The Transmitted Signals of OTFS and VOFDM Are Exactly the Same

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Abstract

This short note connects OTFS and single transmit antenna VOFDM systems and explains why VOFDM is good to time varying channels with Doppler spread, which also explains why OTFS is so, as well.

Main Description

It is not hard to check that the transmitted signals in either discrete or continuous time format of OTFS [7] and single transmit antenna vector OFDM (VOFDM) [1, 2] are the same. For the discrete time format, for convenience, assume that the pulse g(t) is rectangular. Then, the transmitted sequences of OTFS and VOFDM are the same. If a more general continuous time pulse g(t) is used, the transmitted signal waveform of VOFDM is the same as that of OTFS, i.e., formula (5) in [7], no matter whether channel is stationary or time-varying. Note that the cyclic prefix for VOFDM does not have to be a multiple of vectors and it can be a truncated sequence of length not smaller than the channel length in order to have free interference across vector subchannels [4].

Recently it has been claimed that OTFS is good to deal with Doppler spread for time-varying channels [7]. From the VOFDM point of view, it has been shown in [1, 2, 3, 4] that VOFDM can achieve multipath diversity and/or signal space diversity, even with the MMSE linear receiver in a vectorized subchannel [4]. This is because in VOFDM, at transmit side, a vector of information symbols is DFT (or IDFT) transformed implicitly and then, at receive side, the information symbols in this vector are demodulated together. More specifically, since in VOFDM, a vectorized channel matrix is pseudo-circulant, it can be diagonalized by DFT/IDFT matrix with a phase shift diagonal matrix, see formula (4.1) in [2]. Then, this DFT (or IDFT) of a vector of information symbols is similar to the precoding in single antenna systems to collect signal space diversity to combat wireless fading (Doppler effect) [5] or diagonal space-time block coding in MIMO systems to collect spatial diversity [6]. This can be seen in [3] with a simplified demodulator as well. We believe that it is the main reason why OTFS (or VOFDM) is good to deal with a time-varying channel with both Doppler and time spreads.

In [10], a more general setting than VOFDM has been studied, where a channel independent precoder G, a matrix of size $M \times K$ with $K \leq M$, is used at the transmitter, where the precoder has K input information symbols and produces M output symbols to transmit. The simplest precoder studied in [10] is the identity matrix (or the $M \times K$ submatrix of the $M \times M$ identity matrix), and when

M = K, it is VOFDM. In [10], this general setting is studied for time-varying channels with both time and Doppler spreads (see the channel equation (7.4.1) on page 172 of [10]) in both theory and simulations (see Section 7.4 of [10] that is attached).

Furthermore, single transmit antenna VOFDM is a bridge between OFDM and single carrier frequency domain equalizer (SC-FDE) [4]. VOFDM converts an intersymbol interference (ISI) channel to multiple vectorized subchannels and there is no ISI across these vectorized subchannels. In each vectorized subchannel, the information symbols inside an information symbol vector may interfer each other, i.e., they may have ISI, but the length of ISI is limited to the vector size. When the vector size is 1 in VOFDM, it is OFDM. In this case, there is no ISI even in each subchannel. When the vector size is not less than a channel length and IFFT size is 1, it is SC-FDE. In this case, the length of ISI is the same as the length of the ISI channel.

For more details about single transmit antenna VOFDM, please see [2], [4]. For quasi-static channels (or stationary channels), it has been already shown in [8], [9] that OTFS is equivalent to VOFDM. Since quasi-static channel is arbitrary, when it is the ideal channel, i.e., $\delta(t)$, this equivalence also implies that the transmitted signals of OTFS and VOFDM are equivalent.

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