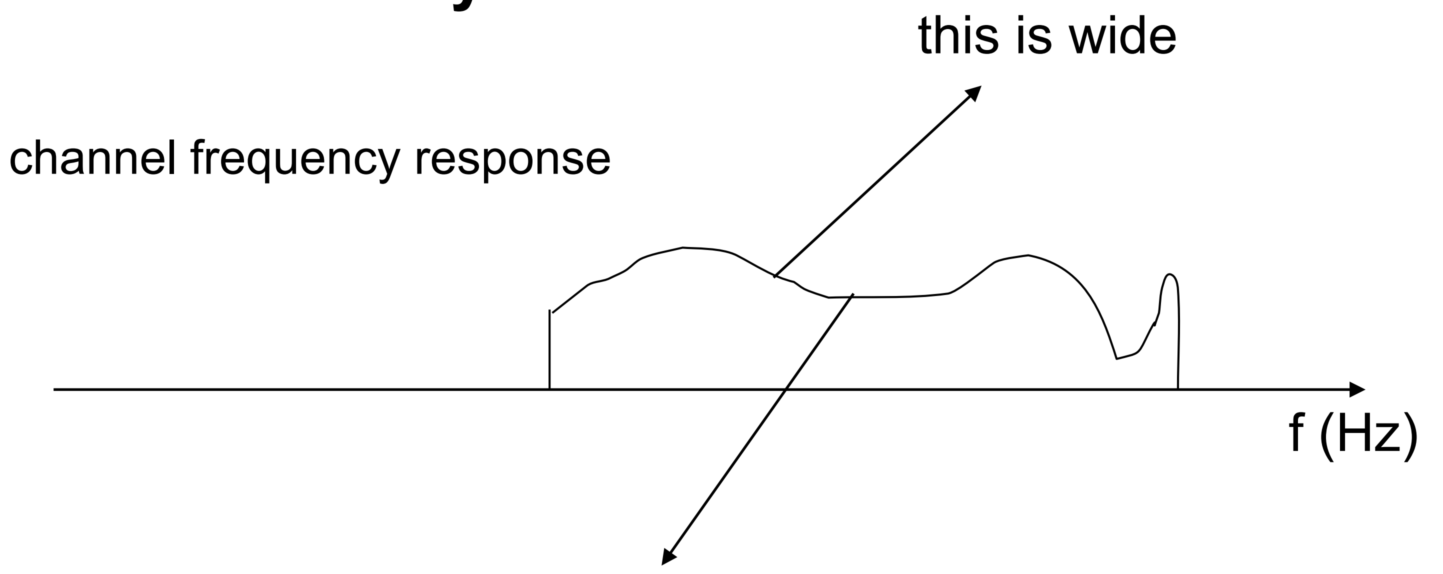
→ Approximately a linear system

system frequency response




signal
cos(p(t))
channel
System/channel frequency response
f (Hz)
(approximately) flat
none flat
additive white Gaussian (AWGN)
intersymbol interference (ISI)

In broadband systems



It is not possible to be flat → ISI occurs

Wireless: Multipath

Wired: None flat ISI channel
None ideal

Wired (modem): Channel is fixed and has high SNR

< 9.6 kbs/s equalization (Lucky 60s)

9.6 kbs/s TCM +equalization (DFE)

Squeeze more bits to a symbol

14.4 kbs/s
28.8 kbs/s TCM + equalization
56 kbs/s

Asymmetric Digital Subscriber Line (ADSL)
6 Mbs/s **orthogonal frequency division multiplexing (OFDM)**
or called discrete multi-tone (DMT)

Use more bandwidth

Data Rate	Wire Size	Distance
1.5 or 2 Mbps	0.5 mm	5.5 Km
1.5 or 2 Mbps	0.4 mm	4.6 Km
6.1 Mbps	0.5 mm	3.7 Km
6.1 Mbps	0.4 mm	2.7 Km

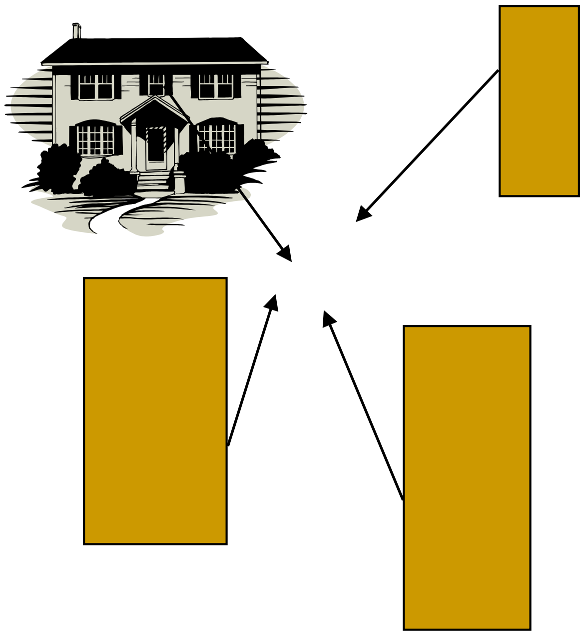
The physical layer was mainly on dealing with ISI

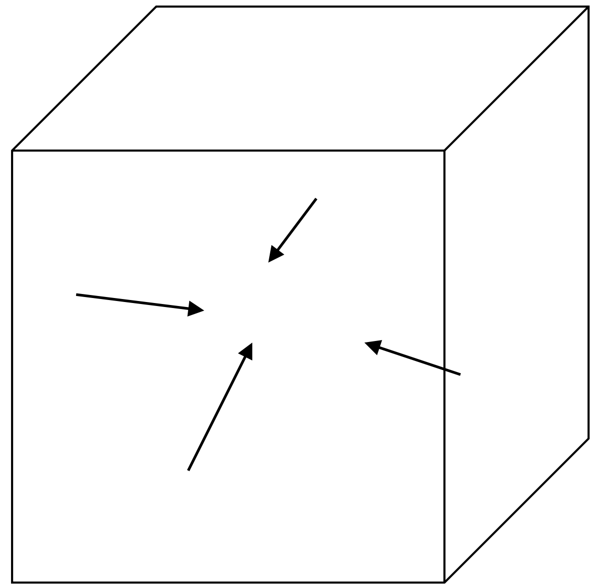
Wireless Systems: Channel varies/fades and not high SNR

outdoor

indoor

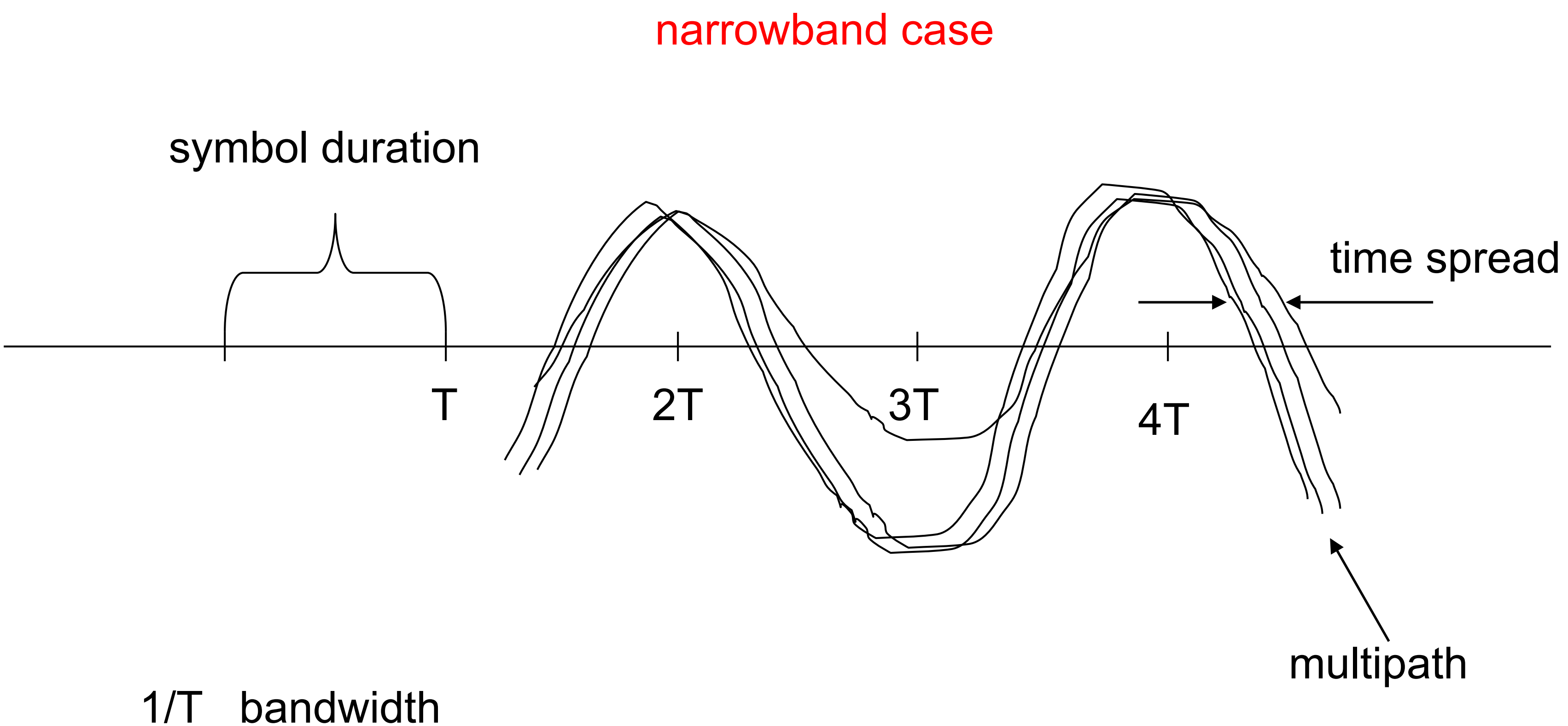
multipaths





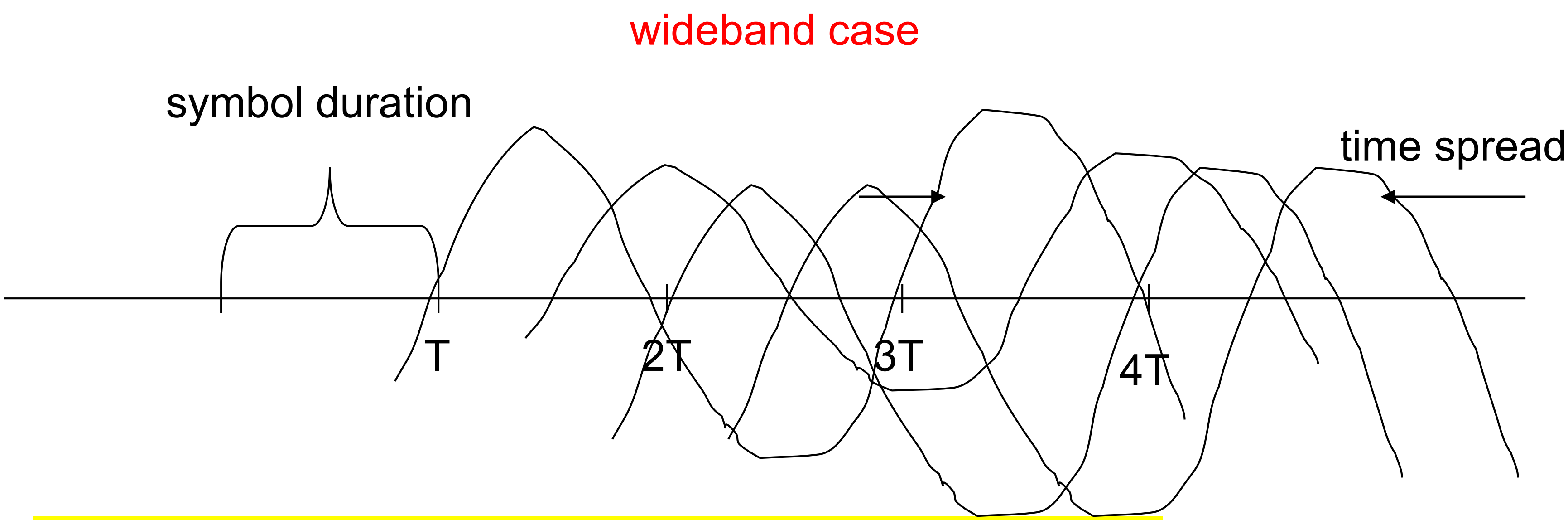
multiple reflections

Multipath



No intersymbol interferences

When the bandwidth is too wide or T is too small, the time spread may be across over multiple symbols. In this case, intersymbol interference (ISI) occurs.



x(t): transmitted signal; *r(t)*: received signal

$$r(t) = \sum_{n=0}^{L} h(n)x(t-nT) + W(t)$$

ISI; AWGN

In practice, *h(n)* may have Doppler spread, i.e., time-varying

Number of Multipath vs. Modulation Methods in Wireless Applications

2G (IS-95)	1.23 MHz	Almost optimal for single path (or equivalent)
3G (WCDMA CDMA2000)	< 11 MHz	6--8 multipaths (or equivalent) almost the break point to use CDMA
IEEE 802.11b (LAN)	similar to 3G	
IEEE 802.11a (LAN)	20 MHz	16 multipaths (or equivalent) OFDM
IEEE 802.11n (LAN)	20 & 40MHz	40MHz doubles everything in 20MHz OFDM
4G LTE	20 MHz	16 multipaths (or equivalent) OFDM and SC-FDE
5G	100 MHz or more	OFDM
6G	????	????

Digital Wireless Standards vs. Bandwidth (#of Multipaths)

- A standard is determined by a bandwidth (so far)
- 2G: 1.23MHz, almost the highest for non-ISI (or highest for TDMA in cellular systems)
  - Both TDMA and CDMA (DS spread spectrum) work well
- 3G: ~10 MHz, a few multipaths, highest for CDMA
  - Due to the ISI and wireless varying channels, time domain equalization may not work well, TDMA is not used, but CDMA (DS spread spectrum) is used in all standards since it is good to resist a few chip level time delays (RAKE receiver)
- 4G: 20 MHz, more multipaths
  - **Even CDMA RAKE receiver may not work well**
  - **OFDM is adopted (down link)**
    - Due to wireless channel varying, the number of subcarriers, N=64, is used, 25% data overhead for the cyclic prefix (CP) to deal with the multipaths
- 5G: 100 MHz, OFDM

Some Comments on These Standards

- The modulation schemes for all these standards are determined by the way to deal with ISI.
  - In my opinion, multi-access or multi-cells is NOT the problem to determine which basic modulation is used.
  - Adding more antennas or not is the hardware choice and may not determine a basic modulation (?)
  - A basic modulation has to be simple.
- Dealing with ISI is the key !

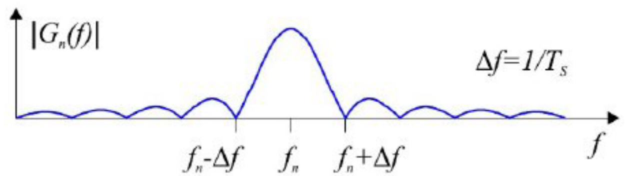
6G: Bandwidth >>20 MHz (?)

- Can OFDM Still Work?
  - Much more multipaths exist
    → much large CP length to deal with multipaths
    → much large number *N* of subcarriers/IFFT_size
    → may lead to break down OFDM??
      - High PAPR (?)
      - Time varying channels (?)
- Is multiband OFDM bandwidth efficient?
  - Five 20 MHz bandwidth OFDM systems to form 100 MHz band
- What bandwidth will be the breakpoint for OFDM in **cellular systems**? How large will a bandwidth go?
  - Can we make it work with a **fixed *N*** while it still can deal with the increased # of multipaths?
    - **We next think about single antenna VOFDM** [Xia, TCOM, August, 2001, also ICC 2000]

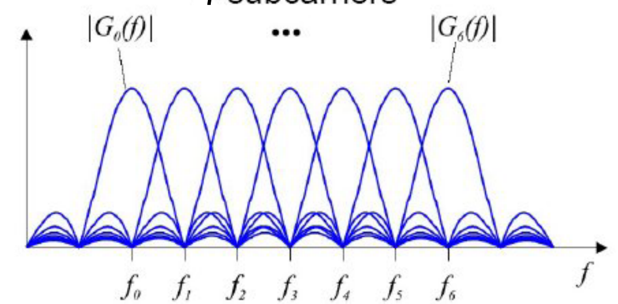
OFDM and VOFDM

- OFDM: orthogonal frequency division multiplexing

1 subcarrier

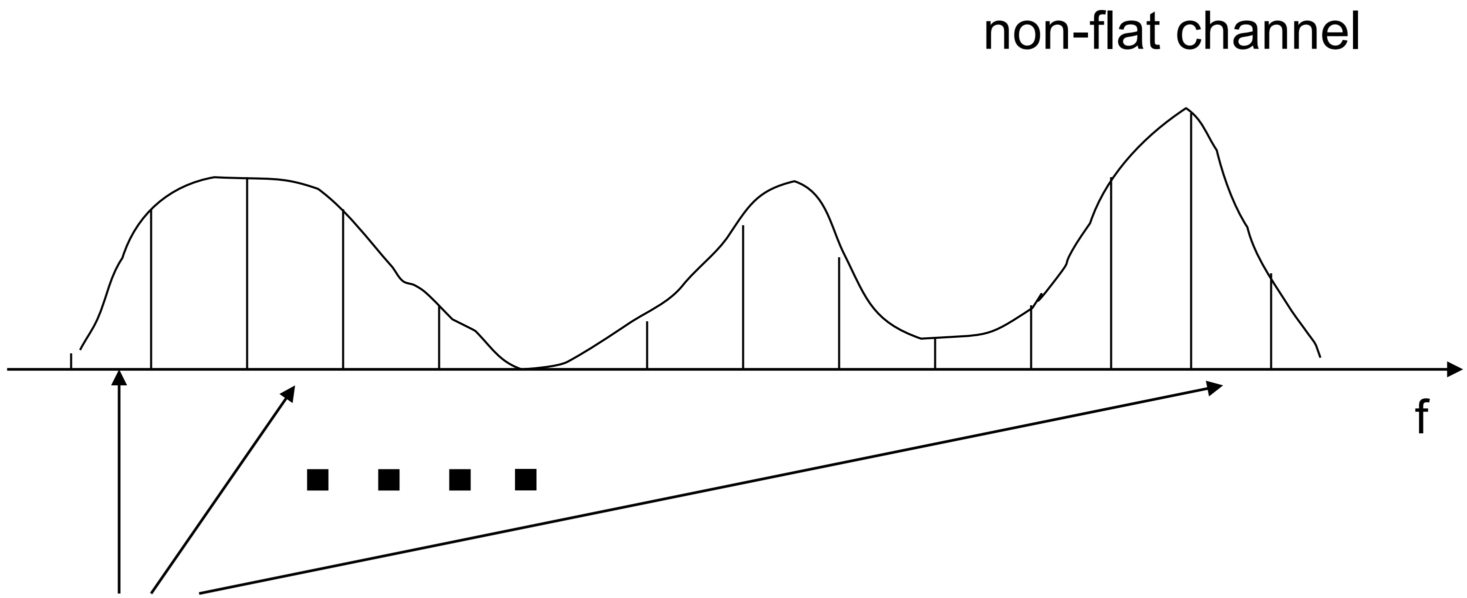


7 subcarriers



- VOFDM: vector OFDM
  - It is **NOT** Cisco's for multiple antennas, where every antenna employs OFDM
    - Cisco's is MIMO-OFDM that is a trivial concept
  - It is not trivial for single transmit antenna systems
    - **Today's focus**

Why OFDM ? -------- Rough Idea



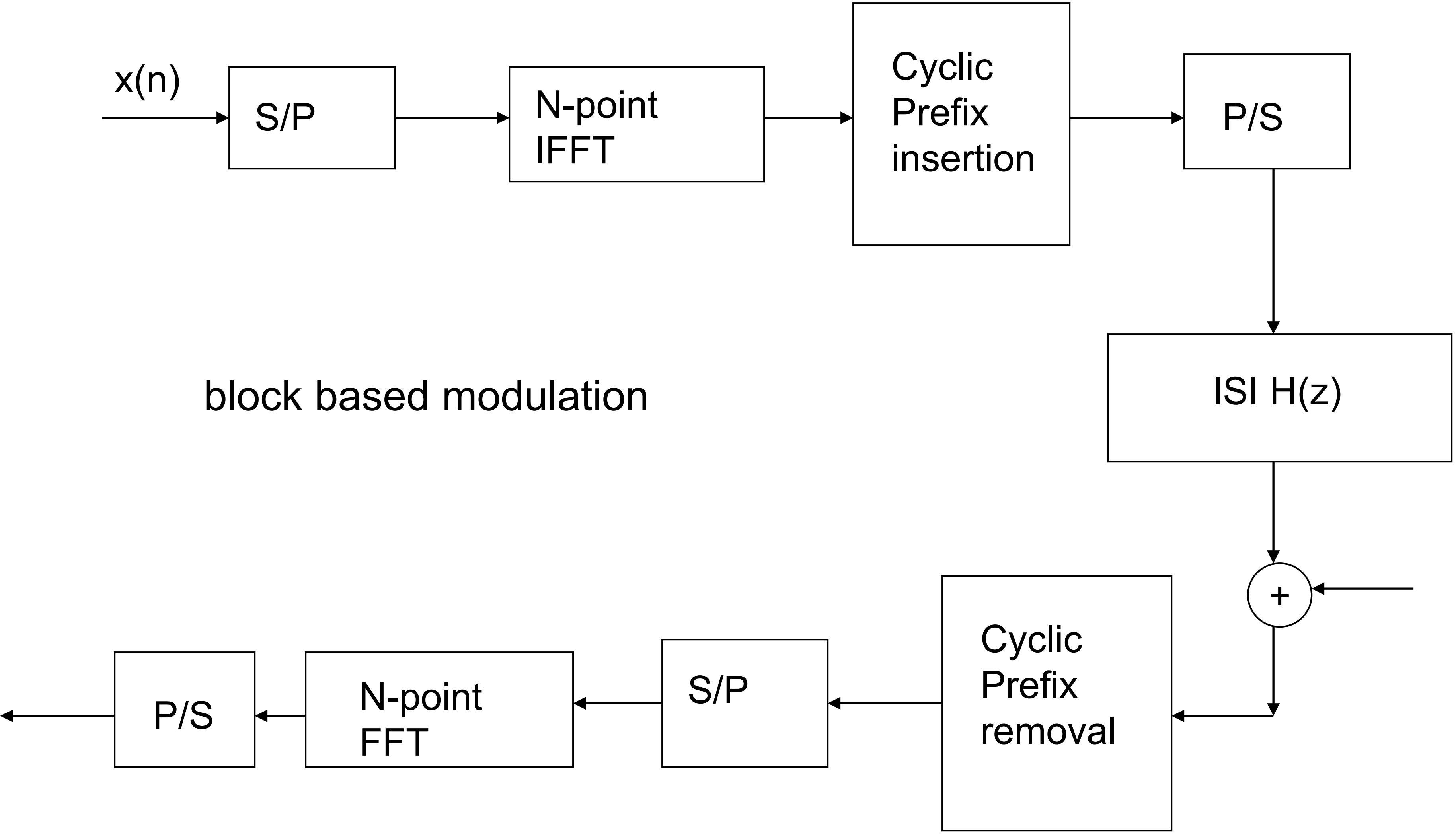
Narrow subchannels using multiple subcarriers

- These subchannels may have overlapped spectrums. So, OFDM is bandwidth efficient
  - The analog signals in these subchannels are not orthogonal each other.
  - **Their discrete/sampled signals are orthogonal each other.**

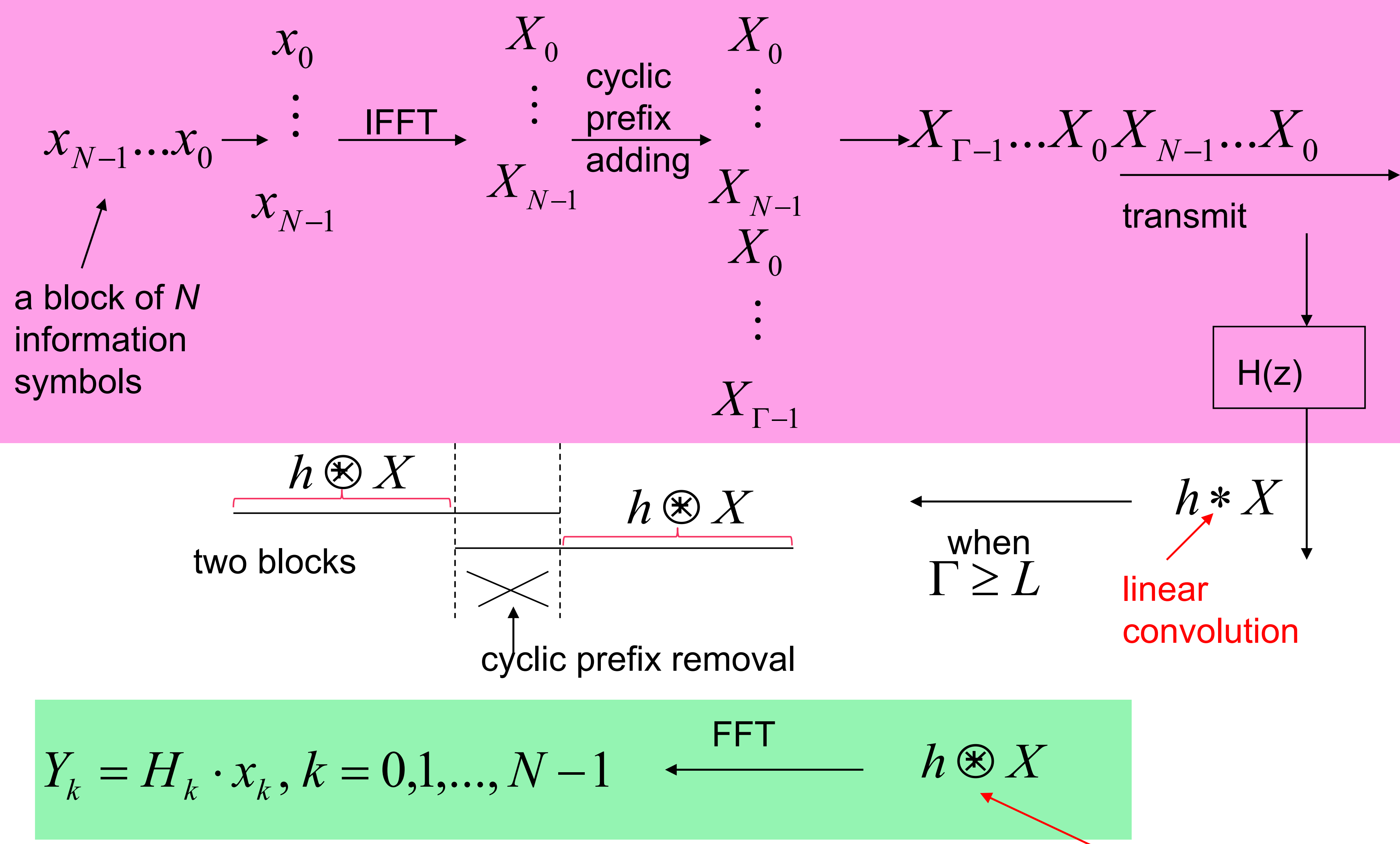
Each subchannel is narrow and therefore more flat

- It does not have ISI.

OFDM



block based modulation



$$x_{N-1}\ldots x_0 \rightarrow \begin{matrix} x_0 \\ \vdots \\ x_{N-1} \end{matrix} \xrightarrow{\text{IFFT}} \begin{matrix} X_0 \\ \vdots \\ X_{N-1} \end{matrix} \rightarrow X_{\Gamma-1}\ldots X_0 X_{N-1}\ldots X_0$$

$$h * X \quad \text{when } \Gamma \geq L \quad h \circledast X$$

$$Y_k = H_k \cdot x_k,\ k = 0,1,\ldots,N-1 \xleftarrow{\text{FFT}} h \circledast X$$

$$H_k = H(e^{j2\pi\frac{k}{N}}) = \sum_{n=0}^{L} h(n) e^{-j\frac{2\pi nk}{N}}$$

ISI channel

$$y(k) = \sum_{n=0}^{L} h(n)x(k-n) + \eta(k)$$

adding cyclic prefix as an additional data rate overhead

N ISI-free subchannels

$$Y_k = H_k \cdot x_k + \eta_k, \quad k = 0,1,\ldots,N-1$$

Each subchannel corresponds to a DFT component H_k of the ISI channel.
If the frequency component H_k is small (bad), then this subchannel is bad.

For 20 MHz Channel, OFDM

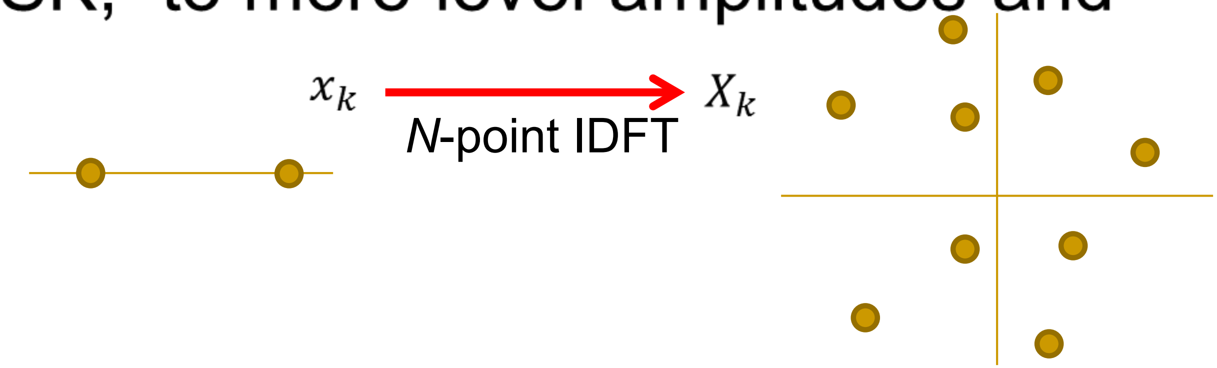
L <= 16

N=64

Γ=L=16, 25% data rate overhead

What Really Matters for OFDM??

- When the DFT size *N* is large, the CP is negligible. Although we may not have the ISI free identities $y_k = H_k x_k$, we may get the approximate ISI free equations: $y_k \approx H_k x_k$
  - CP insertion is not the most essential part of the OFDM
- OFDM converts less finite contellation, such as BPSK, QPSK, to more level amplitudes and phases



- High peak-to-average power ratio (PAPR): PAPR=*N*
  - When all signals x_k are the same, the power is N^2

Single Antenna Vector OFDM System

x_0 ⋮ x_{N-1} → N-point IFFT → X_0 ⋮ X_{N-1} → cyclic prefix adding → X_0 ⋮ X_{N-1} X_0 ⋮ $X_{\Gamma-1}$ → P/S →

scalar

OFDM, when

data rate overhead $\Gamma \geq L$

Receiver:

$$Y_k = H_k x_k + \eta_k$$

N many scalar channels/equations

$\underline{x}_0$ ⋮ $\underline{x}_{N-1}$ → N-point IFFT Component wise → $\underline{X}_0$ ⋮ $\underline{X}_{N-1}$ → cyclic prefix adding → $\underline{X}_0$ ⋮ $\underline{X}_{N-1}$ $\underline{X}_0$ ⋮ $\underline{X}_{\tilde{\Gamma}-1}$ → P/S →

M by 1 vector

VOFDM, when

data rate overhead $\tilde{\Gamma} \geq \tilde{L} \approx \frac{L}{M}$

Receiver:

$$\underline{Y}_k = \underline{H}_k \underline{x}_k + \underline{\eta}_k$$

N many M by 1 vector channels/equations

This may be a simple way to see the transmission of OTFS.

Comments on VOFDM and OTFS

- An analog pulse $g(t)$ is skipped here but can be always added to VOFDM signals in real transmission similar to OFDM.

- The CP length does not have to be exactly a multiple of the vector size *M*: $\tilde{\Gamma}M$.
  - The CP part can be truncated to any length that is not less than the channel length *L* to avoid the inter-block-interference.

- The transmission of VOFDM is exactly the same as that of OTFS.

VOFDM: Vectorized Channel

- The ISI channel *H(z)* is converted into *N* vector channels

M symbols in each vector are in ISI

$$\underline{Y}_k = \underline{H}_k \underline{x}_k + \underline{\eta}_k \ , \quad k=0,1,\ldots,N-1, \quad (1)$$

where $\underline{H}_k$ is the *M* by *M* blocked version of the original frequency responses of the ISI *H(z)*:

$$\underline{H}_k = \underline{H}(e^{j2\pi k/N}),$$

$$\underline{H}(z) = \begin{bmatrix} H_0(z) & z^{-1}H_{M-1}(z) & \cdots & z^{-1}H_1(z) \\ H_1(z) & H_0(z) & \cdots & z^{-1}H_2(z) \\ \vdots & \vdots & \vdots & \vdots \\ H_{M-1}(z) & H_{M-2}(z) & \cdots & H_0(z) \end{bmatrix}$$

$$H_m(z) = \sum_{l=0}^{\tilde{L}'} h(Ml+m)z^{-l}, \quad 0 \le m \le M-1.$$

*m*th polyphase component of *H(z)*

$$\tilde{L}' = \left\lfloor \frac{L}{M} \right\rfloor$$

Vectorized Channel Example

If $H(z) = 1 + 0.9z^{-1} - 0.8z^{-2} + 0.6z^{-3} + 0.5z^{-4} - 0.4z^{-5}$, vector size M=2,

then, its polyphase components are

$$H_0(z) = 1 - 0.8z^{-1} + 0.5z^{-2}, \quad H_1(z) = 0.9 + 0.6z^{-1} - 0.4z^{-2}$$

and the vector channel coefficient matrices are

$$L = 5$$

$$\tilde{L} = \left\lceil \frac{L}{M} \right\rceil = \left\lceil \frac{5}{2} \right\rceil = 3$$

$$\underline{H}(z) = \begin{bmatrix} H_0(z) & z^{-1}H_1(z) \\ H_1(z) & H_0(z) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0.9 & 1 \end{bmatrix} + \begin{bmatrix} -0.8 & 0.9 \\ 0.6 & -0.8 \end{bmatrix} z^{-1} + \begin{bmatrix} 0.5 & 0.6 \\ -0.4 & 0.5 \end{bmatrix} z^{-2} + \begin{bmatrix} 0 & -0.4 \\ 0 & 0 \end{bmatrix} z^{-3}$$

Why VOFDM Is Good for Channels with Doppler Spread

- The vectorized channel matrix $\underline{H}(z)$ is pseudo-circulant and can be diagonalized by *M*-point DFT matrix $\mathbf{W}_M$ with a diagonal phase shift matrix $\Lambda(z) = \mathrm{diag}(1, z^{-1}, \cdots, z^{-M+1})$ as follows:

$$\underline{H}(z^M) = \left(\mathbf{W}_M^* \Lambda(z)\right)^{-1} \mathrm{diag}(H(z), H(zW_M), \cdots, H(zW_M^{M-1}))\mathbf{W}_M^* \Lambda(z)$$

where $W_M = e^{-j\frac{2\pi}{M}}$. Thus, matrices $\underline{H}_k = \underline{H}(W_N^{-k})$ can be diagonalized by $\mathbf{W}_M^* \Lambda(W_N^{-k})$, *k=0,1,…,N-1*.

- The receiver equation (1) becomes

$$\underline{\tilde{Y}}_k = \mathrm{diag}(H(W_{MN}^{-k}), H(W_{MN}^{-k} W_M), \ldots, H(W_{MN}^{-k} W_M^{M-1})) \quad \mathbf{W}_M^* \mathrm{diag}(1, W_{MN}^{k}, \ldots, W_{MN}^{k(M-1)}) \underline{x}_k + \underline{\tilde{\eta}}_k$$

This frequency domain part is similar to the channel in time domain for single antenna systems **Or** diagonal space-time coded MIMO systems

This part is similar to the precoding to achieve signal space diversity for time-varying channel **Or** diagonal space-time block coding to achieve spatial diversity

When channel varies with Doppler spread, it can collect multipath diversity and/or signal space diversity. This can be seen later even with the MMSE receiver.

VOFDM vs OTFS

- The VOFDM receiver equation to demodulate

$$\underline{Y}_k = \underline{H}_k \underline{x}_k + \underline{\eta}_k$$

  - It coincides with that of OTFS when the channel is stationary/quasi-static at both Tx and Rx.

Y. Ge, Q. Deng, P. C. Ching, and Z. Ding, “OTFS Signaling for Uplink NOMA of Heterogeneous Mobility Users,” *IEEE Trans. on Commu.*, vol. 69, no. 5, pp. 3147-3161, May 2021.

P. Raviteja, Y. Hong, and E. Viterbo, “OTFS Performance on Static Multipath Channels”, *IEEE Wireless Commu. Lett*., vol. 8, no. 3, pp. 745 – 748, 2019.

- In fact, the transmission of OTFS is the same as that of VOFDM, no matter the channel is stationary or not.
  - **The transmitted signals of OTFS and VOFDM are the same in either discrete-time sequence or continuous-time waveform.**
  - **VOFDM has also been studied over time-varying channels in my book *Modulated Coding for Intersymbol Interference Channels*, New York, Marcel Dekker, 2000: Section 7.4**
- Some other names proposed later in the literature:

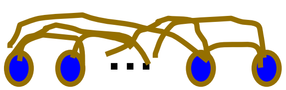
OSDM, Quadrature OFDMA (or A-OFDM)

VOFDM, OFDM, SC-FDE

- When M=1, VOFDM=OFDM
- When M>L and the FFT size N is 1, VOFDM=SC-FDE:
  - at the transmitter, no IFFT is implemented (so the PAPR is not changed) but just CP of the information symbols is inserted; low PAPR.
  - at the receiver, both FFT and IFFT, and frequency domain equalizer are implemented.
- VOFDM is a bridge between OFDM and SC-FDE.
  - Its ML receiver complexity is also in the middle.

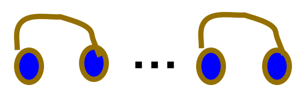
Time domain single carrier vs. equalization

Maximum # symbols in ISI



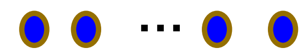
VOFDM

No, or 2, or 3, …, or Maximum # (**you choose**) symbols in ISI



Frequency domain OFDM

No ISI



Single antenna VOFDM is in the middle between single carrier and OFDM in terms of dealing with ISI

VOFDM: Some Other Advantages

- Cyclic prefix data rate overhead reduction when the FFT/IFFT size is fixed
  - For OFDM, it is $\frac{L}{N}$
  - For VOFDM, it is $\frac{L}{MN}$
- For fixed cyclic data rate overhead, the FFT/IFFT size can be reduced by M times
  - The IFFT size reduction reduces the peak-to-average power ratio (PAPR), which is important in cellular communications.

VOFDM: Some Other Advantages

- VOFDM can be combined with matrix modulation: at the receiver $\underline{Y}_k = \underline{H}_k \underline{x}_k + \underline{W}_k$ where $\underline{x}_k$ are vectors of information bits or symbols. By grouping two vectors of size 2 together considering BPSK for each information symbol, these vectors become 16 matrices

$$\left\{ \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} : x_{ij} \in \{1, -1\} \right\}$$

- These 16 matrices are not good in terms of matrix modulation due to the channel matrices have random components (fading).
  - These 16 matrices can be replaced by the ones with the best known diversity product.

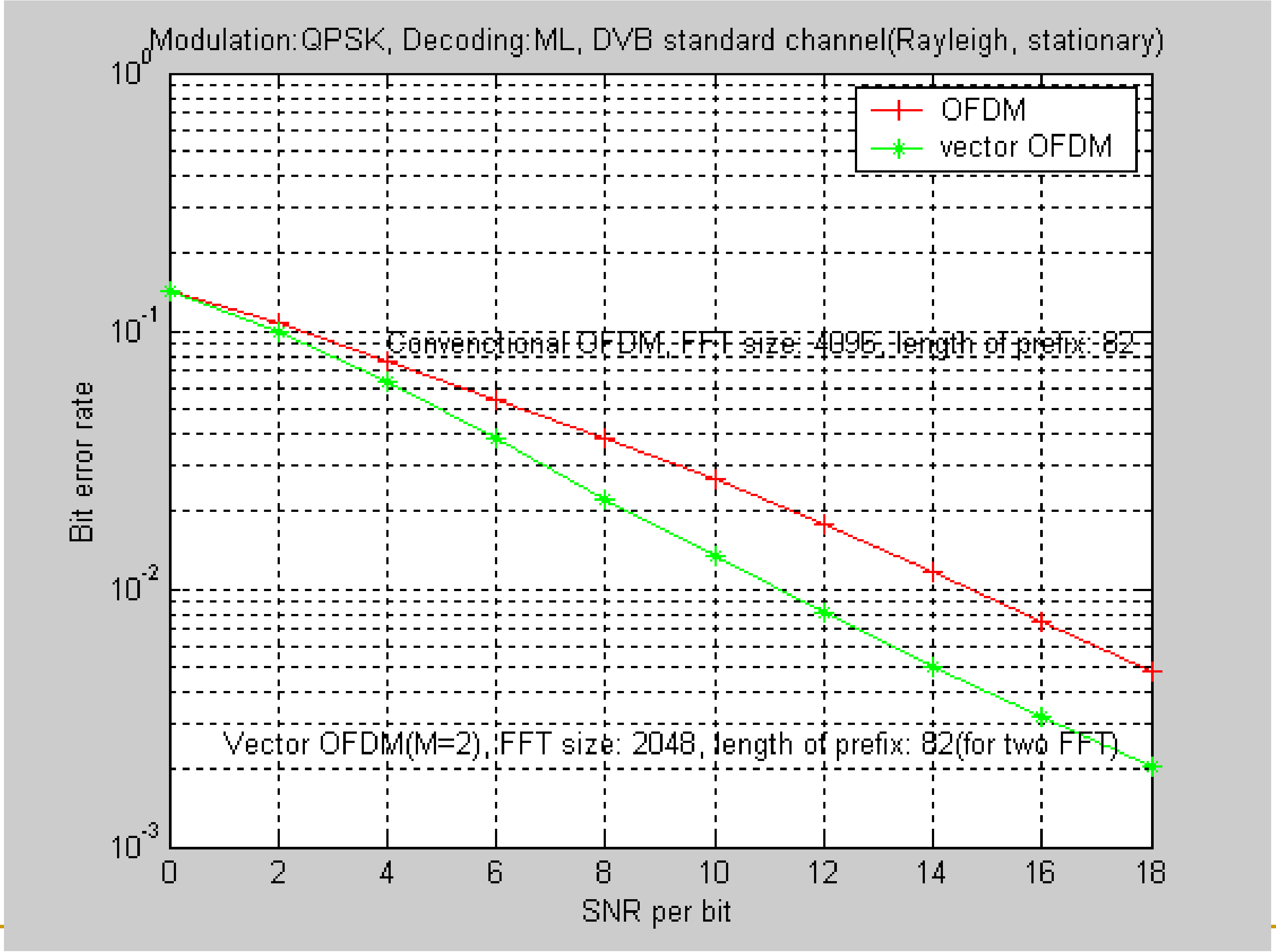
Unitary Matrix Modulation

- 16 best known 2 by 2 unitary matrices in the literature (Liang-Xia, IEEE Trans. Information Theory, Aug. 2002) with the best known diversity product (*the minimum absolute value of all the determinants of difference matrices of any two distinct matrices*):

$$\left\{ \begin{bmatrix} e^{jl\pi/8} & 0 \\ 0 & e^{jl3\pi/8} \end{bmatrix} \begin{bmatrix} \cos\frac{l\pi}{2} & \sin\frac{l\pi}{2} \\ -\sin\frac{l\pi}{2} & \cos\frac{l\pi}{2} \end{bmatrix} \begin{bmatrix} e^{jl\pi/4} & 0 \\ 0 & e^{-jl\pi/4} \end{bmatrix} : 0 \le l \le 15 \right\}$$

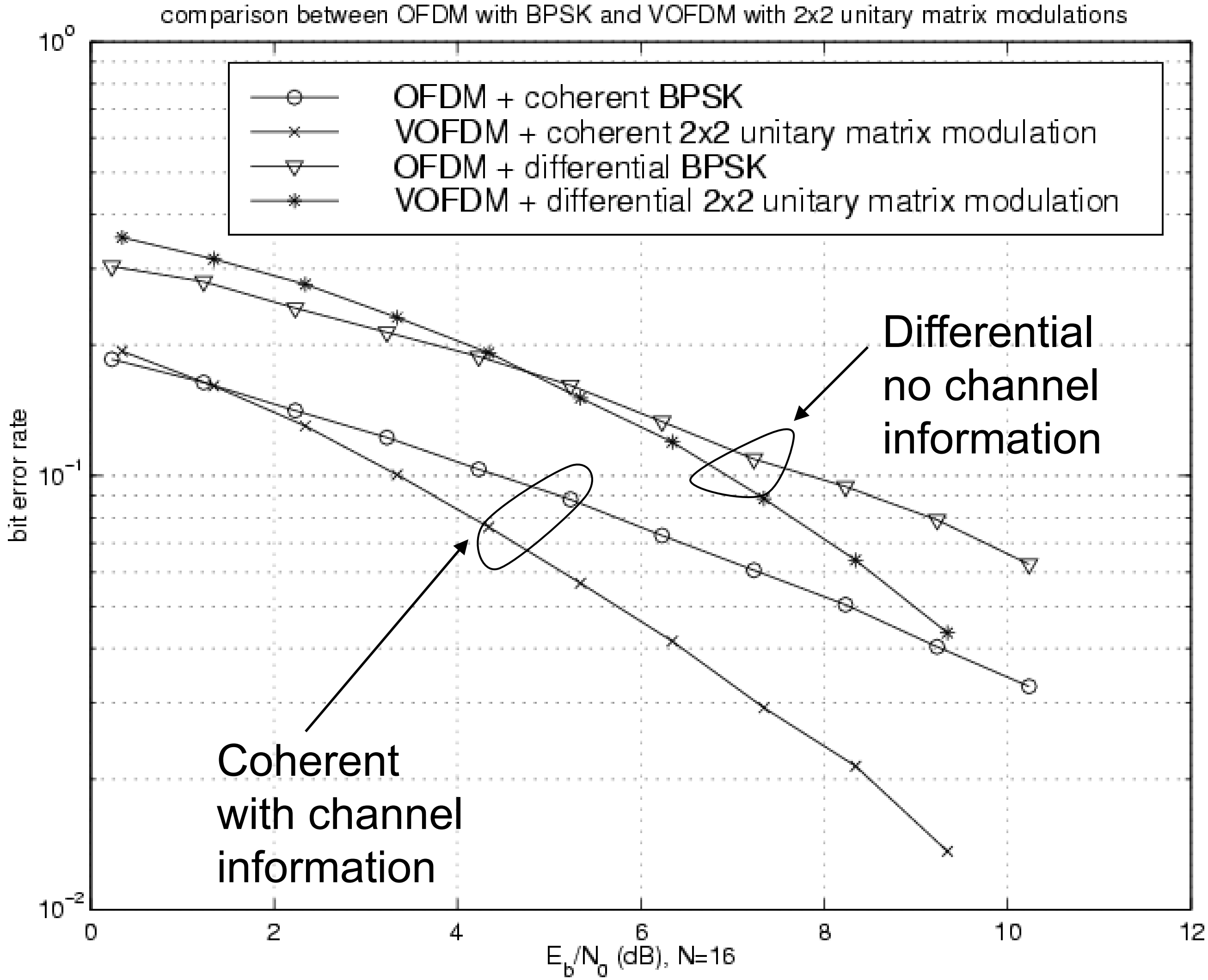
Simulations

DVB



CP data rate overhead is the same for the two curves, matrix modulation is not used.

ML receivers



Linear Receivers for Single Antenna VOFDM (Yabo Li, Ngebani, Xia and Host-Madsen, *IEEE Trans. on Signal Processing*, Oct. 2012)

- Zero-Forcing (ZF) receiver
- Minimum mean square error (MMSE) receiver

Detection SNR Gap Between ZF and MMSE Receivers

Theorem *1: Denote the m-th column of* $\mathbf{H}_l$ *as* $\mathbf{h}_{l,m}$ *and the matrix of* $\mathbf{H}_l$ *after deleting the m-th column as* $\mathbf{H}_{l,m}$, *which is an* $M \times (M-1)$ *matrix. When* $\rho \to \infty$, *the gap between the detection SNRs of the ZF-V-OFDM and the MMSE-V-OFDM can be written as*

$$\lim_{\rho \to \infty} \left(\rho_l^{MMSE} - \rho_l^{ZF} \right) = \left\| \mathbf{h}_l^H \mathbf{H}_{l,m} \left(\mathbf{H}_{l,m}^H \mathbf{H}_{l,m} \right)^{-1} \right\|^2, \tag{12}$$

which is independent of m. As $\rho \to \infty$, *the ratio between* ρ_l^{MMSE} *and* ρ_l^{ZF} *approaches* 1, *i.e.,*

$$\lim_{\rho \to \infty} \frac{\rho_l^{MMSE}}{\rho_l^{ZF}} = 1.$$

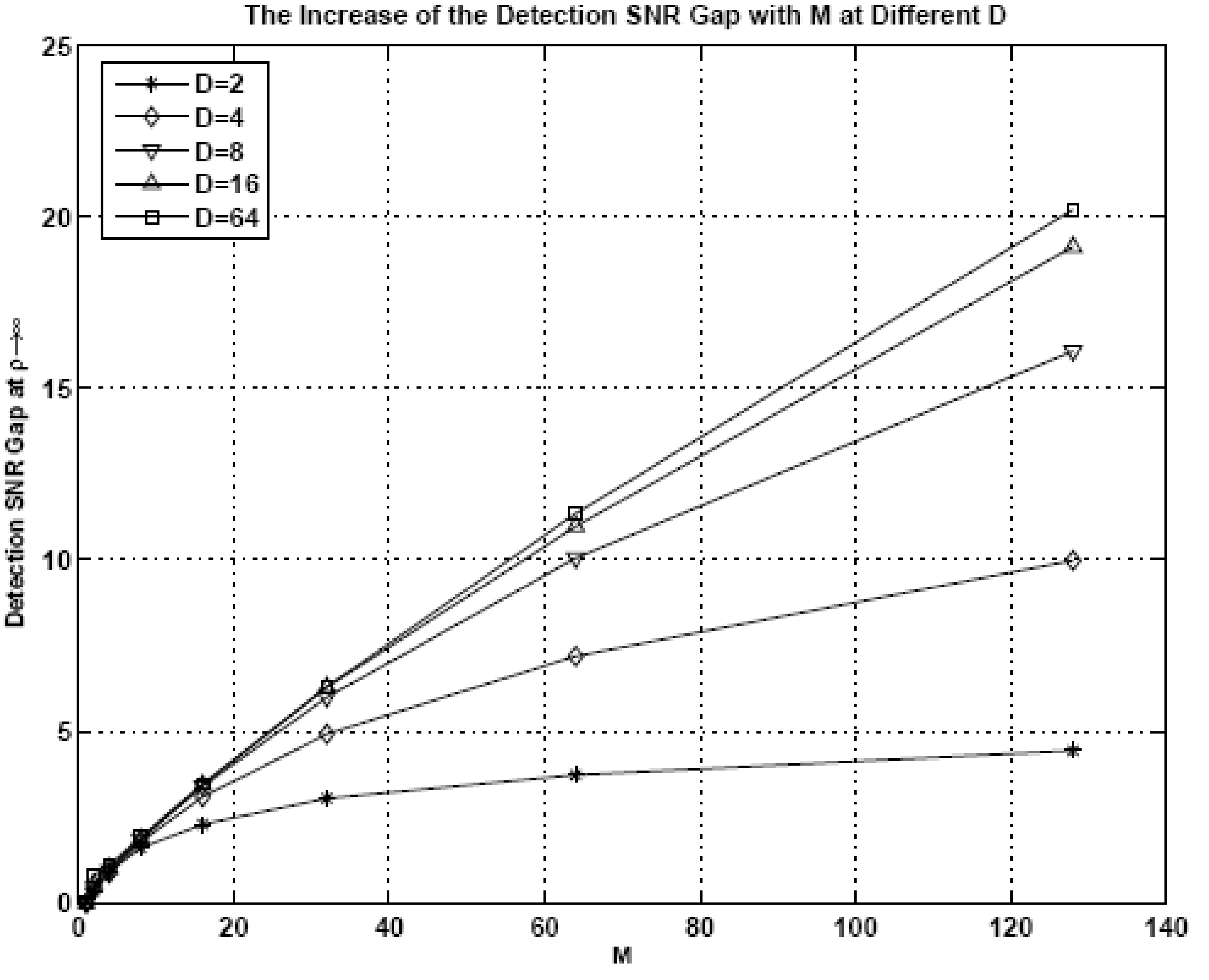
Detection SNR Gap Between ZF and MMSE Receivers

- For V-OFDM, the SNR gap between ZF and MMSE detections doesn’t approach zero as SNR approaches infinity.

- On average, the performance gap increases with the vector block (VB) block size *M* and the maximum delay *L (*or *D* used as below*)* of the channel.

Detection SNR Gap Between ZF and MMSE Receivers

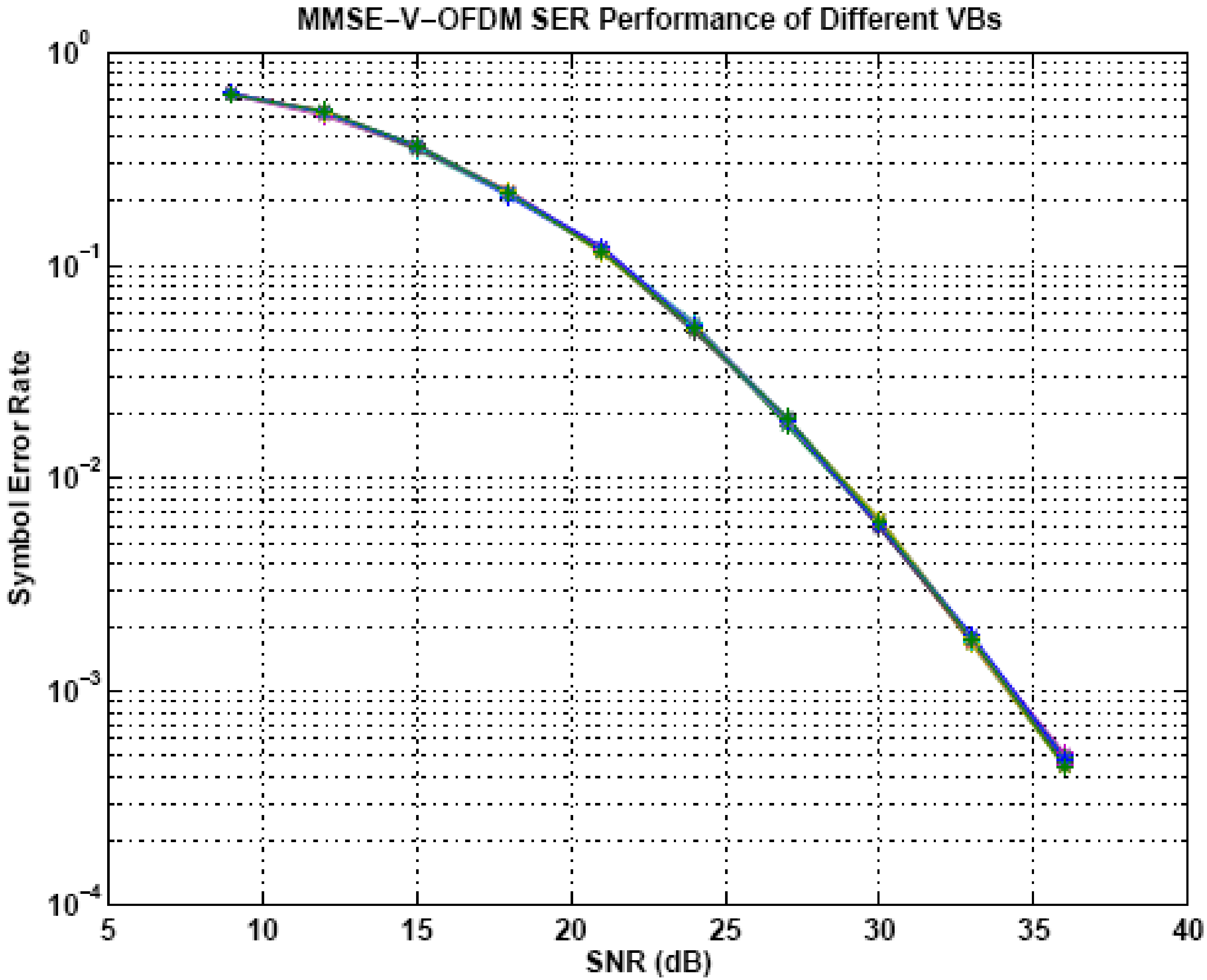
D=L



The Performance Independence of Vector Block Index

- **Theorem 2**: For ZF-VOFDM and MMSE-VOFDM, after averaging over all the channel, the *NM* transmitted symbols have the same error rate performance.
  - For VOFDM with ML receiver (i.e., ML-VOFDM), different VBs may have different performances (See Han *et al* 2010 and Cheng *et al.* 2011).
  - However, for VOFDM with ZF and MMSE receivers, all the VBs have the same performance.

The Performance Independence of Vector Block Index



Diversity Order of MMSE and ZF Receivers

D=L

- Definition of the diversity order

$$d\left(R,M,D,N\right) = -\lim_{\rho\to\infty}\frac{\log P_{ser}(R,M,D,N)}{\log\rho}.$$

  - *R* is the spectrum efficiency defined as bits/sec/Hz

- **Theorem** 5: *For MMSE-V-OFDM, the diversity* $d^{MMSE}(R,M,D,N)$ *equals*

$$d^{MMSE}(R,M,D,N) = \min\left\{\lfloor M2^{-R}\rfloor, D\right\} + 1.$$

- **Theorem** 6: *For ZF-V-OFDM, the diversity order* $d^{ZF}(R,M,D,N) = 1.$

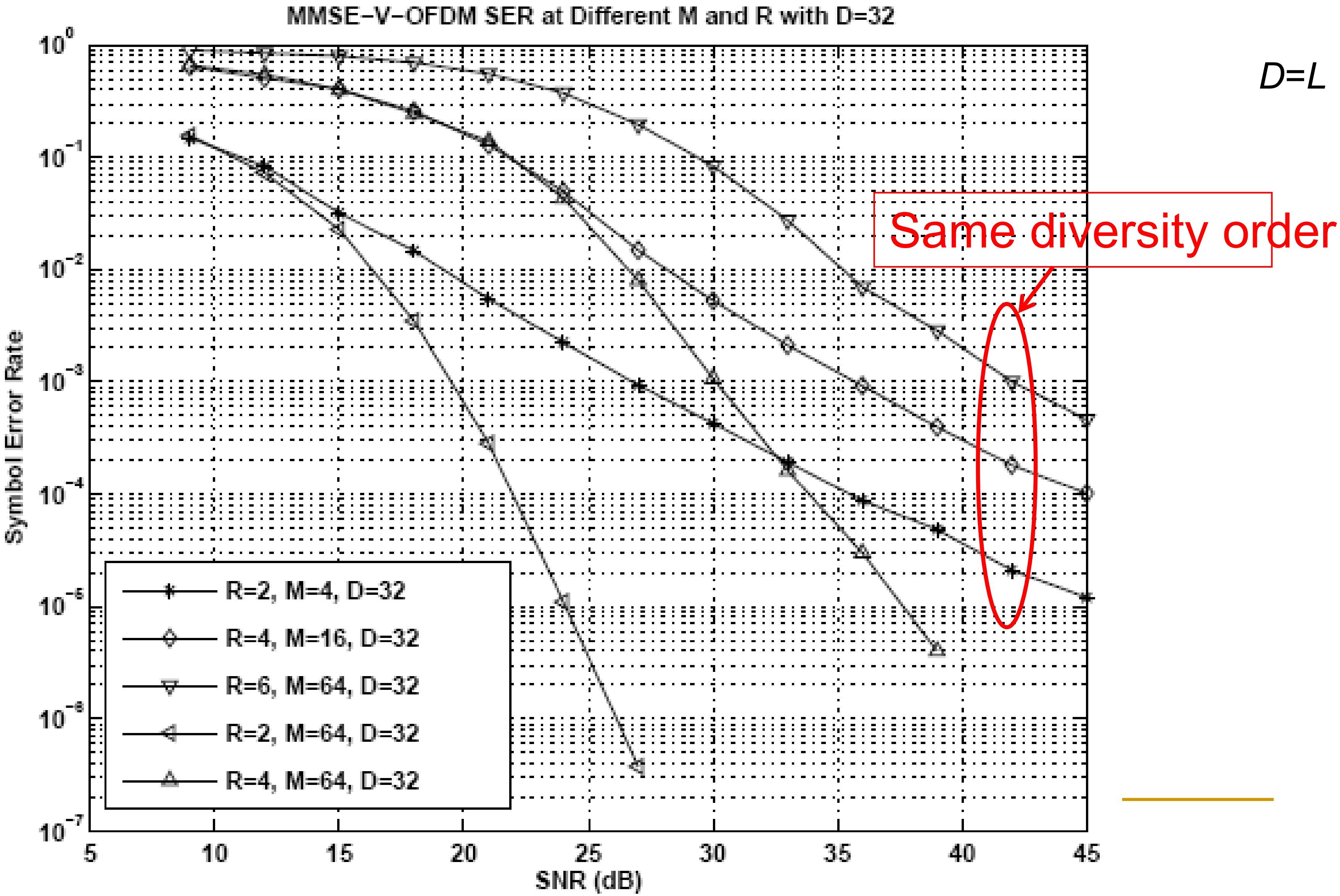
Diversity Order of MMSE and ZF Receivers

- Both ZF and MMSE detections are scalar detections, they have the similar complexities.
- However, the MMSE detection can exploit the diversity inside the VOFDM, while ZF detection cannot.
- The only required extra information for MMSE detection is the channel SNR, which can be obtained at the receiver.

Diversity Order of MMSE Receiver

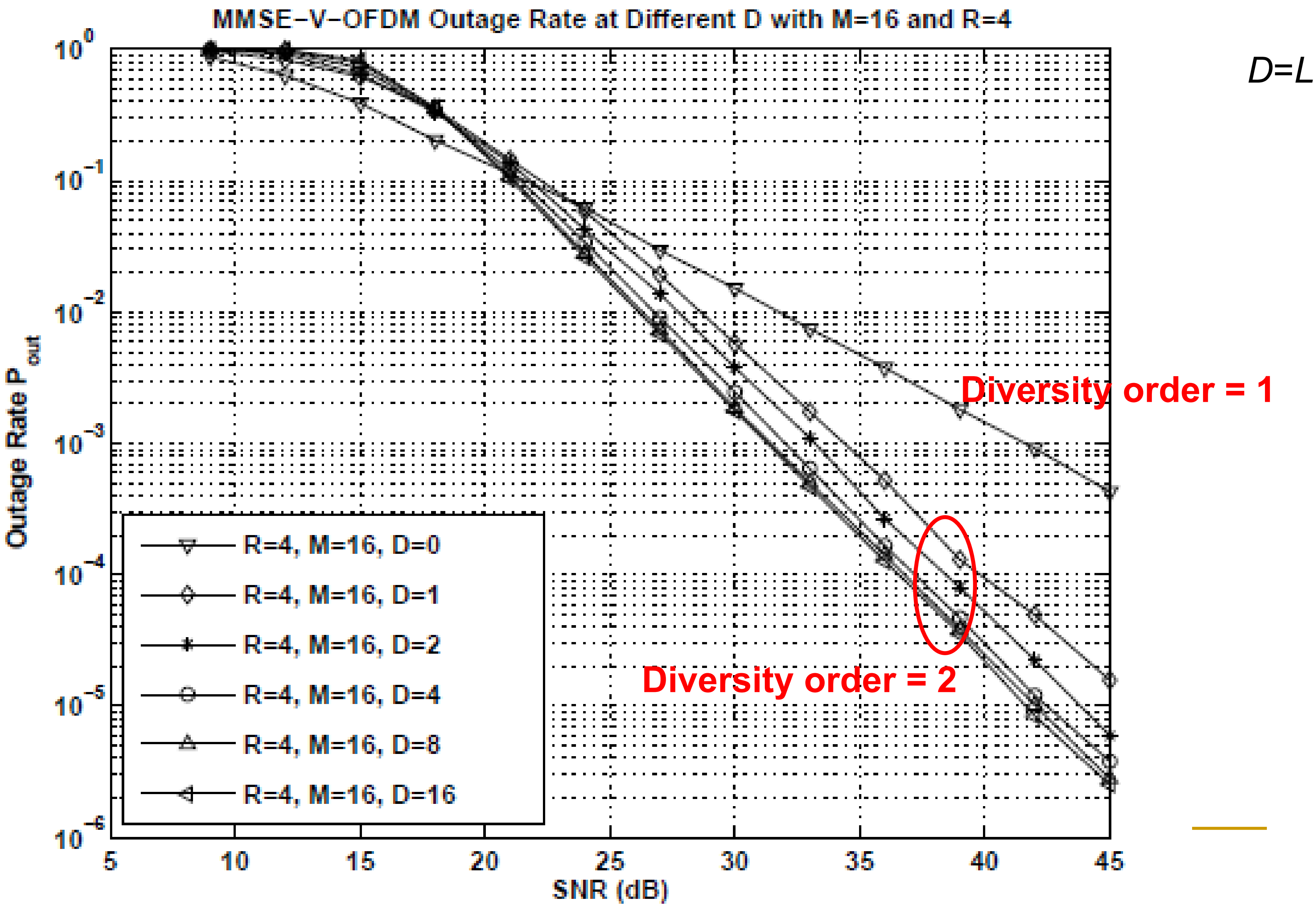
$$\text{diversity order} = \min\{\lfloor M 2^{-R} \rfloor, D\} + 1$$

D=L



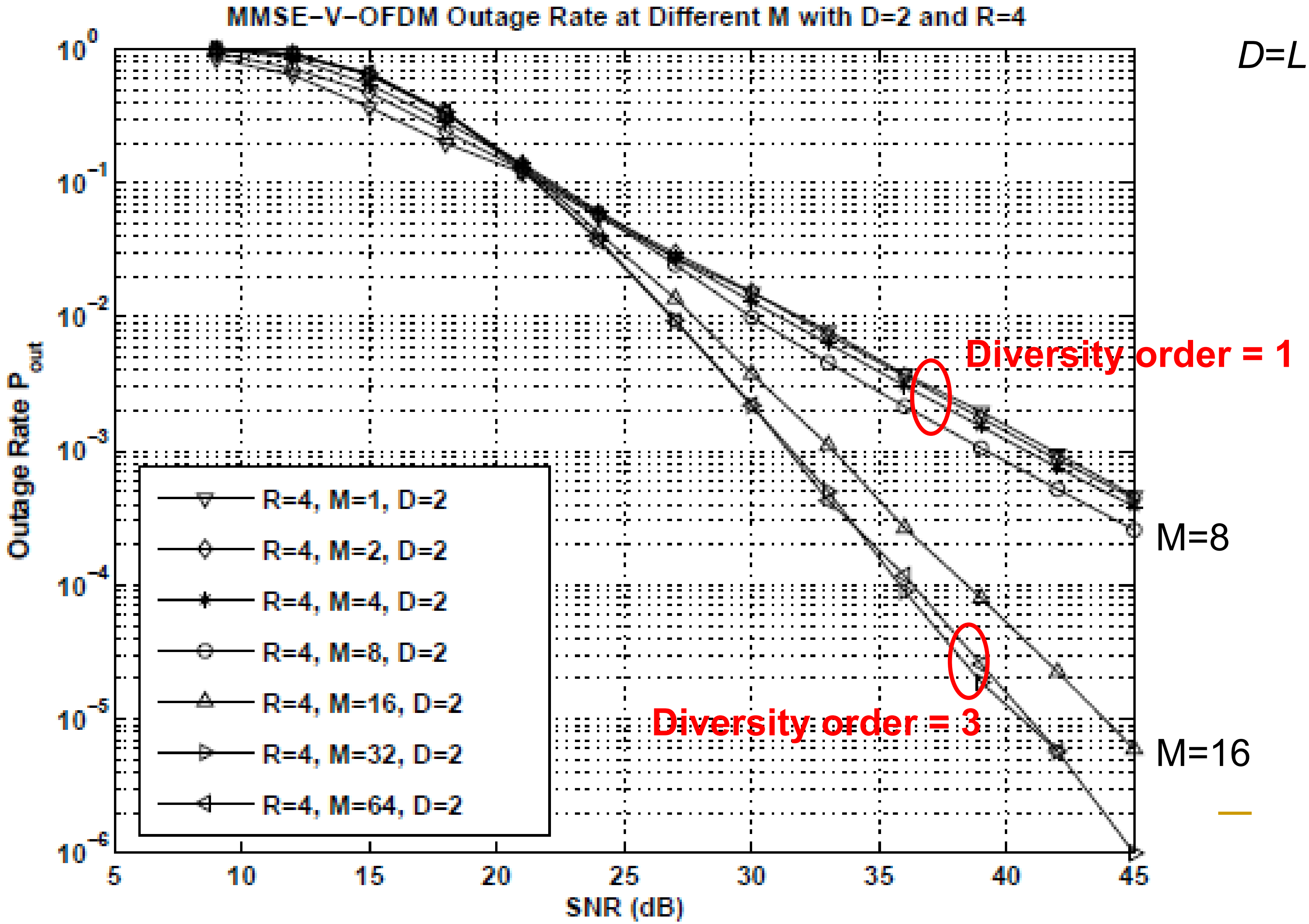
Diversity Order of MMSE Receiver

$$\text{diversity order} = \min\left\{\left\lfloor M 2^{-R} \right\rfloor, D\right\} + 1$$



Diversity Order of MMSE Receiver

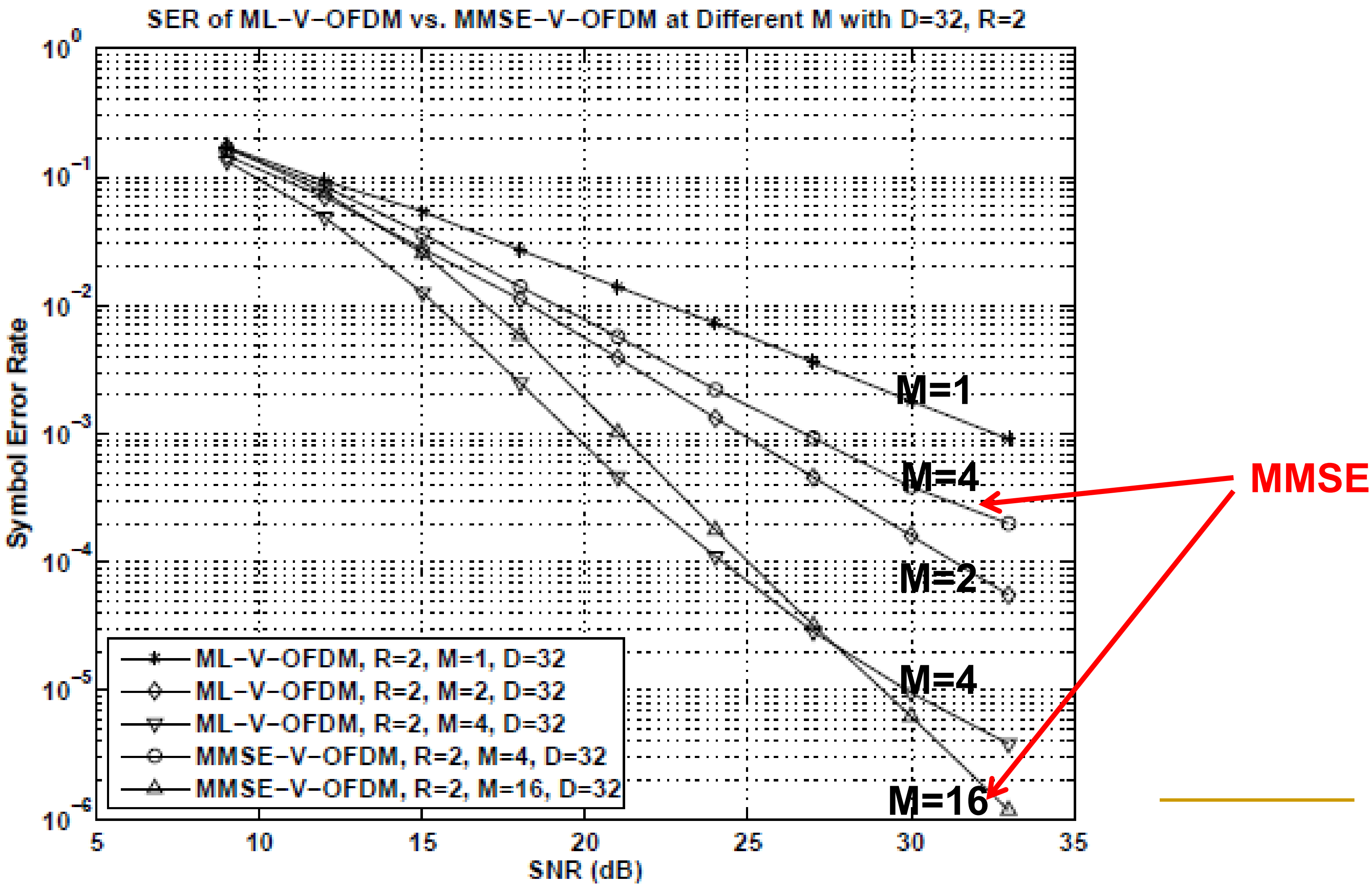
$$\text{diversity order} = \min\{\lfloor M 2^{-R} \rfloor, D\} + 1$$



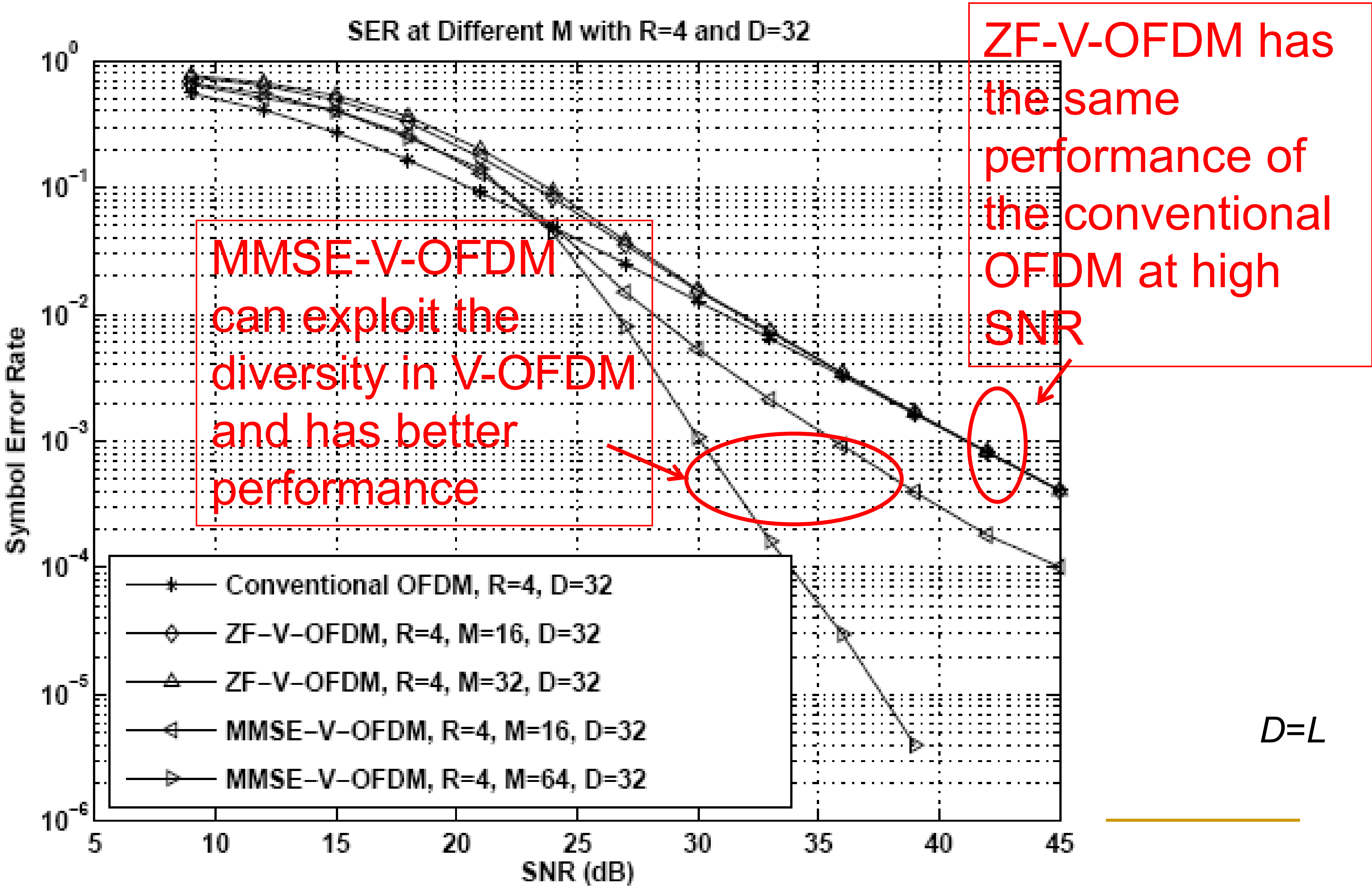
Performances for ML and MMSE Receivers

D=32, R=2

D=L



Diversity Order of ZF Receiver



D=L

Multiple Antenna VOFDM Using Cyclic Delay Diversity (CDD)

- CDD can be used to collect both spatial and multipath diversities in a MIMO-OFDM systems

$$\overrightarrow{h_{11}, h_{12}, \cdots, h_{1L}}$$

$$\vdots$$

$$\overrightarrow{h_{n_t 1}, h_{n_t 2}, \cdots, h_{n_t L}}$$

After CDD

It is equivalent to

$$\overrightarrow{h_{11}, \cdots, h_{1L}, \cdots\cdots, h_{n_t 1}, \cdots, h_{n_t, L}}$$

if $N \geq n_t L$

When the bandwidth is larger, the number *L* of multipaths will be larger too. Then, CDD in this case may not be able to collect full spatial and multipath diversities anymore.

Multiple Antenna VOFDM Using Cyclic Delay Diversity (CDD)

- CDD VOFDM can collect both spatial and multipath diversities despite of a large bandwidth

$$\underrightarrow{H_{11}, H_{12}, \cdots, H_{1\frac{L}{M}}}$$

$$\vdots$$

$$\underrightarrow{H_{n_t 1}, H_{n_t 2}, \cdots, H_{n_t \frac{L}{M}}}$$

After CDD $\longrightarrow$

$$\underrightarrow{H_{11}, \cdots, H_{1\frac{L}{M}}, \cdots\cdots, H_{n_t 1}, \cdots, H_{n_t \frac{L}{M}}}$$

It is equivalent to

if $N \geq n_t \frac{L}{M}$

The number of multipaths is equivalently reduced by *M* times for VOFDM with a vector size *M*

Conclusion and Future Research

- VOFDM can be used either to reduce the PAPR by reducing the IFFT size while at a fixed the CP data rate overhead; or reduce the CP data rate overhead while at a fixed the IFFT size.
- VOFDM provides a tradeoff between the receiver complexity, performance, PAPR, CP overhead for an ISI channel.
- VOFDM is in the middle between single carrier and OFDM systems in terms of dealing with ISI.
- The transmitted signals of OTFS and VOFDM are identical.
  - **Good for channels with both time and Doppler spreads.**
  - **VOFDM was also studied over time-varying channels in Section 7.4:**

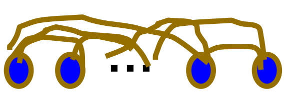
  X.-G. Xia, *Modulated Coding for Intersymbol Interference Channels*, New York, Marcel Dekker, 2000.
- CDD VOFDM for multi-antennas can collect both spatial and multipath diversities, where CDD OFDM is not be able to do so in a large bandwidth system.

Recall Physical Layer Communications Developments in Recent Decades for Both Wireless and Wired Systems

- It has been always on dealing with ISI

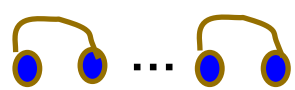
Time domain single carrier vs. equalization

Maximum # symbols in ISI



VOFDM

No, or 2, or 3, …, or Maximum # (**you choose**) symbols in ISI



Frequency domain OFDM

No ISI

Is this VOFDM something to think about after OFDM?

Or what's next???

References

[1] X.-G. Xia, "Precoded and Vector OFDM Robust to Channel Spectral Nulls and with Reduced Cyclic Prefix Length in Single Transmit Antenna Systems," *IEEE Trans. on Communications,* vol. 49, pp.1361-1374, Aug. 2001.

[2] H. Zhang, X.-G. Xia, L. J. Cimini, and P. C. Ching, "Synchronization techniques and guard-band-configuration scheme for single-antenna vector-OFDM systems," *IEEE Trans. on Wireless Communications*, vol. 4, no. 5, pp. 2454-2464, Sept. 2005.

[3] H. Zhang and X.-G. Xia, "Iterative decoding and demodulation for single-antenna vector OFDM systems," *IEEE Trans. on Vehicular Technology*, vol. 55, no. 4, pp. 1447-1454, Jul. 2006.

[4] C. Han, T. Hashimoto, and N. Suehiro, "Constellation-rotated vector OFDM and its performance analysis over Rayleigh fading channels," *IEEE Trans. on Communications*, vol. 58, no. 3, pp. 828-837, Mar. 2010.

[5] P. Cheng, M. Tao, Y. Xiao, and W.-J. Zhang, "V-OFDM: On performance limits over multi-path Rayleigh fading channels," *IEEE Trans. on Communications*, vol. 59, no. 7, pp. 1878-1892, Jul. 2011.

[6] Y. Li, I. Ngebani, X.-G. Xia, and A. Host-Madsen, "On performance of vector OFDM with linear receivers," *IEEE Trans. on Signal Processing*, Oct. 2012.

[7] X.-G. Xia, *Modulated Coding for Intersymbol Interference Channels*, New York, Marcel Dekker, 2000.

[8] X.-G. Xia, "The transmitted signals of OTFS and VOFDM are the same," *IEEE Trans. on Wireless Commun.*, 2022, doi: 10.1109/TWC.2022.3190437.

Thank you!