

30 Years of Generalized and Robust Chinese Remainder Theorems

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Abstract

In this short article, I would like to briefly summarize the works on generalized and robust Chinese remainder theorems (CRT) my collaborators and I have carried out over the past 30 years. It is also dedicated to the 30th anniversary of my career at the University of Delaware.

1. Generalized CRT

After I joined the University of Delaware in the Fall of 1996, exactly 30 years ago, I started to involve with Chinese remainder theorem (CRT) that is the most world -wise well-known mathematics result from ancient China. It is to uniquely determine a large unknown non-negative integer from its remainders modulo several smaller moduli. In the first 10 years, I was interested in its generalized version where there are multiple unknown integers to determine from their sets of remainders modulo several moduli. The first paper on this topic is [1] that, although, appeared two years later than [2], while [2] is based on [1]. The reason is that the processing of IEEE Transactions on Signal Processing at that time was slow and 3 years from submitting to publishing was not uncommon. Our results on generalized CRT appeared in [1]-[10].

2. Robust CRT

Since 2006, I started to work on robust Chinese remainder theorem (RCRT) that was motivated from the phase unwrapping in radar signal processing. The name RCRT appeared at the first time in 2007 in [11] in the literature. In many applications including phase unwrapping, only remainders of an unknown value can be detected and the detected remainders are usually erroneous. It is known that the conventional CRT is not robust in the sense that a small error in a remainder may cause a large error in the reconstruction. The question is how to robustly recover the unknown value from the erroneous remainders with small errors. The above mentioned RCRT is to answer this

question. Our results on one dimensional RCRT appeared in [11]-[32] with applications in moving target imaging in radar signal processing using linear antenna arrays and etc., and also in digital communications as error correction coding.

3. MD-CRT and Robust MD-CRT

We then started to work on multidimensional CRT (MD-CRT) and its robust versions, i.e., robust MD-CRT. The first paper on this topic is [33]. For MD-CRT, a critical question is whether it can perform better than several independent one dimensional/conventional CRT, i.e., whether non-separable MD-CRT performs better than separable MD-CRT. [40] has positively answered this question. In [41]-[42], we have further shown that for 3D moving target SAR imaging, using robust MD-CRT, non-separable planar antenna arrays can truly perform better than separable planar antenna arrays. Hopefully, this is just the beginning of the investigations of multidimensional array designs in various applications by using MD-CRT and its robust versions.

4. Conclusion

The summary above covers only research to which I contributed in the last 30 years. This article is not intended to review other researchers' work on these topics.

5. Acknowledgement

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6. List of Publications

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