Polynomial Ambiguity Resistant Precoder (PARP)

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In [1,2,3], we introduced the concept of polynomial resistant precoder (PARP) that can be applied to an intersymbol interference (ISI) channel, either single input single output (SISO) or multi-input multi-output (MIMO) channel. With a PARP, in theory neither transmitter nor receiver needs to know the ISI channel, and the receiver can blindly identify an ISI channel and the transmitted signal up to a constant scaling difference. Below let me briefly introduce PARP.

A polynomial matrix $H(z)$ of order p and size $N \times K$ is an N by K matrix whose all entries are polynomials of z^{-1} of order at most p, where there is at least one nonzero coefficient of the highest order z^{-p} among all the polynomial entries. A polynomial matrix $H(z)$ is called irreducible if it has full rank for all nonzero z including z = infinity. A function matrix $V(z)$ is a matrix where all entries are functions of z^{-1}.

Definition 1: An $N \times K$ irreducible polynomial matrix $G(z)$ is r th order polynomial ambiguity resistant (PAR) if the following equation for a $K \times K$ function matrix $V(z)$ has only trivial solutions of the form $V(z)=a(z)I_K$ for some nonzero polynomial $a(z)$ of order at most r :

$$E(z)G(z) = G(z)V(z)$$

where $E(z)$ is an $N \times N$ nonzero polynomial matrix of order at most r, and I_K is the K by K identity matrix. An r th order PAR polynomial matrix is called an r th order polynomial ambiguity resistant precoder (PARP).

The above polynomial ambiguity resistant property only requires the uniqueness of the right hand side matrix $V(z)$ up to a nonzero polynomial.

Definition 2: An $N \times K$ irreducible polynomial matrix $G(z)$ is ***strong*** r th order polynomial ambiguity resistant if the following equation for an $N \times N$ nonzero polynomial matrix $E(z)$ of

order at most r and a $K \times K$ function matrix $V(z)$ have only trivial solutions of the forms $E(z)=a(z)I_N$ and $V(z)=a(z)I_K$ for some nonzero polynomial $a(z)$ of order at most r :

$E(z)G(z) = G(z)V(z)$.

A strong r th order PAR polynomial matrix is called a strong r th order PARP.

The above strong polynomial ambiguity resistant property requires a uniqueness up to a nonzero polynomial not only for the right-hand side matrix $V(z)$ but also for the left-hand side nonzero polynomial matrix $E(z)$. Obviously, strong PARP are PARP, and a (strong) r th order PARP is also a (strong) $(r-1)$th order (strong) PARP.

Some simple properties for PARP are, for example, K has to be less than N, i.e., $K<N$, and any constant matrix G cannot be PARP. This means that some redundancy and memory have to be added in a PARP. PARP and strong PARP have been applied to blind channel identification and/or equalization for both SISO and MIMO channels, and systematically studied and constructed in [1,2,3,4]. It turns out that a (strong) PARP is necessary and sufficient for the blind identifiability from the output and the precoder. More details are referred to [1,2,3,5]. Moreover, some optimality about PARP has been studied in [5], where a precoder is called modulated code (MC) and a PARP is renamed as PARMC.

References

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