Modulation Designs for Multiple Antennas with Low Complexity Receivers

Xiang-Gen Xia

Department of Electrical and Computer Engineering
University of Delaware
Newark, DE 19716, USA
and
Chonbuk National University
Jeonju, South Korea

Email: xxia@ee.udel.edu

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Yue Shang, Xiaoyong Guo, Wei Zhang, Tianyi Xu

Outline

- Background and Preliminaries
- Linear Receivers
  - Design Criterion
  - Design Examples
- PIC Group Decoding Algorithm
  - Design Criterion
  - Design Examples
- Summary and Future Research

From Wired Modems to Wireless Modems

Wired phones → (In the last twenty years of the last century) → Wired computer modems

Wireless cell-phones → (Now) → Wireless modems

Computer Modems

	Modulation	Bitrate [kbit/s]	Year Released
110 baud Bell 101 modem	FSK	0.1	1958
300 baud (Bell 103 or V.21)	FSK	0.3	1962
1200 modem (1200 baud) (Bell 202)	FSK	1.2	
1200 Modem (600 baud) (Bell 212A or V.22)	QPSK	1.2	1980
2400 Modem (600 baud) (V.22bis)	QAM, TCM	2.4	1984
2400 Modem (1200 baud) (V.26bis)	PSK	2.4	
4800 Modem (1600 baud) (V.27ter)	PSK	4.8	
9600 Modem (2400 baud) (V.32)	QAM, TCM	9.6	1984
14.4k Modem (2400 baud) (V.32bis)	QAM, TCM	14.4	1991
28.8k Modem (3200 baud) (V.34)	QAM, TCM	28.8	1994
33.6k Modem (3429 baud) (V.34)	QAM, TCM	33.6	
56k Modem (8000/3429 baud) (V.90)		56.0/33.6	1998
56k Modem (8000/8000 baud) (V.92)		56.0/48.0	2000

US Robotics 14.4kbits/s

V.34

Key Elements in Wired Modems (Impact of Coding): Two Ways to Improve Data Rates

< 9.6 kbs/s — equalization (Lucky 60s)

9.6 kbs/s 1984 — TCM +equalization (DFE)

Bandwidth efficient coding
Several bits/s/Hz

14.4 kbs/s — TCM
19.20 kbs/s — TCM
28.8 kbs/s — high dim TCM
33.6 kbs/s — high dim TCM
56 kbs/s — high dim TCM

+ equalization

Squeeze more bits to a symbol

better

Higher SNR

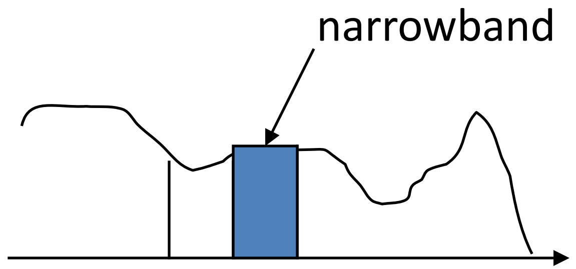
Asymmetric Digital Subscriber Line (ADSL)

6 Mbs/s

orthogonal frequency division multiplexing (OFDM)

or called discrete multi-tone (DMT)

More advanced DSP



Use more bandwidth

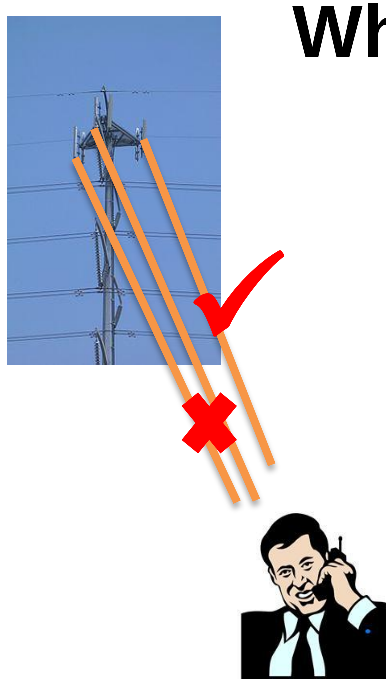
What Happens to Wireless Modems

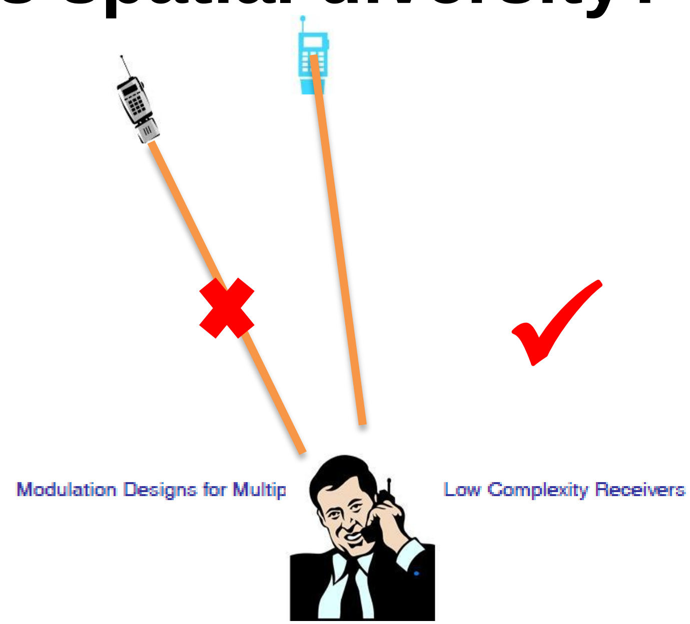
- Fading, interferences, low SNR
  - TCM has not been used successfully in wireless systems
  - OFDM systems are only successful in LAN not WAN yet
- Can we overcome these difficulties?
  - Maybe! To sufficiently use spatial diversities to overcome wireless fading
  - Beginning standards: IEEE 802.11n, 802.16d/e, 3GPP/LTE
  - More standards are coming (similar to wired modems)

Exploit Spatial Diversity

- Two ways to exploit spatial diversity:
  - Multi-antenna systems
  - Cooperative communication systems
- Benefits:
  - Potential high capacity gain

What is spatial diversity?





Diversity Order and Full Diversity

- If the symbol/bit error rate of a system satisfies

$$P_{\text{error}}(\text{SNR}) \sim c \cdot \text{SNR}^{-d}$$

  for some positive constant c, then we say that the system achieves **diversity order** d.
  - The larger d is, the smaller the error probability is, for a given channel SNR.
- For an N transmiter (MIMO system or cooperative system) and M receiver wireless system, its highest spatial diversity order is MN that is also the number of freedoms of the system.
- If the maximal diversity order is achieved, we call the system achieves **full diversity**.

Exploit Spatial Diversity: MIMO Systems

- Multiple antennas are co-located.
- Space-time coding plays an important role to achieve the spatial diversity.
- They are (or will be) adopted in the current and future high speed wireless systems.

Space-Time Codes/Modulations

- A space-time code (or modulation) is a collection (set) of matrices (one dimension is space and the other dimension is time).
- A space-time code achieves full diversity when ML receiver is used, if it has the **full rank property**, i.e., the rank of difference matrix of any two distinct matrices in the code is full rank.
- The most well-known space-time code is Alamouti code

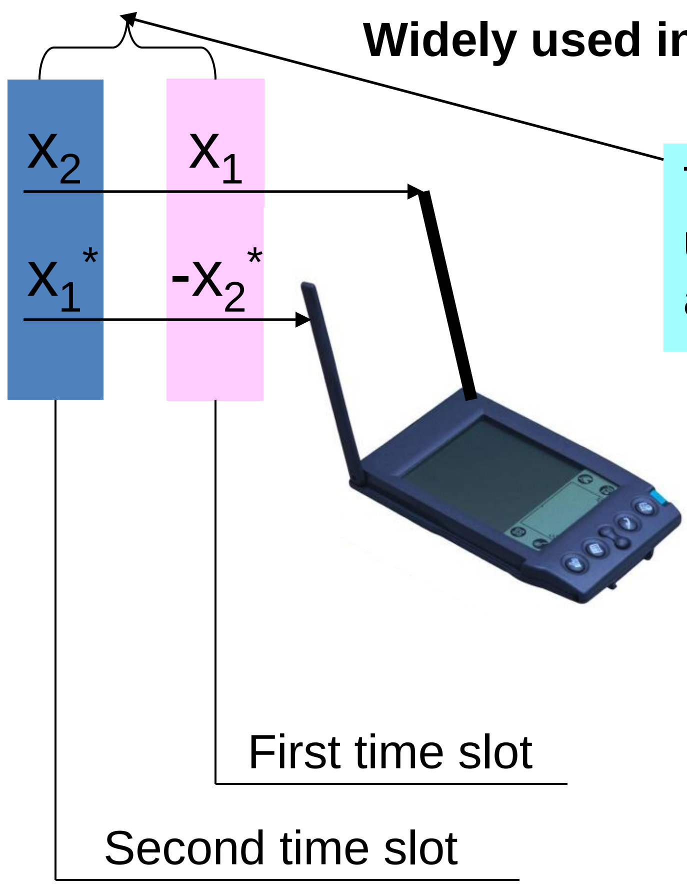
$$\begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix},$$

where x_1 and x_2 are two modulated information symbols.

Multiple Antenna Systems: Spatial Diversity Using Space-Time Coding

- Alamouti Code: Achieve Full Spatial Diversity

Widely used in multiple antenna systems



The 2 by 2 matrix is always full rank unless two information symbols are all zero.

Background

(Zheng-Tse)

- The diversity and multiplexing tradeoff (DMT) has been well-understood and linear lattice codes, such as perfect codes, have been designed to achieve the DMT tradeoff, when ML receiver is used.
  - With ML receiver, a space-time code can be full rate, i.e., the number of transmit antennas, to achieve full diversity.
- Not much is known when the decoding complexity is concerned while the decoding complexity is so important in practical applications.

Background

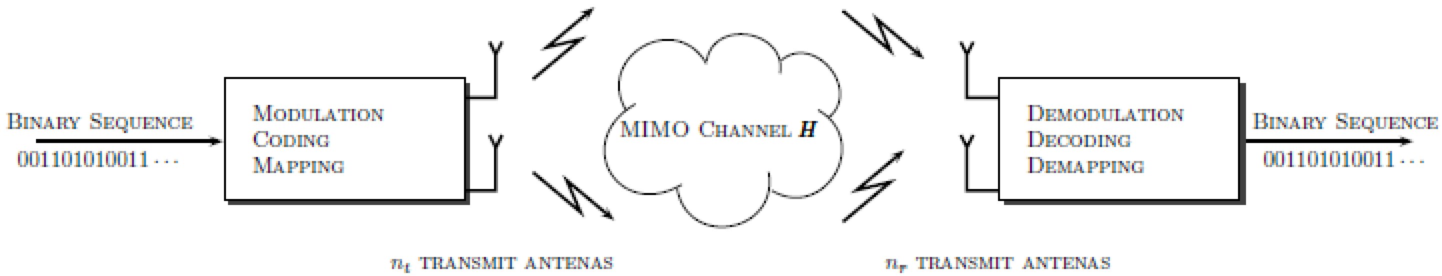
- The most-well known Alamouti code and orthogonal codes
  - They have the fastest ML receiver that achieves full diversity, where ML receiver is the same as linear receiver.
  - Unfortunately, their symbol rates are upper bounded by 3/4 for more than two transmit antennas and approach 1/2 when the transmit antenna number goes large. (Wang-Xia'03)

Questions

- Are there codes beyond orthogonal codes that achieve full diversity with a low complexity receiver, such as linear receiver?
- What will be a design criterion for such codes?
- How to design these codes?
- How to deal with Complexity-Rate-Performance (full diversity)?

Channel Model

n_t: Tx number $\qquad$ n_r : Rx number



MIMO Channel Model for STBC:

$$\boldsymbol{Y} = \sqrt{\frac{\mathsf{SNR}}{n_t}}\boldsymbol{H}\boldsymbol{X} + \boldsymbol{W}$$

$\boldsymbol{Y} = (y_{i,j}) \in \mathbb{C}^{n_r \times t}$ is the received signal matrix that is received in t time slots

$\boldsymbol{H} = (h_{i,j}) \in \mathbb{C}^{n_r \times n_t}$ is the channel matrix

n_t is the number of transmit antennas

SNR is the average signal-to-noise-ratio (SNR) at the receiver

$\boldsymbol{X} \in \mathbb{C}^{n_t \times t}$ is the codeword matrix

$\boldsymbol{W} \in \mathbb{C}^{n_r \times t}$ is the additive white Gaussian noise matrix

with i.i.d. entries $w_{i,j} \sim \mathcal{CN}(0,1)$

LD-STBC

- $\mathcal{A}$: A symbol constellation
- General form of a linear dispersion (LD) STBC:

$$\mathcal{X} = \left\{ \boldsymbol{X} = \sum_{i=0}^{n-1} x_i \boldsymbol{A}_i + x_i^* \boldsymbol{B}_i, x_i \in \mathcal{A}, 0 \leq i < n \right\}$$

- The equivalent channel model for LD-STBC

$$\boldsymbol{y} = \sqrt{\mathsf{SNR}}\, \boldsymbol{G}(\boldsymbol{h}) \boldsymbol{x} + \boldsymbol{w}$$

$\boldsymbol{G}(\boldsymbol{h})$ Equivalent Channel. $\boldsymbol{h} = \mathrm{vec}(\boldsymbol{H})$ Each entry of $\boldsymbol{G}(\boldsymbol{h})$ is a linear combination of h_i and $h_i^*, 0 \leq i \leq n_r n_t - 1$

$\boldsymbol{x}$ $\boldsymbol{x} = [x_0, x_1, \ldots, x_{n-1}]^{\mathsf{T}}$ is the symbol vector embedded in the codeword $\boldsymbol{X}$

$\boldsymbol{w}$ White Gaussian Noise

LD-STBC

- Example: Alamouti Code with ONE receive antenna

$$\begin{bmatrix} y_{0,0} & y_{0,1} \end{bmatrix} = \sqrt{\frac{\text{SNR}}{2}} \begin{bmatrix} h_{0,0} & h_{0,1} \end{bmatrix} \begin{bmatrix} x_0 & -x_1^* \\ x_1 & x_0^* \end{bmatrix} + \begin{bmatrix} w_{0,0} & w_{0,1} \end{bmatrix}$$

- The equivalent channel model:

$$\begin{bmatrix} y_{0,0} \\ y_{0,1}^* \end{bmatrix} = \sqrt{\text{SNR}} \left(\frac{1}{\sqrt{2}} \begin{bmatrix} h_{0,0} & h_{0,1} \\ h_{0,1}^* & -h_{0,0}^* \end{bmatrix} \right) \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} + \begin{bmatrix} w_{0,0} \\ w_{0,1}^* \end{bmatrix}$$

LD-STBC

The equivalent channel model for Alamouti Code with two TWO receive antennas

$$\begin{bmatrix} y_{0,0} \\ y_{0,1}^* \\ y_{1,0} \\ y_{1,1}^* \end{bmatrix} = \sqrt{\mathsf{SNR}} \left(\frac{1}{\sqrt{2}} \begin{bmatrix} h_{0,0} & h_{0,1} \\ h_{0,1}^* & -h_{0,0}^* \\ h_{1,0} & h_{1,1} \\ h_{1,1}^* & -h_{1,0}^* \end{bmatrix} \right) \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} + \begin{bmatrix} w_{0,0} \\ w_{0,1}^* \\ w_{1,0} \\ w_{1,1}^* \end{bmatrix}$$

Revisit of ML Receiver

- Full full diversity is achieved when a STBC/modulation $\boldsymbol{X}$ has the full rank property
- (Guo-Xia'09) The full rank property of $\boldsymbol{X}$ is equivalent to that all the column vectors, $\boldsymbol{g}_1, \boldsymbol{g}_2, \cdots, \boldsymbol{g}_n$, of the equivalent channel matrix $\boldsymbol{G}(\boldsymbol{h})$ are linearly independent over $\Delta\mathcal{A} = \{a_1 - a_2 | a_1, a_2 \in \mathcal{A}\}$, i.e., $\sum_{i=1}^{n} \alpha_i \mathbf{g}_i = 0$ for $\alpha_i \in \Delta\mathcal{A}$ if and only if $\alpha_i = 0$ for all i.

Linear Receivers: ZF and MMSE Receivers

- From the equivalent channel model (if existed), we have
  - ZF receiver

$$\hat{s}_{ZF} = \sqrt{\frac{\text{SNR}}{n_t}} (\boldsymbol{G}^{\dagger}(\boldsymbol{h})\boldsymbol{G}(\boldsymbol{h}))^{-1}\boldsymbol{G}^{\dagger}(\boldsymbol{h})\boldsymbol{y}$$

  - MMSE receiver

$$\hat{s}_{MMSE} = \sqrt{\frac{\text{SNR}}{n_t}} (I_{n_t} + \frac{\text{SNR}}{n_t}\boldsymbol{G}^{\dagger}(\boldsymbol{h})\boldsymbol{G}(\boldsymbol{h}))^{-1}\boldsymbol{G}^{\dagger}(\boldsymbol{h})\boldsymbol{y}$$

- The components of the $\hat{s}_{ZF}$ or $\hat{s}_{MMSE}$ can be treated as the soft outputs of the information symbols in $\boldsymbol{s}$

Question: **What property should code *X* have in order to achieve full diversity when ZF or MMSE receiver is used?**

Criterion

- **Theorem** (Shang-Xia'08): STBC $\boldsymbol{X}$ can achieve full diversity with ZF or MMSE receiver, if an equivalent channel matrix $\boldsymbol{G}(\boldsymbol{h})$ exists and satisfies

$$\|\boldsymbol{G}(\boldsymbol{h})\| \leq c_1 \|\boldsymbol{H}\| \text{ and } \det(\boldsymbol{G}^\dagger(\boldsymbol{h})\boldsymbol{G}(\boldsymbol{h})) \geq c_2 \|\boldsymbol{H}\|^{2n}$$

  for any realization of the original channel matrix $\boldsymbol{H}$, where c_1 and c_2 are positive constants independent of $\boldsymbol{H}$, and n is the number of symbols encoded in each codeword matrix of $\boldsymbol{X}$, i.e., the length of $\boldsymbol{s}$.

- **Equivalent form**: STBC $\boldsymbol{X}$ achieves full diversity with ZF or MMSE receiver, if all the column vectors of the equivalent channel matrix $\boldsymbol{G}(\boldsymbol{h})$ are linearly independent.
  - The above linear independence is the conventional one that is over the complex field.

Maximum Symbol Rates

- **Corollary** (Shang-Xia'08): The symbol rates of the STBC that satisfy the above criterion and thus achieve full diversity with ZF or MMSE receiver can not be above 1, i.e., they are upper bounded by 1.
- The rates of Alamouti code and Toeplitz codes are asymptotically optimal

Y. Shang and X.-G. Xia, Space-Time Block Codes Achieving Full Diversity with Linear Receivers, *IEEE Trans. on Information Theory*, Oct. 2008.

Code Examples Achieving Full Diversity with Linear Receivers

- Toeplitz codes (Liu-Zhang-Wong'08)
- Overlapped Alamouti codes (Shang-Xia'08)
- Combined Toeplitz and Alamouti codes (Wang-Xia-Yin'09)

Toeplitz Codes (Delay Diversity Codes)

- the codeword matrix for n_t transmit antennas with n information symbols $s_0, s_1, \cdots, s_{n-1}$ embedded is

$$\begin{pmatrix} s_0 & 0 & \cdots & 0 \\ s_1 & s_0 & \cdots & 0 \\ s_2 & s_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ s_{n-1} & s_{n-2} & \cdots & s_0 \\ 0 & s_{n-1} & \cdots & s_1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & s_{n-1} \end{pmatrix}$$

that has size of $(n + n_t - 1) \times n_t$

Overlapped Alamouti Codes: Examples for 3 and 4 Tx

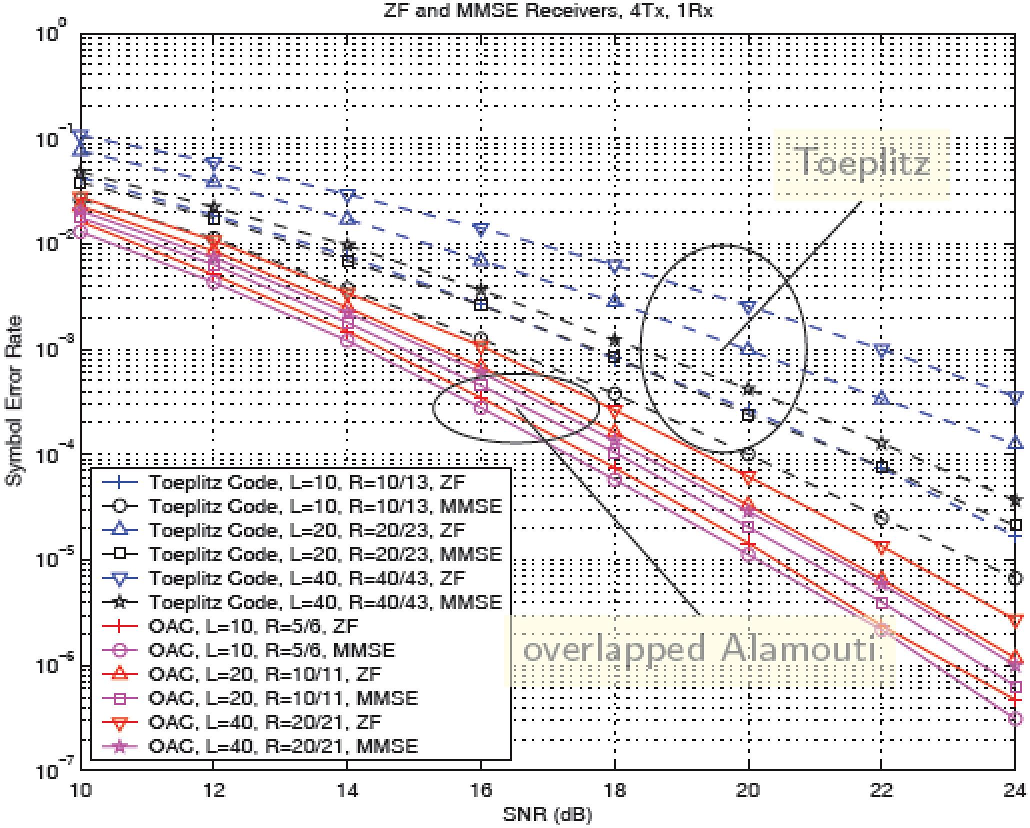
Codeword of $\mathcal{O}_{3,L}$

$$\begin{pmatrix} s_1^* & 0 & 0 \\ 0 & s_1 & s_2 \\ s_3^* & -s_2^* & s_1^* \\ s_2 & s_3 & s_4 \\ s_5^* & -s_4^* & s_3^* \\ s_4 & s_5 & s_6 \\ \vdots & \vdots & \vdots \\ s_{L-4} & s_{L-3} & s_{L-2} \\ s_{L-1}^* & -s_{L-2}^* & s_{L-3}^* \\ s_{L-2} & s_{L-1} & s_L \\ 0 & -s_L^* & s_{L-1}^* \\ s_L & 0 & 0 \end{pmatrix}$$

Codeword of $\mathcal{O}_{4,L}$

$$\begin{pmatrix} s_1 & 0 & 0 & s_2 \\ 0 & s_1^* & -s_2^* & 0 \\ s_3 & s_2 & s_1 & s_4 \\ -s_2^* & s_3^* & -s_4^* & s_1^* \\ s_5 & s_4 & s_3 & s_6 \\ \vdots & \vdots & \vdots & \vdots \\ -s_{L-4}^* & s_{L-3}^* & -s_{L-2}^* & s_{L-5}^* \\ s_{L-1} & s_{L-2} & s_{L-3} & s_L \\ -s_{L-2}^* & s_{L-1}^* & -s_L^* & s_{L-3}^* \\ 0 & s_L & s_{L-1} & 0 \\ -s_L^* & 0 & 0 & s_{L-1}^* \end{pmatrix}$$

Overlapped Alamouti codes vs. Toeplitz codes
$M = 4, N = 1$



- 4-QAM
- The performance loss for overlapped Alamouti codes as L increases is much smaller than that for Toeplitz codes
- The performance improvement of MMSE receiver over ZF receiver for overlapped Alamouti codes is not as significant as that for Toeplitz codes

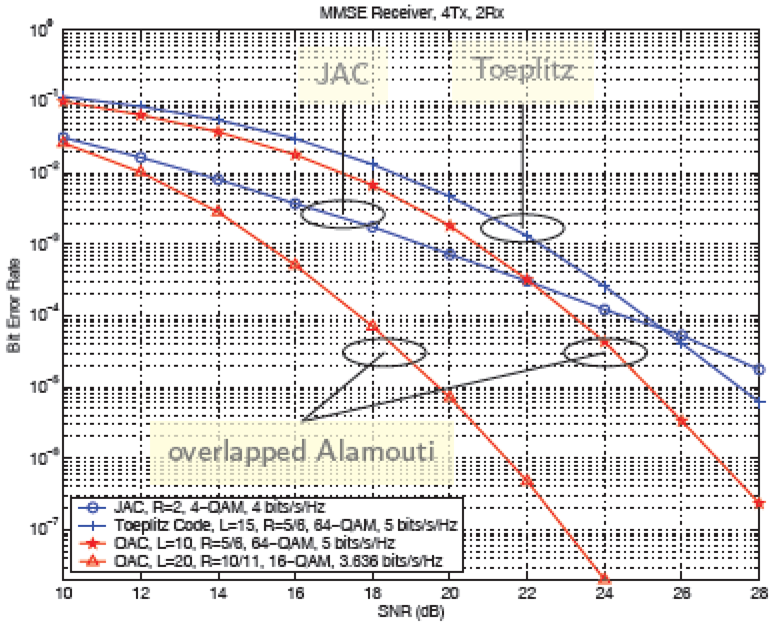
Overlapped Alamouti codes vs. Juxtaposed Alamouti codes
$M = 4, N = 2$

- Juxtaposed Alamouti codes (JAC) proposed in 3GPP for 4 transmit antennas have the codeword matrix in the form of

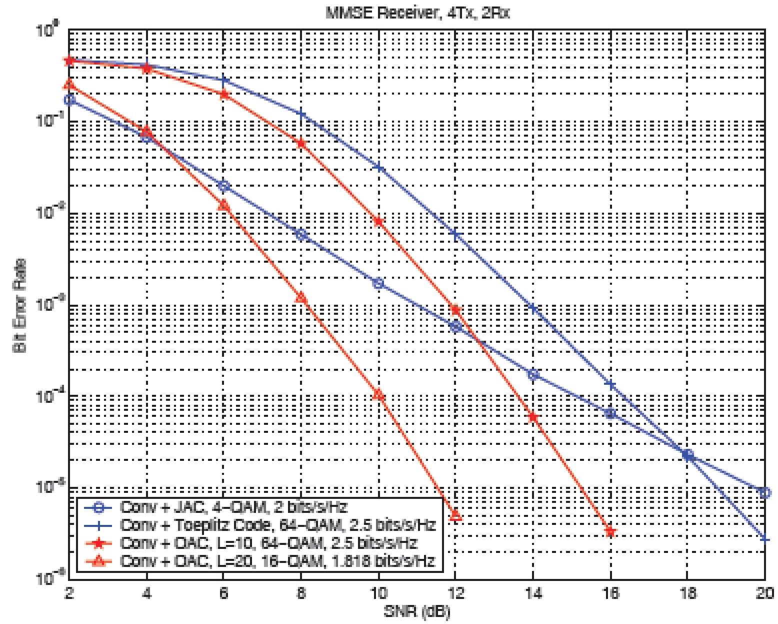
$$X(\mathbf{s}) = \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \end{pmatrix}.$$

- Symbol rate 2 for JAC
- JAC has diversity 2 for ML receiver, so not full diversity achieving for linear receivers

Overlapped Alamouti codes vs. juxtaposed Alamouti codes (Contd.)
$M = 4, N = 2$



(a). without outer convolutional codes

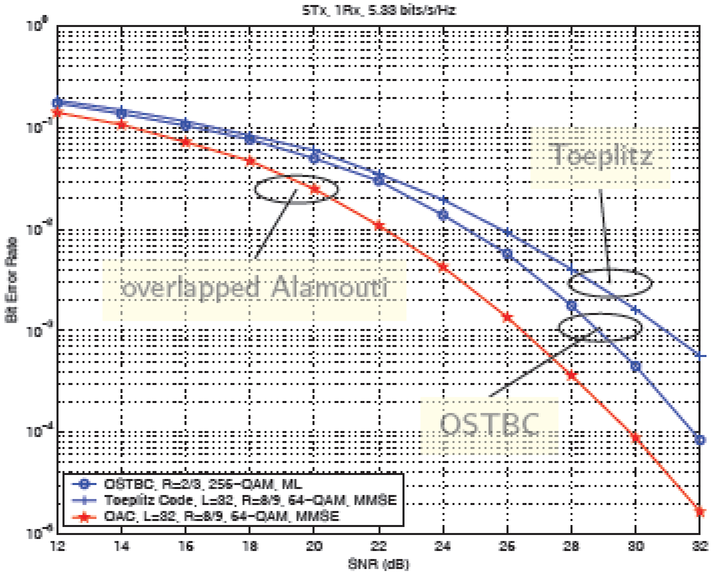


(b). with outer convolutional codes

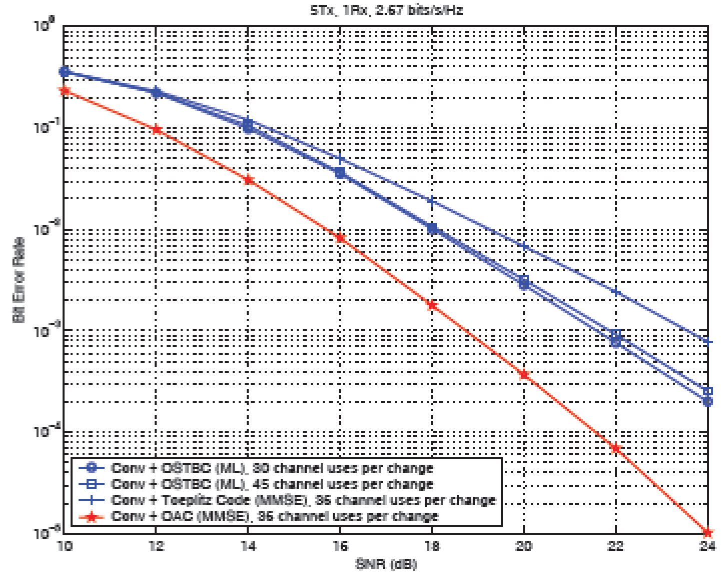
- Overlapped Alamouti code has significant diversity gains over JAC
- The relative trend among the performance curves keeps the same when there is an outer convolutional code

Overlapped Alamouti codes vs. OSTBC
$M = 5, N = 1$

- Overlapped Alamouti codes can have higher rates than OSTBC for $M > 2$, which may lead to performance gains

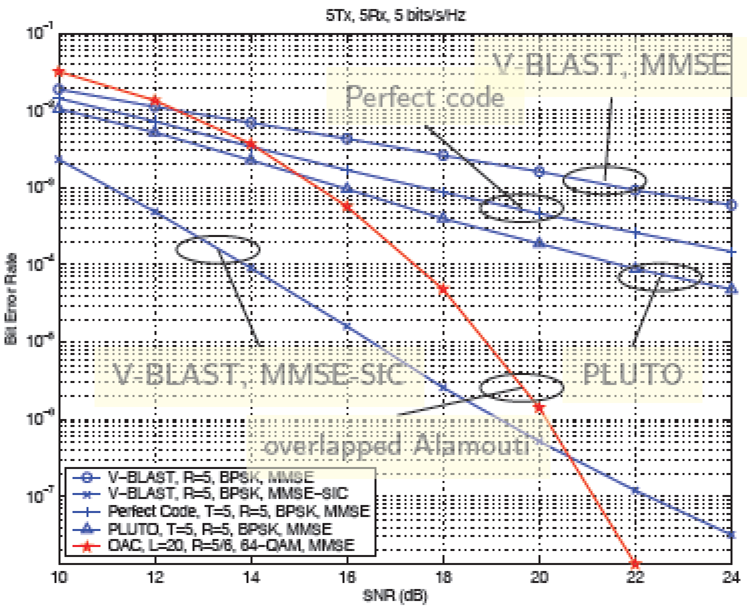


(a). without outer convolutional codes

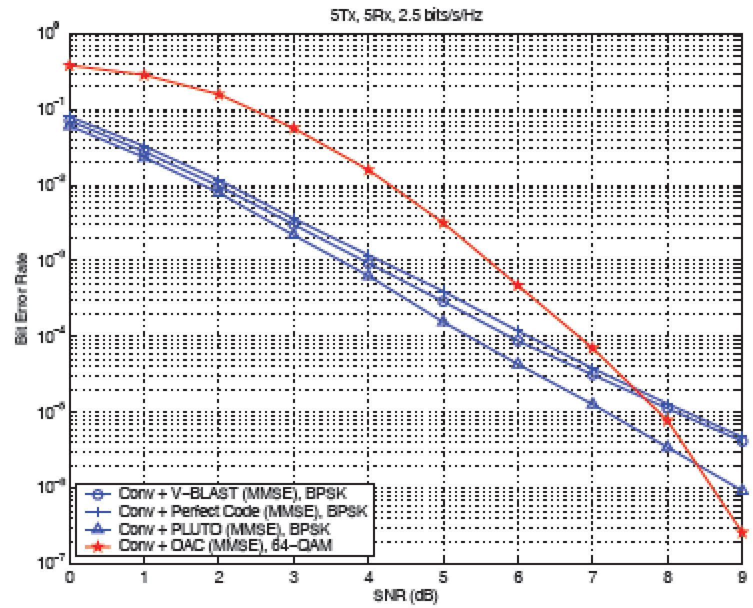


(b). with outer convolutional codes

- OSTBC has rate 2/3 (the highest known rate for $M = 5$) and block length 15
- Similar performance improvements for $M > 5$, where overlapped Alamouti codes have shorter block lengths than OSTBC



(a). without outer convolutional codes



(b). with outer convolutional codes

- The PLUTO codes are designed in terms of minimizing BER for MMSE receiver at a very high symbol rate
- The overlapped Alamouti code has significant diversity gains over others
- The overlapped Alamouti code even outperforms V-BLAST with MMSE-SIC (ordered) receiver

What Happens in Between Linear Receiver and ML Receiver?

- We next propose partial interference cancellation (PIC) group decoding that serves as an intermediate decoding between the ML and linear receivers

Notations

- n: number of information symbols in $\boldsymbol{X}$ or $\boldsymbol{x}$;
- m: the dimension of $\boldsymbol{y}$, equals to $n_t \times t$;
- Write equivalent channel $\boldsymbol{G}(\boldsymbol{h}) \in \mathbb{C}^{m \times n}$ as

$$\boldsymbol{G}(\boldsymbol{h}) = [\boldsymbol{g}_0, \boldsymbol{g}_1, \ldots, \boldsymbol{g}_{n-1}]$$

- Index set $\mathcal{I}$:

$$\mathcal{I} = \{0, 1, 2, \ldots, n-1\}$$

Notations

- Grouping Scheme $\mathcal{I}$:

$$\mathcal{I} = \{\mathcal{I}_0, \mathcal{I}_1, \ldots, \mathcal{I}_{N-1}\}$$

- Each index subset $\mathcal{I}_k$ can be written as follows,

$$\mathcal{I}_k = \{i_{k,0}, i_{k,1}, \ldots, i_{k,n_k-1}\}, n_k \triangleq |\mathcal{I}_k|$$

- For a legitimated grouping scheme, the following equations must be satisfied:

$$\mathcal{I} = \bigcup_{i=0}^{N-1} \mathcal{I}_i \text{ and } \sum_{i=0}^{N-1} n_i = n$$

Notations

- $\boldsymbol{x}_{\mathcal{I}_k}$: sub-vector of $\boldsymbol{x}$,

$$\boldsymbol{x}_{\mathcal{I}_k} = \left[x_{i_{k,0}}, x_{i_{k,1}}, \ldots, x_{i_{k,n_k-1}}\right]^{\mathsf{T}}.$$

- $\boldsymbol{G}_{\mathcal{I}_k}$: sub-matrix of $\boldsymbol{G}(\boldsymbol{h})$,

$$\boldsymbol{G}_{\mathcal{I}_k} = \left[\boldsymbol{g}_{i_{k,0}}, \boldsymbol{g}_{i_{k,1}}, \ldots, \boldsymbol{g}_{i_{k,n_k-1}}\right]$$

- Channel Model can be rewritten as

$$\boldsymbol{y} = \sqrt{\mathsf{SNR}} \sum_{i=0}^{N-1} \boldsymbol{G}_{\mathcal{I}_i} \boldsymbol{x}_{\mathcal{I}_i} + \boldsymbol{w}$$

Question: How to decode one symbol group $x_{\mathcal{I}_k}$?

PIC Group Decoding

- Eliminate interferences: find a projection filter $\boldsymbol{P}_{\mathcal{I}_k}$,

$$\boldsymbol{P}_{\mathcal{I}_k}\boldsymbol{G}_{\mathcal{I}_j} = 0, j \neq k$$

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- Channel Model after fitering:

$$\begin{aligned}\boldsymbol{P}_{\mathcal{I}_k}\boldsymbol{y} = \boldsymbol{z}_{\mathcal{I}_k} &= \sqrt{\mathsf{SNR}}\boldsymbol{P}_{\mathcal{I}_k}\sum_{i=0}^{N-1}\boldsymbol{G}_{\mathcal{I}_i}\boldsymbol{x}_{\mathcal{I}_i} + \boldsymbol{P}_{\mathcal{I}_k}\boldsymbol{w} \\ &= \sqrt{\mathsf{SNR}}\boldsymbol{P}_{\mathcal{I}_k}\boldsymbol{G}_{\mathcal{I}_k}\boldsymbol{x}_{\mathcal{I}_k} + \boldsymbol{P}_{\mathcal{I}_k}\boldsymbol{w}\end{aligned}$$

PIC Group Decoding

- Eliminate interferences: find a projection filter $\boldsymbol{P}_{\mathcal{I}_k}$,

$$\boldsymbol{P}_{\mathcal{I}_k}\boldsymbol{G}_{\mathcal{I}_j} = 0, j \neq k$$

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$$\boldsymbol{P}_{\mathcal{I}_k}\boldsymbol{y} = \boldsymbol{z}_{\mathcal{I}_k} = \sqrt{\mathsf{SNR}}\boldsymbol{P}_{\mathcal{I}_k}\sum_{i=0}^{N-1}\boldsymbol{G}_{\mathcal{I}_i}\boldsymbol{x}_{\mathcal{I}_i} + \boldsymbol{P}_{\mathcal{I}_k}\boldsymbol{w}$$

$$= \sqrt{\mathsf{SNR}}\boldsymbol{P}_{\mathcal{I}_k}\boldsymbol{G}_{\mathcal{I}_k}\boldsymbol{x}_{\mathcal{I}_k} + \boldsymbol{P}_{\mathcal{I}_k}\boldsymbol{w}$$

Dimension-reduced system

PIC Filter: Least Square Solution

$$\boldsymbol{P}_{\mathcal{I}_k} = \boldsymbol{I}_m - \boldsymbol{G}^c_{\mathcal{I}_k} \left(\left(\boldsymbol{G}^c_{\mathcal{I}_k} \right)^{\mathsf{H}} \boldsymbol{G}^c_{\mathcal{I}_k} \right)^{-1} \left(\boldsymbol{G}^c_{\mathcal{I}_k} \right)^{\mathsf{H}},$$

where

$$\boldsymbol{G}^c_{\mathcal{I}_k} = \left[\boldsymbol{G}_{\mathcal{I}_0}, \boldsymbol{G}_{\mathcal{I}_1}, \ldots, \boldsymbol{G}_{\mathcal{I}_{k-1}}, \boldsymbol{G}_{\mathcal{I}_{k+1}}, \ldots, \boldsymbol{G}_{\mathcal{I}_{N-1}} \right].$$

PIC Group Decoding

- Reduce the decoding complexity by reducing the dimension of the decoding problem.
- It can be proved that the ML decoding rule for the dimension-reduced system is

$$\hat{\boldsymbol{x}}_{\mathcal{I}_k} = \arg \min_{\bar{\boldsymbol{x}} \in \mathcal{A}^{n_k}} \left\| \boldsymbol{z}_{\mathcal{I}_k} - \sqrt{\mathsf{SNR}} \boldsymbol{P}_{\mathcal{I}_k} \boldsymbol{G}_{\mathcal{I}_k} \bar{\boldsymbol{x}} \right\|,$$

Example

Consider the quasi-orthogonal STBC proposed by Tirkkonen-Boariu-Hottinen

$$\boldsymbol{X} = \begin{bmatrix} x_0 & -x_1^* & x_2 & -x_3^* \\ x_1 & x_0^* & x_3 & x_2^* \\ x_2 & -x_3^* & x_0 & -x_1^* \\ x_3 & x_2^* & x_1 & x_0^* \end{bmatrix}$$

Example

- In the case $n_r = 1$, the equivalent channel matrix $\boldsymbol{G}(\boldsymbol{h})$ is

$$\begin{bmatrix} h_0 & h_1 & h_2 & h_3 \\ h_1^* & -h_0^* & h_3^* & -h_2^* \\ h_2 & h_3 & h_0 & h_1 \\ h_3^* & -h_2^* & h_1^* & -h_0^* \end{bmatrix}$$

Example

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- Let $\mathcal{I}_0 = \{0, 2\}$ and $\mathcal{I}_1 = \{1, 3\}$:

$$\hat{\boldsymbol{x}}_{\mathcal{I}_0} = \arg \min_{\bar{\boldsymbol{x}}_{\mathcal{I}_0} \in \mathcal{A}^2} \left\| \boldsymbol{y} - \sqrt{\mathsf{SNR}} \boldsymbol{G}_{\mathcal{I}_0} \bar{\boldsymbol{x}}_{\mathcal{I}_0} \right\| .$$

Example

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$$\hat{\boldsymbol{x}}_{\mathcal{I}_0} = \arg \min_{\bar{\boldsymbol{x}}_{\mathcal{I}_0} \in \mathcal{A}^2} \left\| \boldsymbol{y} - \sqrt{\mathsf{SNR}} \boldsymbol{G}_{\mathcal{I}_0} \bar{\boldsymbol{x}}_{\mathcal{I}_0} \right\| .$$

- In this case the PIC decoding is equivalent to ML decoding.

Connections with Linear and ML Receivers

- When each group contains only one variable, i.e., $n_i = 1$, the PIC group decoding is reduced to the linear ZF receiver.
- When there is only one group, i.e., $N = 1$ and $n_1 = n$, the PIC group decoding is the ML receiver.
- The PIC group decoding is a bridge between the simplest receiver, linear receiver, and the most complicated receiver, ML receiver.
- We may expect that design criterion for the PIC group decoding should be between the ones for linear receiver and ML receiver.

PIC-SIC Group Decoding

- In the PIC group decoding, we may use *successive interference cancellation (SIC)* strategy to aid the decoding process called PIC-SIC group decoding.
- The basic idea of SIC is simple: remove the already-decoded symbols from the received signals to reduce the interferences.
- Different decoding orders will result in different SER performances.

PIC-SIC Group Decoding

Suppose the ordered symbol sets are as follows,

$$\boldsymbol{x}_{\mathcal{I}_{i_0}}, \boldsymbol{x}_{\mathcal{I}_{i_1}}, \ldots, \boldsymbol{x}_{\mathcal{I}_{i_{N-1}}}.$$

The ordered PIC-SIC group decoding algorithm is then:

1) Decode the first set of symbols $x_{\mathcal{I}_{i_0}}$ using the PIC group decoding algorithm;
2) Let $k = 0$, $\boldsymbol{y}_0 = \boldsymbol{y}$;

PIC-SIC Group Decoding

3) Remove the components of the already-detected symbol set $\boldsymbol{x}_{\mathcal{I}_{i_k}}$

$$\boldsymbol{y}_{k+1} \triangleq \boldsymbol{y}_k - \sqrt{\mathsf{SNR}}\boldsymbol{G}_{\mathcal{I}_{i_k}}\boldsymbol{x}_{\mathcal{I}_{i_k}}$$
$$= \sqrt{\mathsf{SNR}} \sum_{j=k+1}^{N-1} \boldsymbol{G}_{\mathcal{I}_{i_j}}\boldsymbol{x}_{\mathcal{I}_{i_j}} + \boldsymbol{w};$$

4) Decode $\boldsymbol{x}_{\mathcal{I}_{i_{k+1}}}$ in the above equation using the PIC group decoding algorithm;

5) If $k < N-1$, then set $k := k+1$, go to Step 3; otherwise stop the algorithm.

Full Diversity Criterion for LD-STBC with PIC Group Decoding

Power Gain Order

- Power gain P for the system is defined as:

$$P = \min_{\Delta x \in \Delta \mathcal{A}^n} \frac{\|\boldsymbol{G}(\boldsymbol{h})\Delta x\|^2}{\|\Delta x\|^2}$$

roughly distance gain

- Power gain order: if P satisfies

$$P \geq c \cdot \sum_{i=0}^{l-1} |h_i|^2 ,$$

for some positive constant c, then we say that the system achieves power gain order l.

- Achieving power gain order $l \Longrightarrow$ the system achieves diversity order l.

Full Diversity Criterion (Guo-Xia'09)

A linear dispersion STBC coded MIMO system with the PIC group decoding has power gain order $n_r \cdot n_t$ (full order) if and only if the following two conditions are satisfied:

- for any two different codewords $\boldsymbol{X}, \tilde{\boldsymbol{X}} \in \mathcal{X}$, $\Delta \boldsymbol{X} \triangleq \boldsymbol{X} - \tilde{\boldsymbol{X}}$ has the full rank property, which is equivalent to the condition that $\boldsymbol{g}_0, \boldsymbol{g}_1, \ldots, \boldsymbol{g}_{n-1}$ are linearly independent over $\Delta\mathcal{A}$;

- for a fixed k, $0 \le k \le N-1$, any non-zero linear combination over $\Delta\mathcal{A}$ of the vectors in the kth group $\boldsymbol{G}_{\mathcal{I}_k}$ does not belong to the space linearly spanned by all the vectors in the remaining vector groups, $\mathbb{V}_{\mathcal{I}_k}$, i.e., for any $a_0, a_1, \ldots, a_{n_k-1} \in \Delta\mathcal{A}$, not all zero,

$$\sum_{j=0}^{n_k-1} a_j \boldsymbol{g}_{i_{k,j}} \notin \mathbb{V}_{\mathcal{I}_k},$$

as long as $\boldsymbol{h} \neq \boldsymbol{0}$.

Full Diversity Criterion for PIC-SIC Group Decoding (Guo-Xia'09)

When the received signals are decoded using the PIC-SIC group decoding with the previous ordering, each dimension-reduced system derived during the decoding process (i.e., the STBC with the PIC-SIC group decoding) has power gain order $n_r \cdot n_t$ if and only if the following two conditions are satisfied:

- for any two different codewords $\boldsymbol{X}, \tilde{\boldsymbol{X}} \in \mathcal{X}$, $\Delta \boldsymbol{X} \triangleq \boldsymbol{X} - \tilde{\boldsymbol{X}}$ has the full rank property, i.e., the code $\mathcal{X}$ achieves full diversity with the ML receiver;
- at each decoding stage, for $\boldsymbol{G}_{\mathcal{I}_{i_k}}$, which corresponds to the current to-be decoded symbol group $\boldsymbol{x}_{i_k}$, any non-zero linear combination over $\Delta\mathcal{A}$ of the vectors in $\boldsymbol{G}_{\mathcal{I}_{i_k}}$ does not belong to the space linearly spanned by all the vectors in the group $[\boldsymbol{G}_{\mathcal{I}_{i_{k+1}}}, \ldots, \boldsymbol{G}_{\mathcal{I}_{i_{N-1}}}]$ as long as $\boldsymbol{h} \neq 0$.

Some Corollaries

- **Corollary**: If a linear dispersion STBC achieves full diversity with the PIC group decoding for n_r receiver antenna, then it does too for 1 receive antenna.
- **Corollary**: In the PIC group decoding, let each group have K symbols. If a code satisfies the full diversity conditions for the PIC group decoding, then the maximum symbol rate of this code is upper bounded by K.

Discussions

- Achieving full diversity with ML receiver has the weakest condition that is the linear independence of the equivalent channel column vectors $\{\mathrm{g}_i\}$ over a finite constellation difference $\Delta\mathcal{A}$.
- Achieving full diversity with linear receiver has the strongest condition that is the linear independence of the equivalent channel column vectors $\{\mathrm{g}_i\}$ over the complex number field.
  - The orthogonality of $\{\mathrm{g}_i\}$ is not necessary to achieve full diversity with linear receiver (linear receiver is the same as ML receiver when the orthogonality holds).

Discussions

- The rates for STBC achieving full diversity with PIC group decoding may be between 1 and n_t.
- A tradeoff of complexity-diversity-multiplexing/rate.

X. Guo and X.-G. Xia, On Full Diversity Space-Time Block Codes with Partial Interference Cancellation Group Decoding, *IEEE Trans. on Information Theory*, Oct. 2009.

X. Guo and X.-G. Xia, Correction to ``On Full Diversity Space-Time Block Codes with Partial Interference Cancellation Group Decoding'', *IEEE Trans. on Information Theory*, July. 2010.

Code Design Examples (Guo-Xia'09)

- Codeword: (2 Tx and 3 time slots)

$$X = \begin{bmatrix} cx_0 + sx_1 & cx_2 + sx_3 & 0 \\ 0 & -sx_0 + cx_1 & -sx_2 + cx_3 \end{bmatrix},$$

where $c = \cos\theta, s = \sin\theta, \theta \in [0, 2\pi)$ but $\tan\theta$ is not a rational number

- X can be viewed as a stacked diagonal code
- The symbol rate of this code is $\frac{4}{3} > 1$
- Grouping scheme: $\mathcal{I}_0 = \{0, 1\}, \mathcal{I}_1 = \{2, 3\}$.

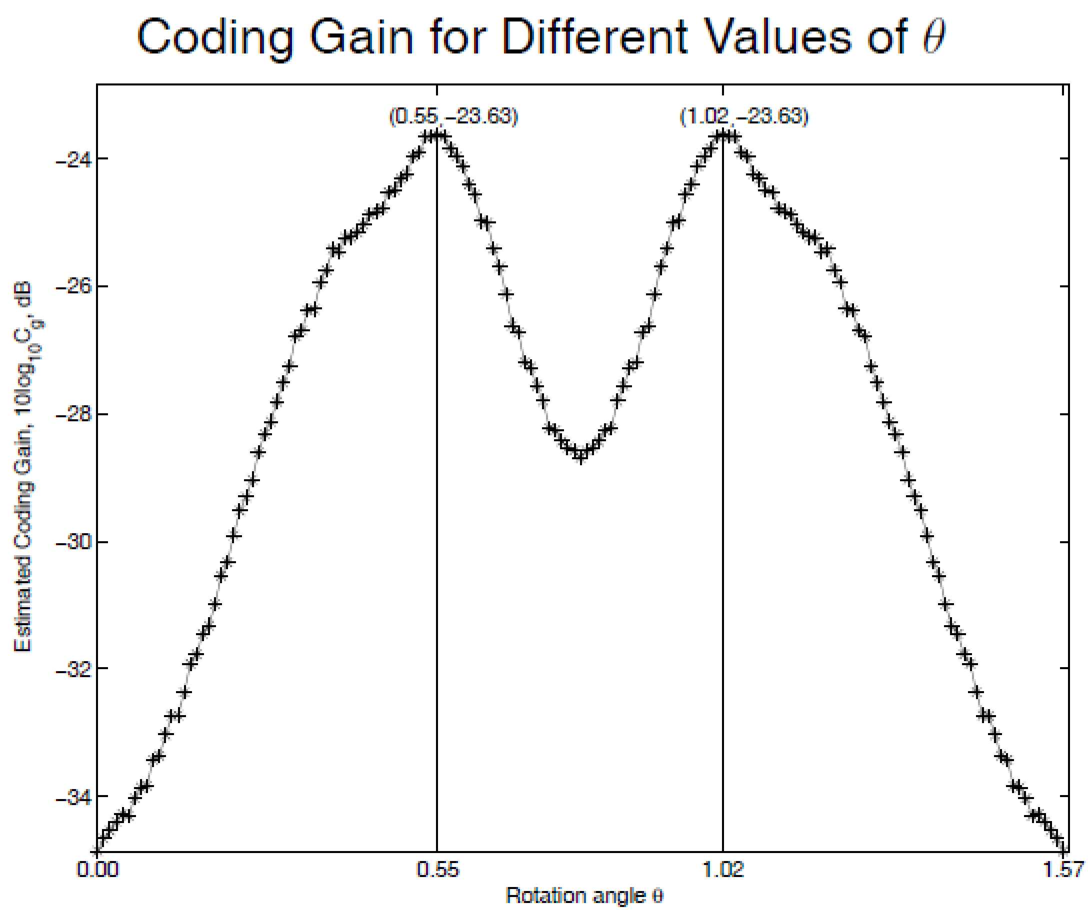
Code Design Example for 4 Tx (Guo-Xia'09)

$$\left[\begin{array}{ccc} cx_0 + sx_1 & -cx_2^* - sx_3^* & cx_4 + sx_5 \\ 0 & 0 & cx_0 + sx_1 \\ cx_2 + sx_3 & cx_0^* + sx_1^* & cx_6 + sx_7 \\ 0 & 0 & cx_2 + sx_3^* \end{array}\right.$$

$$\left.\begin{array}{ccc} -cx_6^* - sx_7^* & 0 & 0 \\ -cx_2^* - sx_3^* & cx_4 + sx_5 & -cx_6^* + sx_7^* \\ cx_4^* + sx_5^* & 0 & 0 \\ cx_0^* + sx_1^* & cx_6 + sx_7 & cx_4^* + sx_5^* \end{array}\right]$$

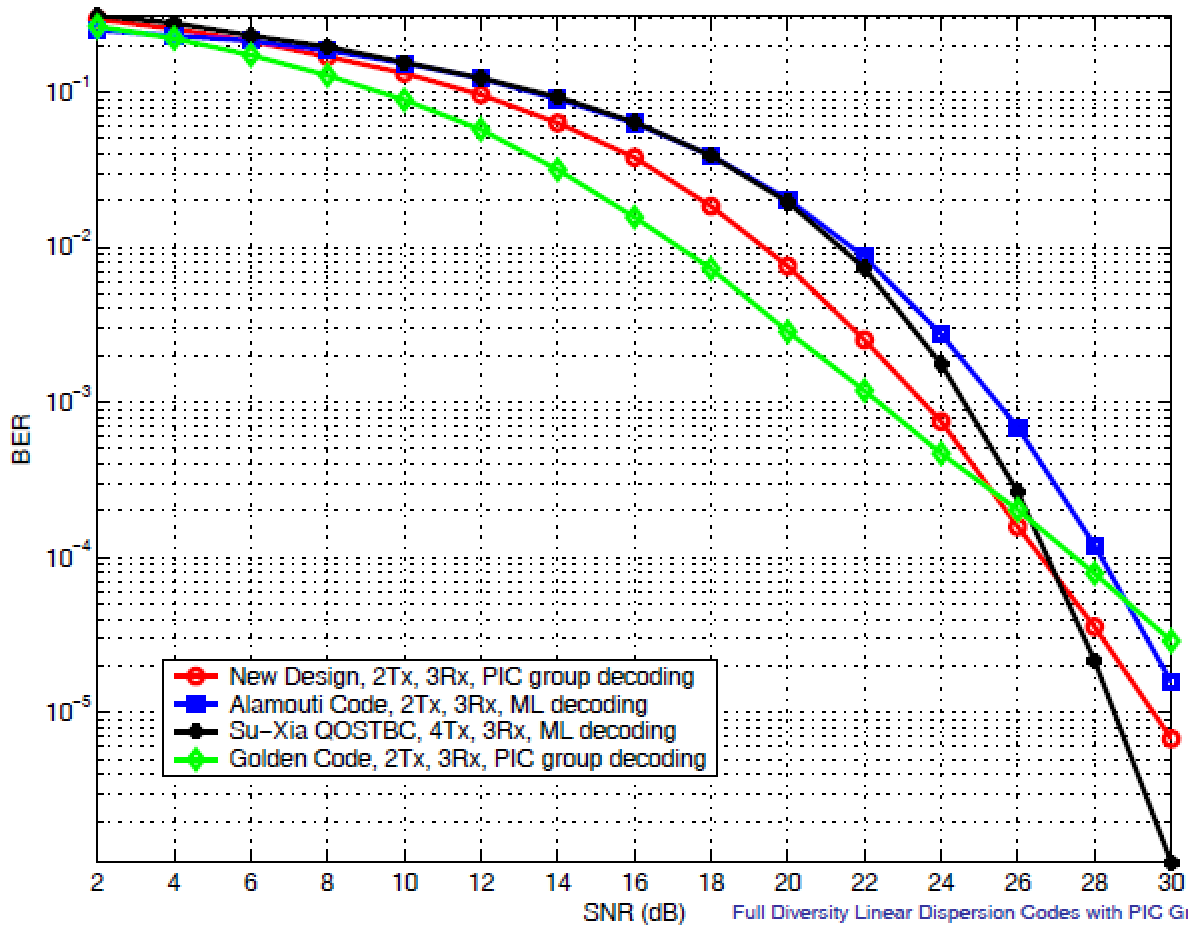
where $c = \cos\theta$, $s = \sin\theta$, $\theta \in [0, 2\pi)$ but $\tan\theta$ is not a rational number. Symbol rate $r = \frac{4}{3} > 1$. Grouping scheme: $\mathcal{I}_0 = \{0,1\}$, $\mathcal{I}_1 = \{2,3\}$, $\mathcal{I}_2 = \{4,5\}$, $\mathcal{I}_3 = \{6,7\}$.

Searching Results for optimal θ



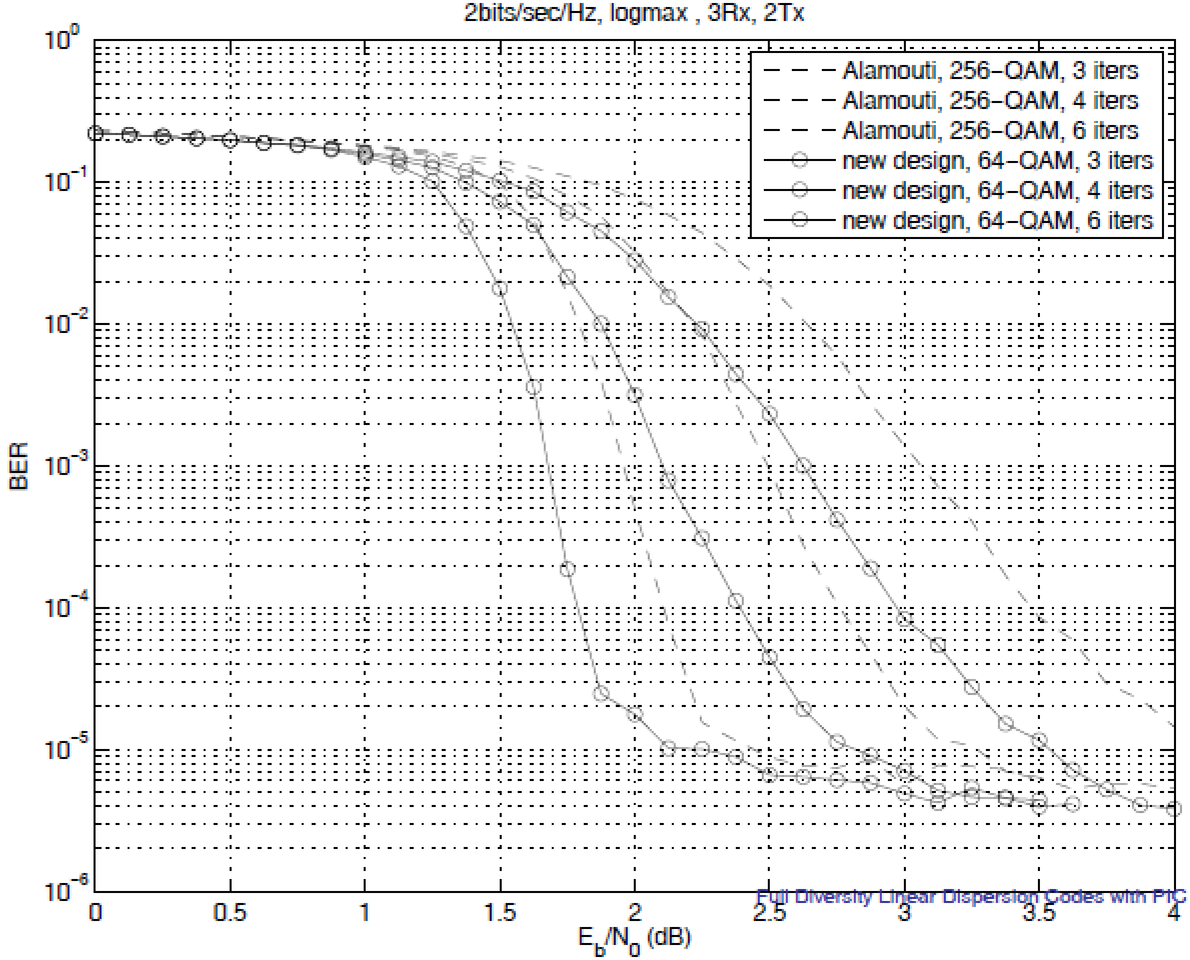
Simulation Results

8bits/sec/Hz, QOSTBC 4Tx, 3Rx, New Design, Alamouti Code, Golden



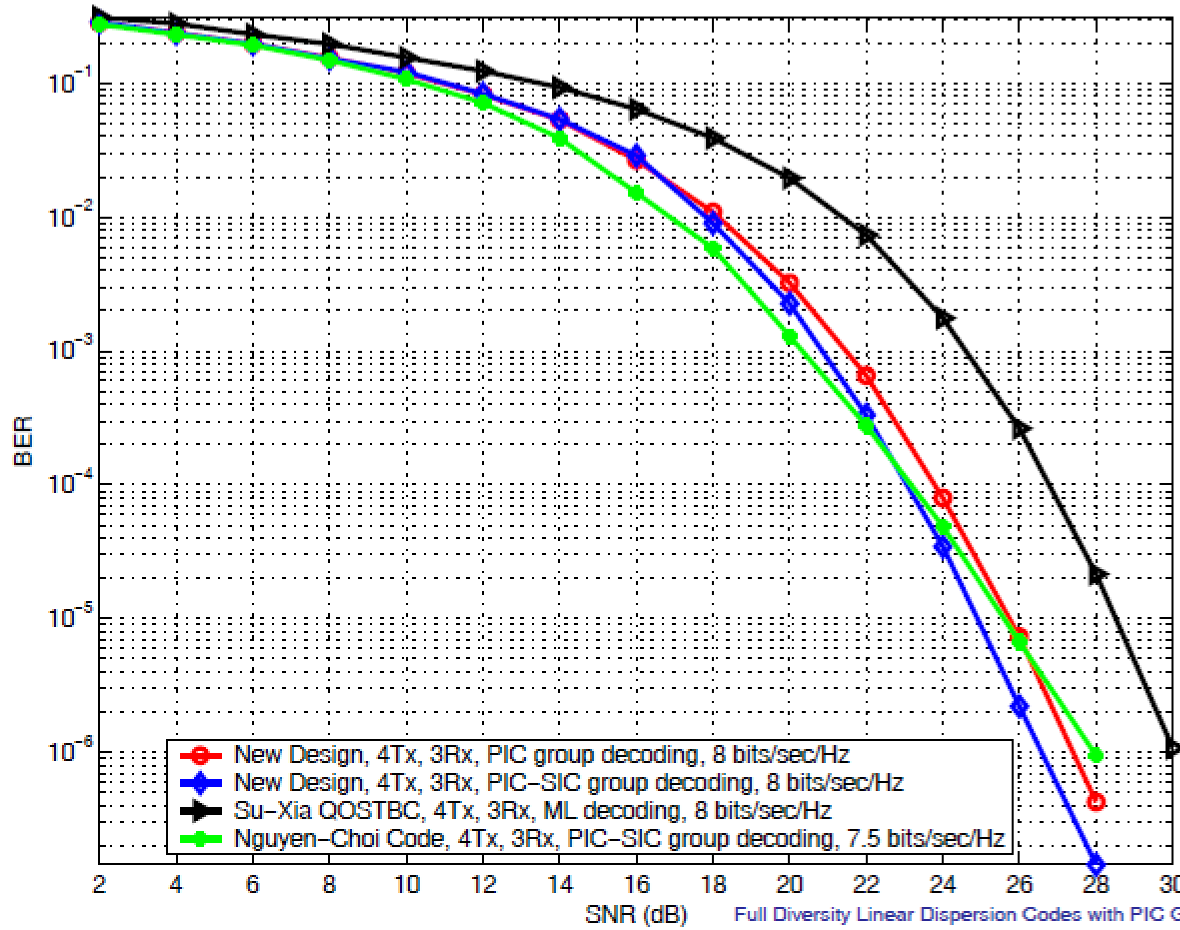
Simulation Results

2 bits/sec/Hz, 2Tx, 3Rx, Rate $1/4$ turbo code



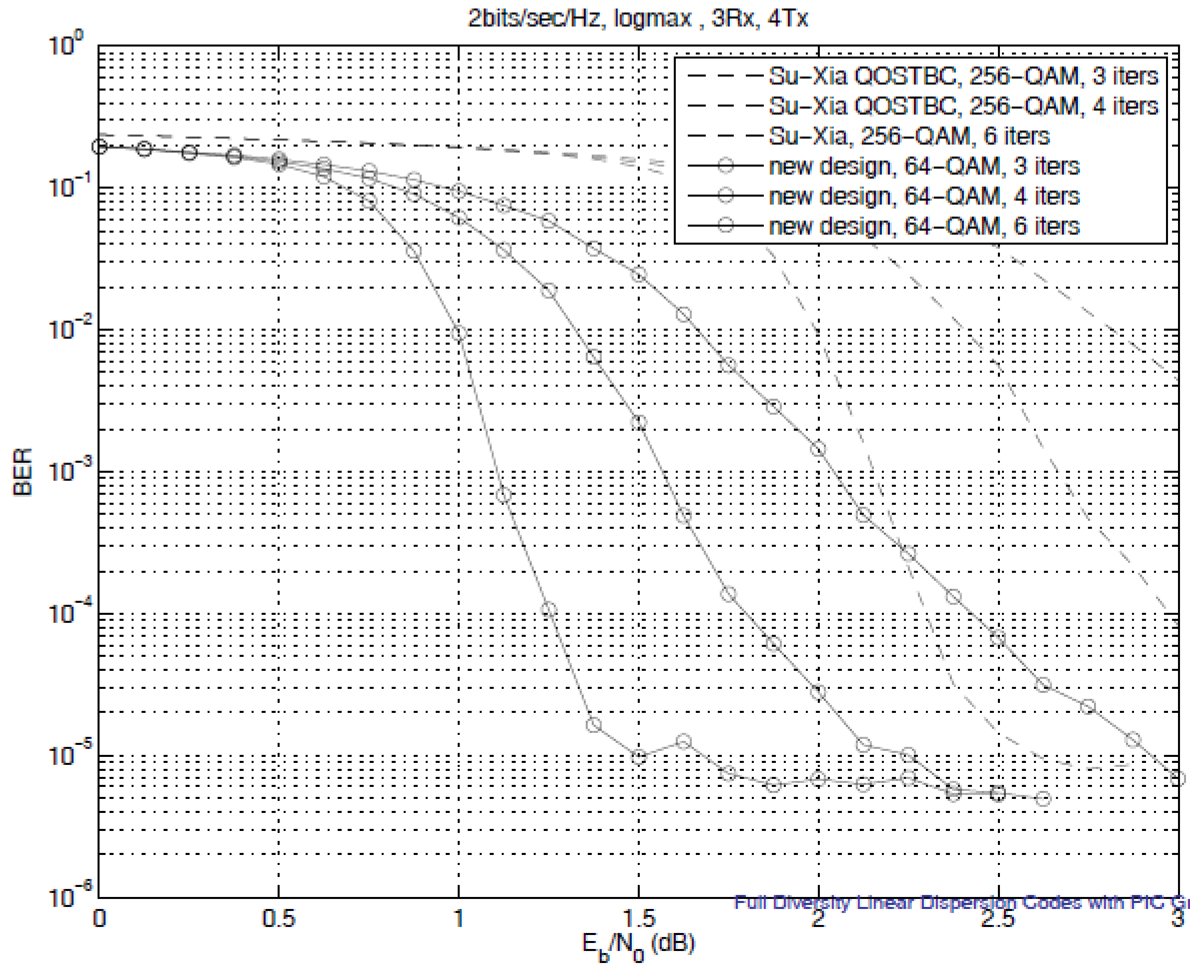
Simulation Results

4Tx, 3Rx, New Design, QOSTBC 8bits/sec/Hz; Nguyen-Choi Code 7.5b



Simulation Results

2 bits/sec/Hz, 4Tx, 3Rx, Rate $1/4$ turbo code



More Designs

- Zhang-Xia (‘09), Globecom 2009;
- Basar-Aygolu (‘09), ISWCS 2009;
  - Obtained a design for two Tx that achieves full diversity with single-complex-symbol-wise decoding but symbol rate higher than 1 (for OSTBC, the highest rate is 1)
- Zhang-Shi-Xia (‘10), ISIT 2010
- Xu-Xia (‘10), ISIT 2010, Conditional PIC Group Decoding, 2011 TIT
- Shi-Zhang-Xia (2010, ArXiv, 2011 TCOM)
- Zhang-Xu-Xia (2011, TIT, rates >2)
- Natarajan-Rajan (2010, ArXiv, 2011 TWC)

n_t transmit antennas

Decoding algorithms	Complexity	Rates	Full diversity criterion
Maximum-likelihood (ML)	Highest	Highest (n_t)	Weakest: Full rank criterion Linear independence of equivalent channel column vectors over signal constellation
Conditional PIC group decoding			
PIC group decoding		Group size(K)	
Linear receiver (ZF/MMSE)	Lowest	Lowest (1)	Strongest Linear independence of equivalent channel column vectors
Orthogonal codes	Un-necessary	1/2	Orthogonal

Conclusion and Future Research

- Presented a bridge between the most complicated but optimal ML receiver and the simplest linear receivers: PIC group decoding
- Presented code/modulation design criterion to achieve full diversity with PIC group decoding
- A trade-off of complexity-rate-performance/diversity
- Some future research problems
  - How to systematically design these codes for an arbitrarily given grouping scheme
  - How to design these codes with optimal coding gains
  - Space-Time-Frequency code/modulation designs with low decoding complexity for MIMO-OFDM systems
  - Distributed space-time codes for multi-user and cooperative systems
  - Space-time code/modulation research is NOT finished yet

Thank You