Filterbank Precoders for Blind Equalization: Polynomial Ambiguity Resistant Precoders (PARP)

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***Abstract*—Filterbank precoding in the intersymbol interference (ISI) mitigation has recently attracted much attention. Two main areas of such research have been explored. One of them is on filterbank precoding when the ISI channel is known to both the transmitter and the receiver, while the other is on filterbank precoding when the ISI channel is not known to the transmitter or the receiver. This paper is in the second area and the aim is two-fold. We first summarize some recent results on *ambiguity resistant* filterbank precoders for the ISI mitigation when the ISI channel is not known at the transmitter or the receiver, i.e., for blind equalization. We then present some *new* results on the construction and characterization of such precoders. The theory presented in this paper applies to both single antenna (SISO) systems and multiple antenna (MIMO) systems as space–time precoding.**

***Index Terms*—Blind equalization, filterbank precoding, intersymbol interference mitigation, polynomial ambiguity resistant precoders, space–time precoding.**

I. INTRODUCTION

DUE TO the intersymbol interference (ISI), channel equalization is one of the most important tasks in digital communications which becomes more and more important in high-speed communication systems. There have been extensive studies on this problem in the last several decades, most of which focus on the following three areas:1) post equalization techniques, such as zero-forcing (ZF) and decision feedback equalization (DFE) [32], [33]; 2) precoding techniques, such as Tomlinson–Harashima (TH) precoding and trellis precoding [38]–[47]; and 3) multicarrier modulation techniques [34]–[37]. Although many of these techniques have found successful applications in practical systems, their performance usually degrades significantly when the channels have spectrum nulls, and in particular when the SNR is not high. Recently, a new filterbank precoding method shown in Fig. 1(a) was proposed in [1], where the filterbank precoder is channel independent, linear (unlike the TH and trellis precoding, no modulo operation is needed), and more importantly, enables an ideal FIR equalizer for any FIR ISI channel and any kind of signal symbols at the expense of a minimum amount of bandwidth expansion. To construct an equalizer using such an approach, however, the knowledge of an ISI channel is needed at the receiver. This precoding scheme has been generalized to the level of error correction coding (ECC) [15], [16], [52], [17], [50] and is named as modulated coding (MC), i.e., ECC over the complex field. The advantage of MC is that it can be naturally combined with an ISI channel and therefore optimally designed for the ISI mitigation. As a result, the ISI in this case is no longer distortion but a gain. It is shown [15] that for any finite tap ISI channel there always exists MC such that it has coding gain in the ISI channel compared with the uncoded ideal additive white Gaussian noise (AWGN) channel. For the filterbank precoding when the ISI channel is known, see also [11] and [12], where the minimum mean square error (MMSE) criterion for the optimal precoder design is used. The disadvantage of this approach is that both the transmitter and the receiver need to know the ISI channel.

A. Previous Work

As a part of postequalization techniques, blind equalization has attracted much attention lately due to the recent advances in channel identification using output diversities (for example, multiple receivers) [18]–[20]. Spatial diversity (antenna arrays) and temporal diversity (fractional sampling) are the mostly studied ones among possible others. Many blind identification algorithms exploiting either second-order cyclostationary statistics [18]–[31] or algebraic structures (often referred to as the deterministic solutions) [22], [23] have been proposed. However, the use of output diversities inevitably multiplies the number of data samples and therefore causes additional computations at the receiver. A new transmitter-assisted (precoded) blind equalization method has been studied lately in [2], [3], [51], and [4]–[7] as explained below, where the overall data rate expansion over the baud rate is not an integer multiple but a fractional number. The filterbank precoding in [1] is generalized to the blind equalization in [2] without much analysis on a precoder. Later, in [7] some precoding analysis in the time domain is introduced. In [3] and [51], the concept of *ambiguity resistant* precoders (ARP) is first introduced in the z-transform domain for the blind identification by injecting a minimum amount of *structured* redundancy at the transmitter. The paper [3], [51] addresses the blind equalization problem for both a baud-rate sampled single-receiver system and an undersampled multi-receiver system by casting them into a multi-input/multi-output (MIMO) framework with more out-

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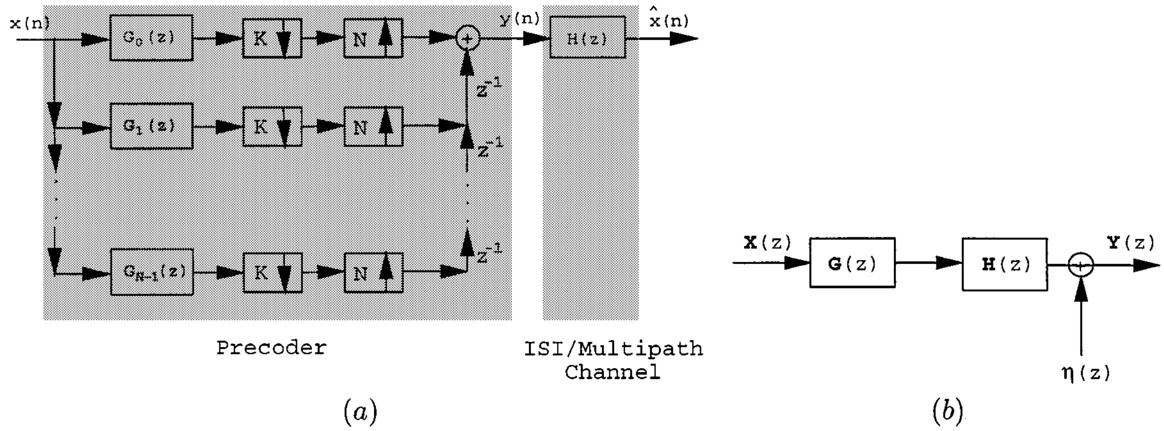

Fig. 1. (a) Filterbank precoding. (b) A general filterbank precoded system of matrix form.

puts than inputs. With existing MIMO identification methods, for example [22], [23], [9], [28], and [29], the multi-input signal can be identified up to a nonsingular constant matrix from the multi-output signal. The ambiguity resistant precoders proposed in [3] and [51] are capable of removing the constant matrix ambiguity directly from the receiver outputs. These precoders can be thought of as a family of the precoders proposed in [1] with an additional ambiguity resistant capability (by adding memory to the precoding), which is essential to the blind identifiability. In [5], ARP are systematically studied and characterized and constructed. To resist an ISI channel, an ARP is sufficient. However, in practical communication systems, the additive noise has to be taken into the account. Therefore, a natural question is which ARP is more robust to the additive noise. In [6], such an issue is addressed, where an optimality on ARP is introduced and some optimal ARP are characterized and constructed. In [4], the concept of the ambiguity resistance is generalized from resisting only constant matrices to any FIR polynomial matrices as shown in Fig. 1(b). For obvious reasons, the precoders studied here are called *(strong) polynomial ambiguity resistant* precoders (PARP). Based on the definitions in [4], strong PARP not only resist the ambiguity in the input signals but also in the FIR channel inverse, while regular PARP only resist the ambiguity in the input signal. In this paper, we shall use the notations and the terminologies used in [4].

B. Outline of This Paper

As one can see, the filterbank precoding is a transmitter-assisted approach and there have been two main areas of research on filterbank precoders in an ISI channel. They are i) MC, when an ISI channel is known at the transmitter and the receiver, where the performance is the key factor and ii) PARP, when an ISI channel is not known at the transmitter or the receiver, where the channel information is the key factor. This paper is focused on the second area, i.e., PARP. The aim of this paper is two-fold. In the first part of this paper (Section II), we want to review the concepts of PARP and strong PARP. We also review the related applications and the blind identifiability in an ISI channel. We show that, for the blind identifiability of the input signal in the precoded system, it is necessary and sufficient for a precoder to be a PARP. The theory developed in this paper applies to both single antenna (SISO) systems and multiple antenna (MIMO) systems as space–time precoding. In the second part (Section III), we present some new properties and constructions of (strong) PARP, such as a new connection between PARP and strong PARP and a new sufficient condition for strong PARP. In Section IV, we present some simple simulation results.

II. Polynomial Ambiguity Resistant Precoders (PARP)

In this section, we review the concept of (strong) PARP and its applications in blind signal identification introduced and studied in [3], [51], [4]. By using the polyphase representation of a filterbank, the precoded system in Fig. 1(a) can be recast into the general one in Fig. 1(b), where $\mathbf{G}(z)$ is the polyphase matrix of $G_0(z), \cdots, G_{N-1}(z)$, and $\mathbf{H}(z)$ corresponds to the pseudo-circulant matrix blocked from $H(z)$; see for example [48]. In what follows, we focus on the general MIMO system in Fig. 1(b), where $\mathbf{G}(z)$ and $\mathbf{H}(z)$ are two polynomial matrices, and the problems of interest are: What is the condition on a precoder $\mathbf{G}(z)$ such that the receiver is able to blindly recover an input signal $\mathbf{X}(z)$ and/or an MIMO channel inverse $\mathbf{H}^{-1}(z)$ given $\mathbf{G}(z)$ and a received signal $\mathbf{Y}(z)$? How to construct such a precoder? We will answer the first question in this section, i.e., $\mathbf{G}(z)$ is PARP, and study the second question, i.e., the construction of PARP, in the next section.

A. Definitions

A *polynomial matrix* $\mathbf{H}(z)$ of size $N \times K$ has the following form:

$$\mathbf{H}(z) = \sum_{m=0}^{Q_h} H(m) z^{-m} \tag{II.1}$$

where $H(m)$ are $N \times K$ constant matrices. $\mathbf{H}(z)$ is also referred to as a matrix polynomial in some literature; see for example, [48]. A function matrix $\mathbf{V}(z)$ is a matrix where all entries are functions of z^{-1}. If $H(Q_h) \neq 0$, the integer Q_h is defined as the *order* of $\mathbf{H}(z)$. A polynomial matrix $\mathbf{H}(z)$ is *invertible* if it has full rank for some value z, whereas $\mathbf{H}(z)$ is *irreducible* if it has full rank for all $z \neq 0$ including $z = \infty$, which is equivalent to the conventional definition of the nonfactorizability [49]. A square polynomial matrix is *unimodular* if its determinant is a nonzero constant. When $N = K$, the irreduciblility of $\mathbf{H}(z)$ is equivalent to the unimodularity of $\mathbf{H}(z)$, i.e., its determinant is a nonzero constant. $\mathbf{H}(z)$ has FIR inverse if and only if $\mathbf{H}(z)$ has determinant cz^{-n_0} for some nonzero constant c and integer n_0. $\mathbf{H}(z)$ is irreducible implies that it has FIR inverse. Clearly, the probability of an $N \times N$ polynomial matrix having FIR inverse is zero. On the other hand, when $N > K$, $\mathbf{H}(z)$ is irreducible if and only if all the determinants of all the $K \times K$ submatrices of $\mathbf{H}(z)$ are coprime, which holds with probability 1 for an arbitrarily given $N \times K$ polynomial matrix $\mathbf{H}(z)$. It is clear that an $N \times K$ irreducible polynomial matrix $\mathbf{H}(z)$ with $K < N$ has an $K \times N$ irreducible polynomial matrix inverse $\mathbf{H}^{-1}(z)$, i.e., $\mathbf{H}^{-1}(z)\mathbf{H}(z) = I_K$, where $\mathbf{H}^{-1}(z)$ may not be unique. For more about unimodular and irreducible polynomial matrices, we refer the reader to Vaidyanathan [48].

We are now ready to define (strong) polynomial ambiguity resistant precoders. First, let us define polynomial ambiguity resistant precoders (PARP).

Definition 1: An $N \times K$ irreducible polynomial matrix $\mathbf{G}(z)$ is rth-order polynomial ambiguity resistant if the following equation for a $K \times K$ function matrix $\mathbf{V}(z)$ has only trivial solutions of format $\mathbf{V}(z) = \alpha(z)I_K$ for some nonzero polynomial $\alpha(z)$ of order at most r:

$$\mathbf{E}(z)\mathbf{G}(z) = \mathbf{G}(z)\mathbf{V}(z) \tag{II.2}$$

where $\mathbf{E}(z)$ is an $N \times N$ nonzero polynomial matrix of order at most r.

The above polynomial ambiguity resistant property only requires the uniqueness of the right-hand side matrix $\mathbf{V}(z)$ up to a nonzero polynomial. Strong PARP are defined as follows.

Definition 2: An $N \times K$ irreducible polynomial matrix $\mathbf{G}(z)$ is *strong* rth-order polynomial ambiguity resistant if the following equation for an $N \times N$ nonzero polynomial matrix $\mathbf{E}(z)$ of order at most r and a $K \times K$ function matrix $\mathbf{V}(z)$ has only trivial solutions of format $\mathbf{E}(z) = \alpha(z)I_N$ and $\mathbf{V}(z) = \alpha(z)I_K$ for some nonzero polynomial $\alpha(z)$ of order at most r:

$$\mathbf{E}(z)\mathbf{G}(z) = \mathbf{G}(z)\mathbf{V}(z).$$

The above strong polynomial ambiguity resistant property requires the uniqueness up to a nonzero polynomial not only for the right-hand side matrix $\mathbf{V}(z)$ but also for the left-hand side nonzero polynomial matrix $\mathbf{E}(z)$. Obviously, strong PARP are PARP. The ambiguity resistant precoders studied in [3] and [51] are the (strong) zeroth-order PARP here. It can be easily verified that a (strong) rth-order PARP is also a (strong) $(r-1)$th-order PARP. We will see later in Section II-B-1 that: i) the input $\mathbf{X}(z)$ is blindly identifiable from the output $\mathbf{Y}(z)$ and the precoder $\mathbf{G}(z)$ in the precoded system in Fig. 1(b) *if and only if* the precoder $\mathbf{G}(z)$ is PARP and ii) both the input $\mathbf{X}(z)$ and the ISI channel inverse $\mathbf{H}^{-1}(z)$ are blindly identifiable from the output $\mathbf{Y}(z)$ and the precoder $\mathbf{G}(z)$ in the precoded system in Fig. 1(b) *if and only if* the precoder $\mathbf{G}(z)$ is strong PARP. The following family of *strong* PARP is first presented in [3], [51], [4].

Theorem 1: The following polynomial matrix $\mathbf{G}(z)$ of size $N\times(N-1)$ is strong rth-order polynomial ambiguity resistant:

$$\mathbf{G}(z) = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ z^{-r-1} & 1 & 0 & \cdots & 0 & 0 \\ 0 & z^{-r-1} & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & z^{-r-1} & 1 \\ 0 & 0 & 0 & \cdots & 0 & z^{-r-1} \end{bmatrix}_{N\times(N-1)} \tag{II.3}$$

for an integer $r \geq 0$.

We shall characterize (strong) PARP later in Section III.

B. Applications in Blind Identification

We now discuss the application of the PARP to blind system identification of a MIMO communication system with ISI/multipath channels.

1) Blind Identifiability: A general ISI communication system is shown in Fig. 1(b), where $\mathbf{X}(z)$ is the input signal of size $K \times K$, $\mathbf{G}(z)$ is the precoder of size $N \times K$, $\mathbf{H}(z)$ is an ISI channel transfer matrix of size $M \times N$, $\mathbf{Y}(z)$ is the output signal of size $M \times K$, $K < N < M$, and $\eta(z)$ is the additive noise term of size $M \times K$. Herein, the goal is to identify $\mathbf{X}(z)$ from $\mathbf{Y}(z)$ without knowing the ISI channel characteristics. Note that $\mathbf{G}(z)$ is chosen by the designer and is thus known to the receiver. The techniques presented here concern the exploitation of the precoder structure in removing the unknown channel effects.

Since $\mathbf{H}(z)$ is almost surely irreducible, we assume it is irreducible in the remainder of this paper. The irreducibility of $\mathbf{H}(z)$ ensures that its inverse is also a polynomial matrix and thus input can be perfectly recovered from the output using FIR equalizers.

There are essentially two problems to be studied in blind identification. One on blind identifiability and the other on blind identification algorithm development. For convenience, we assume a noise-free system and set $\eta(z)$ to be zero. In the case of $K = 1$, the overall system in Fig. 1(b) is a single-input/multiple-output (SIMO) system, which has been extensively studied [18]–[31]. Therefore, in the following we only consider the case where $K > 1$. For an input $K \times K$ signal $\mathbf{X}(z)$ with $K > 1$, the greatest common divisor (gcd) of all component polynomials of $\mathbf{X}(z)$ is almost surely a nonzero constant. Such is assumed throughout our discussions. Note that considering $K \times K$ input signals is equivalent to considering $K \times 1$ input signals. Clearly, a column of a $K \times K$ input signal is a $K \times 1$ input signal. Conversely, a $K \times K$ input signal can be obtained by splitting a $K \times 1$ signal into K many $K \times 1$ signals and putting these K many $K \times 1$ signals together.

We first study the blind identifiability for the input signal. Knowing $\mathbf{Y}(z)$, let $\mathbf{X}_1(z)$ and $\mathbf{H}_1(z)$ be the candidate input and channel, respectively. The gcd of the components of $\mathbf{X}_1(z)$ is assumed to be a nonzero constant, whereas $\mathbf{H}_1(z)$ is an $M \times N$ irreducible polynomials as $\mathbf{H}(z)$. Then, the blind identifiability can be described by the following uniqueness:

$$\mathbf{Y}(z) = \mathbf{H}_1(z)\mathbf{G}(z)\mathbf{X}_1(z) = \mathbf{H}(z)\mathbf{G}(z)\mathbf{X}(z)$$

implies

$$\mathbf{X}_1(z) = \alpha \mathbf{X}(z) \tag{II.4}$$

for some nonzero α. The uniqueness (II.4) implies that the input signal $\mathbf{X}(z)$ can be uniquely determined up to a scale from the output signal $\mathbf{Y}(z)$ and the known precoder $\mathbf{G}(z)$. In other words, the input signal $\mathbf{X}(z)$ can be blindly identified. It should be noticed that without the precoder $\mathbf{G}(z)$ in Fig. 1(b), the input signal $\mathbf{X}(z)$ can only be blindly identified up to a $K \times K$ nonsingle constant matrix $\mathbf{T}$ ambiguity by using MIMO blind identification techniques [22], [28]–[30].

In [3] and [51], blind identification is accomplished in two steps. First, existing MIMO blind identification techniques are used to determine the input signal within a matrix ambiguity, $\mathbf{T}$, and then this constant matrix ambiguity $\mathbf{T}$ is resisted through a zeroth-order PAR precoder. In this subsection, we study the possibility of employing a proper order PARP so that the input signal $\mathbf{X}(z)$ can be directly identified from the output signal $\mathbf{Y}(z)$ using a closed-form algebraic algorithm.

The input signal blind identifiability in (II.4) can be reformulated as follows by pre- and post-multiplying $\mathbf{H}_1^{-1}(z)$ and $\mathbf{X}^{-1}(z)$, respectively, to both sides

$$\mathbf{H}_1^{-1}(z)\mathbf{H}(z)\mathbf{G}(z) = \mathbf{G}(z)\mathbf{X}_1(z)\mathbf{X}^{-1}(z)$$

implies

$$\mathbf{X}_1(z)\mathbf{X}^{-1}(z) = \alpha I_K \tag{II.5}$$

for some nonzero constant α, where $\mathbf{H}_1^{-1}(z)$ is a left inverse of $\mathbf{H}_1(z)$, i.e., $\mathbf{H}_1^{-1}(z)\mathbf{H}_1(z) = I_N$. Note that (II.4) is stronger than (II.5) since $\mathbf{H}_1(z)\mathbf{G}(z)\mathbf{X}_1(z) = \mathbf{H}(z)\mathbf{G}(z)\mathbf{X}(z)$ indicates $\mathbf{H}_1^{-1}(z)\mathbf{H}(z)\mathbf{G}(z) = \mathbf{G}(z)\mathbf{X}_1(z)\mathbf{X}^{-1}(z)$ but not vice versa.

The $N \times N$ matrix $\mathbf{H}_1^{-1}(z)\mathbf{H}(z)$ is almost surely a nonzero polynomial matrix. If $\mathbf{H}_1^{-1}(z)\mathbf{H}(z)$ has order at most r, then as long as $\mathbf{G}(z)$ is rth-order polynomial ambiguity resistant, (II.5) implies $\mathbf{X}_1(z)\mathbf{X}^{-1}(z) = \alpha(z)I_K$, i.e., $\mathbf{X}_1(z) = \alpha(z)\mathbf{X}(z)$ for a nonzero polynomial $\alpha(z)$ of order at most r. This implies that a rth-order PARP $\mathbf{G}(z)$ can reduce the $M \times N$ polynomial matrix ambiguity into a scalar polynomial ambiguity. Under the assumption that the gcd of all components of $\mathbf{X}_1(z)$ is a nonzero constant, we can easily reduce $\alpha(z)$ to a constant scalar, α. This proves that if a signal $\mathbf{X}_1(z)$ with the gcd of its all components as a nonzero constant, and $\mathbf{Y}(z) = \mathbf{H}_1(z)\mathbf{G}(z)\mathbf{X}_1(z)$, then $\mathbf{X}_1(z) = \alpha\mathbf{X}(z)$ for a nonzero constant. In other words, the input signal $\mathbf{X}(z)$ is blindly identifiable.

The above discussions imply that when $\mathbf{G}(z)$ is rth-order polynomial ambiguity resistant, the input signal $\mathbf{X}(z)$ can be blindly identified from the output $\mathbf{Y}(z)$ and the precoder $\mathbf{G}(z)$. In order to choose a proper precoder $\mathbf{G}(z)$, it is important to estimate the minimal order r of the polynomial matrix $\mathbf{H}_1^{-1}(z)\mathbf{H}(z)$ given the ISI channel order of $\mathbf{H}(z)$, Q_h.

It is known that the order $Q_{h^{-1}}$ of $\mathbf{H}^{-1}(z)$ satisfies

$$Q_{h^{-1}} \geq \frac{NQ_h + N - M}{M - N}$$

where the lower bound is achievable; see for example [23] and [30]. Therefore, the total order r of $\mathbf{H}_1^{-1}(z)\mathbf{H}(z)$ satisfies

$$r \geq \frac{NQ_h + N - M}{M - N} + Q_h = \frac{NQ_h}{M - N}.$$

Conversely, if $\mathbf{V}(z)$ in (II.2) has a nontrivial solution $\mathbf{V}(z) \neq \alpha(z)I_K$, the inputs $\mathbf{X}(z)$ and $\mathbf{X}_1(z)$ with $\mathbf{X}(z) = \mathbf{V}(z)\mathbf{X}_1(z)$ and $\mathbf{H}_1(z) = \mathbf{H}(z)\mathbf{E}(z)$ satisfy

$$\mathbf{Y}(z) = \mathbf{H}_1(z)\mathbf{G}(z)\mathbf{X}_1(z) = \mathbf{H}(z)\mathbf{G}(z)\mathbf{X}(z).$$

Therefore, it is not possible to identify the input signal.

The above results are summarized in the following theorem.

Theorem 2: Assume the ISI channel $\mathbf{H}(z)$ is an $M \times N$ irreducible polynomial matrix with order Q_h. If $\mathbf{G}(z)$ is a rth-order polynomial ambiguity resistant precoder, then, the input signal $\mathbf{X}(z)$ in Fig. 1(b) is blindly identifiable from the output signal $\mathbf{Y}(z)$ and the precoder $\mathbf{G}(z)$, where

$$r = \left\lceil \frac{NQ_h}{M - N} \right\rceil. \tag{II.6}$$

On the other hand, if the input signal $\mathbf{X}(z)$ in Fig. 1(b) is blindly identifiable from the output signal $\mathbf{Y}(z)$ and the precoder $\mathbf{G}(z)$, $\mathbf{G}(z)$ must be a polynomial ambiguity resistant precoder of a certain order.

Similar arguments apply to the blind identifiability for both the channel inverse $\mathbf{H}^{-1}(z)$ and the input signal $\mathbf{X}(z)$ by using strong PARP: $\mathbf{Y}(z) = \mathbf{H}_1(z)\mathbf{G}(z)\mathbf{X}_1(z) = \mathbf{H}(z)\mathbf{G}(z)\mathbf{X}(z)$ if and only if $\mathbf{H}_1^{-1}(z)\mathbf{H}(z) = \alpha(z)I_N$ and $\mathbf{X}_1(z)\mathbf{X}^{-1}(z) = \alpha(z)I_k$, i.e., $\mathbf{H}_1^{-1}(z) = \alpha(z)\mathbf{H}^{-1}(z)$ and $\mathbf{X}_1(z) = \alpha(z)\mathbf{X}(z)$ for some nonzero polynomial $\alpha(z)$. Following the proof of Theorem 2 about the gcd division, $\alpha(z)$ can be found from $\mathbf{X}_1(z) = \alpha(z)\mathbf{X}(z)$, and then $\mathbf{H}^{-1}(z)$ can be found from $\mathbf{H}_1^{-1}(z) = \alpha(z)\mathbf{H}^{-1}(z)$. The necessity is also similar to the one for Theorem 2. This proves the following result.

Theorem 3: Assume the ISI channel $\mathbf{H}(z)$ is an $M \times N$ irreducible polynomial matrix with order Q_h. If the precoder $\mathbf{G}(z)$ is strong rth-order polynomial matrix ambiguity resistant, then the input signal $\mathbf{X}(z)$ and the ISI channel inverse $\mathbf{H}^{-1}(z)$ in Fig. 1(b) are blindly identifiable from the output signal $\mathbf{Y}(z)$ and the precoder $\mathbf{G}(z)$, where r is defined in (II.6). On the other hand, if the input signal $\mathbf{X}(z)$ and the channel inverse $\mathbf{H}^{-1}(z)$ in Fig. 1(b) are blindly identifiable from the output signal $\mathbf{Y}(z)$ and the precoder $\mathbf{G}(z)$, $\mathbf{G}(z)$ must be a strong polynomial ambiguity resistant precoder of a certain order.

As a remark on the blind identifiability, since $\mathbf{H}(z)$ is not a square matrix, its inverse $\mathbf{H}^{-1}(z)$ is not unique. The above blind identifiability means the unique solution (up to a nonzero constant difference) for the input signal $\mathbf{X}(z)$ and a solution for the inverse $\mathbf{H}^{-1}(z)$ of $\mathbf{H}(z)$. Although the overall solutions for $\mathbf{X}(z)$ and $\mathbf{H}^{-1}(z)$ may not be unique due to the nonuniqueness of $\mathbf{H}^{-1}(z)$, the input signal part $\mathbf{X}(z)$ is always unique. Another remark is that although a PARP is good to resist an ISI channel, its sensitivity to additive noise is not addressed in this paper. A design property was proposed in [6]. As a final remark, the irreducibility of an MIMO channel $\mathbf{H}(z)$ in Theorems 2 and 3 is satisfied almost surely as mentioned before for a randomly given $M \times N$ polynomial matrix $\mathbf{H}(z)$ when $M \neq N$.

2) An Algebraic Blind Identification Algorithm: Results in Section II-B-1 suggest an algebraic algorithm for the blind identification: solve for $\mathbf{X}_1(z)$ in the equation $\mathbf{Y}(z) = \mathbf{H}_1(z)\mathbf{G}(z)\mathbf{X}_1(z)$ from the known output $\mathbf{Y}(z)$ and the precoder $\mathbf{G}(z)$; then remove the scalar polynomial, $\alpha(z)$, from $\mathbf{X}_1(z)$ to obtain $\alpha\mathbf{X}(z)$.

Although the input and output signals $\mathbf{X}(z)$ and $\mathbf{Y}(z)$ are in matrix forms in the previous sections, they can also be column vectors by equating corresponding columns in the matrices. To derive a time-domain closed-form algorithm, we adopt the vector representation for the input and output in the following discussion. More specifically, we consider

$$\mathbf{Y}(z) = \mathbf{H}(z)\mathbf{G}(z)\mathbf{X}(z) \tag{II.7}$$

where $\mathbf{X}(z)$ is of size $K \times 1$ and $\mathbf{Y}(z)$ is of size $M \times 1$. $\mathbf{H}(z)$ is the $M \times N$ irreducible ISI channel of order Q_h, and $\mathbf{G}(z)$ the strong rth-order PARP, where r takes the value in (II.6). The parameters K, N, M satisfy the inequalities $K < N < M$.

It is established in the previous section that solutions of

$$\hat{\mathbf{H}}_1^{-1}(z)\mathbf{Y}(z) = \mathbf{G}(z)\hat{\mathbf{X}}_1(z) \tag{II.8}$$

satisfy $\hat{\mathbf{X}}_1(z) = \alpha_1(z)\mathbf{X}(z)$ and $\hat{\mathbf{H}}_1^{-1}(z)\mathbf{H}(z) = \alpha_1(z)\mathbf{I}(z)$. Replacing $\mathbf{Y}(z)$ with $z^r\mathbf{Y}(z)$ in the above equation yields

$$\hat{\mathbf{H}}_2^{-1}(z)z^r\mathbf{Y}(z) = \mathbf{G}(z)\hat{\mathbf{X}}_2(z) \tag{II.9}$$

where $\hat{\mathbf{H}}_2^{-1}(z)$ and $\hat{\mathbf{X}}_2(z)$ are the solutions corresponding to the received signal $z^r\mathbf{Y}(z)$. Clearly, $\hat{\mathbf{X}}_2(z) = z^r\alpha_2(z)\mathbf{X}(z)$. To exploit the precoder structure and remove the scalar polynomial from the input estimate in one shot, consider the following equation set:

$$\begin{cases} \hat{\mathbf{H}}_1^{-1}(z)\mathbf{Y}(z) = \mathbf{G}(z)\hat{\mathbf{X}}(z) \\ \hat{\mathbf{H}}_2^{-1}(z)z^r\mathbf{Y}(z) = \mathbf{G}(z)\hat{\mathbf{X}}(z) \end{cases}. \tag{II.10}$$

Then $\hat{\mathbf{X}}(z) = \alpha_1(z)\mathbf{X}(z)$ and at the same time, $\hat{\mathbf{X}}(z) = z^r\alpha_2(z)\mathbf{X}(z)$. Since $\alpha_1(z)$ and $\alpha_2(z)$ are of order at most r, it is not difficult to show that $\hat{\mathbf{X}}(z)$ must be of form $\alpha\mathbf{X}(z)$. Hence, the input sequence can be uniquely identified by solving the above linear equation set in the time domain.

Denote $\mathbf{F}(z) = \mathbf{H}^{-1}(z)$. From previous discussion, the minimum achievable order of $\mathbf{F}(z)$, Q_f is given by

$$Q_f = \left\lceil \frac{NQ_h + N - M}{M - N} \right\rceil. \tag{II.11}$$

Let

$$\mathbf{F}(z) = \sum_{m=0}^{Q_f} F(m)z^{-m} \quad \text{and} \quad \mathbf{G}(z) = \sum_{m=0}^{Q_g} G(m)z^{-m}$$
$$\mathbf{X}(z) = \sum_{m=0}^{Q_x} X(m)z^{-m} \quad \text{and} \quad \mathbf{Y}(z) = \sum_{m=0}^{Q_y} Y(m)z^{-m}.$$

Then from $\mathbf{F}(z)\mathbf{Y}(z) = \mathbf{G}(z)\mathbf{X}(z)$ we have

$$\sum_m \left(\sum_{l=0}^{Q_f} F(l)Y(m-l) \right) z^{-m} = \sum_m \left(\sum_{l=0}^{Q_g} G(l)X(m-l) \right) z^{-m}$$

i.e.,

$$\sum_{l=0}^{Q_f} F(l)Y(m-l) = \sum_{l=0}^{Q_g} G(l)X(m-l), \qquad m \in \mathbf{Z} \tag{II.12}$$

where $F(m)$, $0 \le m \le Q_f$, and $X(m)$, $0 \le m \le Q_g$, are unknowns to solve. For each m, let

$$F(m) = \begin{bmatrix} f_{1,m} \\ f_{2,m} \\ \vdots \\ f_{N,m} \end{bmatrix}$$

where $f_{l,m}$ is the lth row of the matrix $F(m)$. Denote $\mathcal{F}$ a *super* column vector containing all unknowns in matrices $F(m)$, $0 \le m \le Q_f$, i.e.,

$$\mathcal{F} = (f_{1,0}, \cdots, f_{N,0}, f_{1,1}, \cdots, f_{N,1}, \cdots, f_{1,Q_f}, \cdots, f_{N,Q_f})^T. \tag{II.13}$$

The size of $\mathcal{F}$ is $(MN(Q_f+1)) \times 1$. Let $\mathcal{Y}(m)$ be the block matrix, shown in (II.14) at the bottom of the next page, of size $N \times (MN(Q_f+1))$ for each integer m.

Then, the time-domain equivalent of (II.10) is given by

$$\mathcal{Y}(m)\mathcal{F}_1 = (G(0)\cdots G(f)) \begin{bmatrix} \hat{X}(m) \\ \vdots \\ \hat{X}(m-Q_g) \end{bmatrix}, \qquad m \ge 0 \tag{II.15}$$

and

$$\mathcal{Y}(m+r)\mathcal{F}_2 = (G(0)\cdots G(f)) \begin{bmatrix} \hat{X}(m) \\ \vdots \\ \hat{X}(m-Q_g) \end{bmatrix}, \qquad m \ge 0. \tag{II.16}$$

Upon defining $\mathcal{Y}_i = [\mathcal{Y}^T(i)\cdots\mathcal{Y}^T(Q_x - r)]^T$, we are able to combine the above equations and establish a linear equation set with respect to all unknowns as follows:

$$\begin{bmatrix} \mathcal{Y}_0 & \mathbf{0} & -\mathcal{G} \\ \mathbf{0} & \mathcal{Y}_r & -\mathcal{G} \\ \mathcal{Y}_0 & -\mathcal{Y}_r & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathcal{F}_1 \\ \mathcal{F}_2 \\ X(0) \\ \vdots \\ X(Q_x - r) \end{bmatrix} = \mathbf{0} \tag{II.17}$$

where $\mathcal{G}$ is the generalized Sylvester matrix shown in (II.18) at the bottom of the next page. Since (II.17) is a typical linear system, it can be solved by using any numerical method of linear equations, which is not the focus of this paper.

The input signal as well as the zero-delay and maximum-delay zero-forcing equalizers can be readily determined. It can be easily verified that when the number of data vectors increases, there are more equations than unknowns in the above linear homogeneous system, which renders an overdetermined system with a unique solution.

C. Applications in Communication Systems

In this section, we will apply the theory previously developed to blind identification of a baud-rate sampled communication system and an undersampled system with multiple receivers (antennas). Contrary to most existing blind identification techniques, the use of PARP allows the blind identification to be accomplished without output diversities.

1) Applications in Single-Receiver, Baud-Rate Sampled Systems: A precoded single-receiver communication system is shown in Fig. 2, where the baud-rate sampled ISI channel is characterized by a polynomial $H(z)$ of order q_h.

To apply the blind techniques developed in the previous section, we need to formulate the above system and transfer it into the one shown in Fig. 1(b). To achieve this, we block the output signal $y[n]$ with block size M (from serial to parallel) into an M-element vector, $Y(n)$. The system in Fig. 2 can then be represented as in Fig. 3, where $\tilde{\mathbf{H}}(z)$ is the blocked version (see [48]) of the channel $H(z)$ in Fig. 2

$$\tilde{\mathbf{H}}(z) = \begin{bmatrix} H_0(z) & z^{-1}H_{M-1}(z) & \cdots & z^{-1}H_1(z) \\ H_1(z) & H_0(z) & \cdots & z^{-1}H_2(z) \\ \vdots & \vdots & \vdots & \vdots \\ H_{M-2}(z) & H_{M-3}(z) & \cdots & z^{-1}H_{M-1}(z) \\ H_{M-1}(z) & H_{M-2}(z) & \cdots & H_0(z) \end{bmatrix} \tag{II.19}$$

where $H_l(z)$ is the lth polyphase component of $H(z)$ as follows:

$$H_l(z) = \sum_n H(Mn+l)z^{-n}, \qquad 0 \le l \le M-1. \tag{II.20}$$

The matrix $\tilde{\mathbf{H}}(z)$ is pseudo-circulant and can be diagonalized as follows (see [42], and [48]): Let $\mathbf{W}_M$ be the $M \times M$ DFT matrix, i.e., $\mathbf{W}_M \triangleq (W_M^{jk})_{0 \le j,\, k \le M-1}$, where $W_M = e^{-2\pi\sqrt{-1}/M}$; $\mathbf{\Lambda}_M(z)$ the diagonal polynomial matrix

$$\mathbf{\Lambda}_M(z) \triangleq \mathrm{diag}(1,\, z^{-1},\, \cdots,\, z^{-M+1})$$

and $\mathbf{V}(z)$ the following diagonal polynomial matrix in terms of the polynomial $H(z)$:

$$\mathbf{V}(z) \triangleq \mathrm{diag}\,(H(z),\, H(zW_M),\, \cdots,\, H(zW_M^{M-1})). \tag{II.21}$$

Then

$$\tilde{\mathbf{H}}(z^M) = (\mathbf{W}_M^*\mathbf{\Lambda}_M(z))^{-1}\mathbf{V}(z)\mathbf{W}_M^*\mathbf{\Lambda}_M(z). \tag{II.22}$$

For a precoder to resist the polynomial ambiguity, $\tilde{\mathbf{G}}(z)$ and $\tilde{\mathbf{H}}(z)$ must be rearranged so that the channel becomes a tall and irreducible polynomial matrix. Clearly, when $H(z)$ is not a nonzero constant, the polynomial matrix $\tilde{\mathbf{H}}(z)$ is not irreducible. Although this is true, it has been proved in [1] that any $M \times N$ submatrix of $\tilde{\mathbf{H}}(z)$ is irreducible as long as two rotations of the zero set of the polynomial $H(z)$ at the angles W_M^m for $0 \le m \le M-1$ do not intersect. Since this condition is satisfied almost surely for a polynomial $H(z)$, we may assume that all $M \times N$ submatrices $\mathbf{H}(z)$ of $\tilde{\mathbf{H}}(z)$ are irreducible when $N < M$. Hence we can design the $M \times K$ precoder in Figs. 2 and 3 to be

$$\tilde{\mathbf{G}}(z) = \begin{bmatrix} I_N \\ 0_{(M-N)\times N} \end{bmatrix} \mathbf{G}(z) \tag{II.23}$$

$$\mathcal{Y}(m) = \begin{bmatrix} Y^T(m) & \cdots & Y^T(m-Q_f) & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \cdots & 0 & Y^T(m) & \cdots & Y^T(m-Q_f) & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & Y^T(m) & \cdots & Y^T(m-Q_f) \end{bmatrix} \tag{II.14}$$

$$\mathcal{G} = \begin{bmatrix} G(0) & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ G(1) & G(0) & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ G(Q_g) & G(Q_g-1) & G(Q_g-2) & \cdots & G(0) & 0 & \cdots & 0 \\ 0 & G(Q_g) & G(Q_g-1) & \cdots & G(1) & G(0) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & G(Q_g) & \cdots & G(0) \end{bmatrix} \tag{II.18}$$

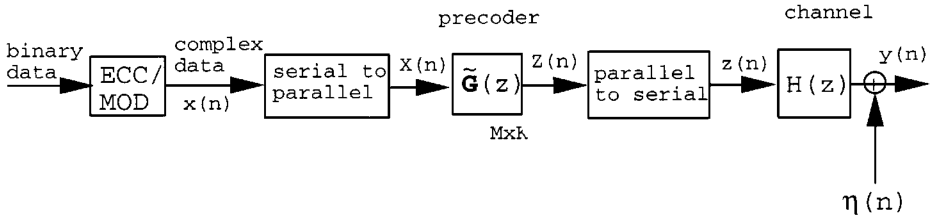

Fig. 2. A single-receiver communication system with baud-rate sampling.

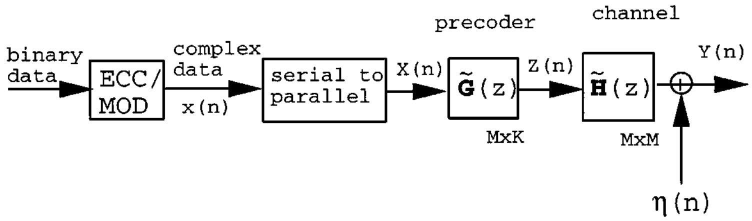

Fig. 3. A blocked single-receiver system with baud-rate sampling.

where $\mathbf{G}(z)$ is an $N \times K$ PARP studied previously. Consequently, the system in Fig. 3 can be described as follows:

$$\mathbf{Y}(z) = \mathbf{H}(z)\mathbf{G}(z)\mathbf{X}(z)$$

and

$$\mathbf{H}(z) = \tilde{\mathbf{H}}(z)\begin{bmatrix} I_N \\ 0_{(M-N)\times N}\end{bmatrix} \tag{II.24}$$

where $\mathbf{H}(z)$ is actually an $M \times N$ submatrix of the $M \times M$ pseudo-circulant matrix $\tilde{\mathbf{H}}(z)$ in (II.19), which is irreducible. From (II.24), it is clear that the system in Fig. 3 is reduced to the one in Fig. 1(b). The theory/algorithm developed in Section II-B becomes readily applicable to the above single-receiver system in Fig. 2.

Given the order of the ISI channel polynomial $H(z)$, q_h, the order of precoder $\mathbf{G}(z)$, r, can be determined as follows. From (II.19) and (II.20), the order of the pseudo-circulant matrix $\tilde{\mathbf{H}}(z)$ and its submatrix $\mathbf{H}(z)$ is

$$Q_h = \left\lceil \frac{q_h}{M} \right\rceil.$$

From (II.6), the corresponding parameters of the precoder in (II.3) can be set as

$$r = \left\lceil \frac{N\left\lceil \frac{q_h}{M}\right\rceil}{M-N} \right\rceil, \qquad K = N-1 \quad \text{and} \quad M > N. \tag{II.25}$$

With these parameters, the output data rate relative to the input signal rate for the above precoded single-receiver system is $(N/K)(M/N) = M/(N-1)$, where M can be chosen as $N+1$. Thus, the relative data rate increase is $2/(N-1)$, which approaches zero, i.e., no expansion, when N is large. This proves the following theorem.

Theorem 4: For any $\epsilon > 0$, there exists a positive integer N for the precoder $\mathbf{G}(z)$ in (II.3) such that the overall data rate expansion for the single antenna receiver system in Fig. 2 is less than ϵ and at the same time, the input signal $\mathbf{X}(z)$ can be blindly identified from the output $\mathbf{Y}(z)$ using the closed-form algorithm in Section II-B-2.

Notice that the existing blind identification techniques require the data rate to be at least twice the input symbol rate at the receiver.

2) Applications in Undersampled Antenna Array Receiver Systems: Having shown that blind identification can be accomplished with a minimum amount of bandwidth expansion using precoding techniques, we now study the possibility of perfect signal recovery when the received signals are undersampled.

Without loss of generality, an undersampled antenna array system can be shown in Fig. 4, where $H_l(z)$ for $l = 1, 2, \ldots, M$ are the ISI channel transfer polynomials of the M antennas, and $N\downarrow$ means downsampling by factor N, i.e., taking one sample from each N samples. Clearly, only partial information of the input is available in each antenna output. It is proved in [3] and [51] that it is impossible to recover the input blindly from the M outputs without using precoding at the transmitter.

The system in Fig. 4 can be converted to the one in Fig. 5, where $\mathbf{H}(z)$ is the $M \times N$ polyphase matrix of the M polynomials $H_l(z)$, $1 \le l \le M$: $\mathbf{H}(z) = (H_{l,n}(z))_{M\times N}$. Here

$$H_{l,n}(z) = \sum_k H_l(Nk+n)z^{-k}$$

is the nth polyphase component of the lth polynomial $H_l(z) = \sum_m H_l(m)z^{-m}$, and $Y(n) = (y_1(n), y_2(n), \cdots, y_M(n))^T$. As discussed in Section II, when $M > N$ this matrix $\mathbf{H}(z)$ is almost surely irreducible. From Fig. 5, one can see that the undersampled antenna array receiver system in Fig. 4 can be cast into the exact same framework in Fig. 1(b), allowing direct applications of the theory/algorithm developed in Section II-B.

Assume q_h is the maximum of the orders of the M polynomials $H_l(z)$ for the M antennas. The order Q_h of the polyphase matrix $\mathbf{H}(z)$ is

$$Q_h = \left\lceil \frac{q_h}{N} \right\rceil.$$

For blind identification, the parameters for the precoder $\mathbf{G}(z)$

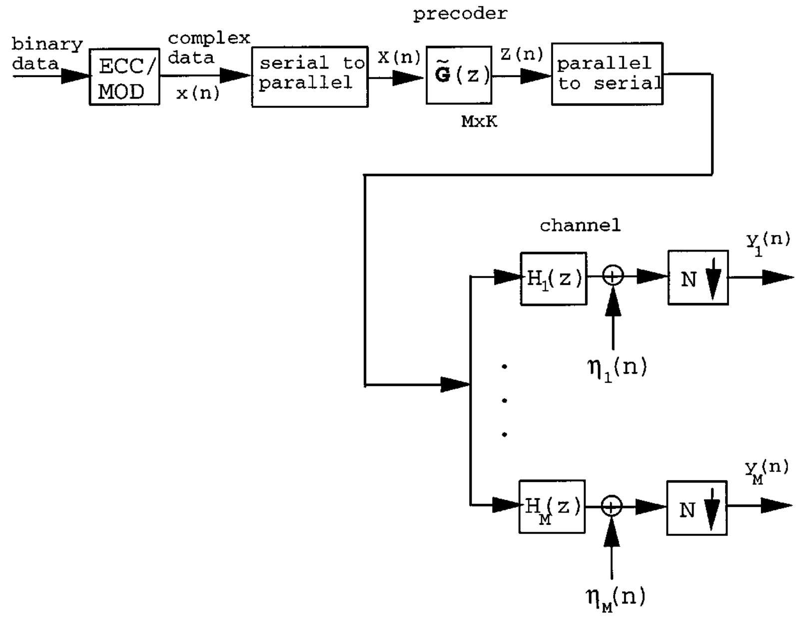

Fig. 4. An undersampled antenna array system.

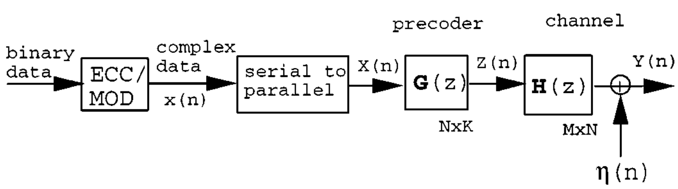

Fig. 5. An equivalent undersampled antenna array system.

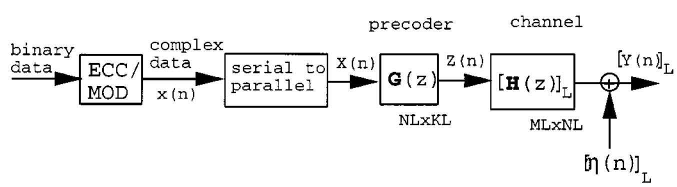

Fig. 6. A blocked undersampled antenna array system.

in (II.3) can be chosen as

$$r = \left\lceil \frac{N\left\lceil \frac{q_h}{N}\right\rceil}{M-N} \right\rceil, \qquad K = N-1 \quad \text{and} \quad M > N. \quad \text{(II.26)}$$

It should be noticed that the number of antennas, M, in a system is usually fixed. Because $N < M$ is required, this seems to provide a lower bound for the data rate expansion in the transmitter, which requires $0 < K < N < M$. With the minimum bandwidth expansion setup: $K = N-1$, $N = M-1$, at least $1/(M-1)$ data rate increase is needed for the blind equalization given the number of antennas, M. In the following, we show that this limitation can be lifted by blocking the vector output sequence $Y(n) = (y_1(n), y_2(n), \cdots, y_M(n))^T$ in Fig. 5 similar to the way for the single antenna system studied in the previous subsection. The blocked equivalent of the undersampled antenna array receiver system in Fig. 5 is shown in Fig. 6, where the block size is L and the matrix $[\mathbf{H}(z)]_L$ is the blocked version of the matrix $\mathbf{H}(z)$ in Fig. 5

$$[\mathbf{H}(z)]_L = \begin{bmatrix} \mathbf{H}_0(z) & z^{-1}\mathbf{H}_{M-1}(z) & \cdots & z^{-1}\mathbf{H}_1(z) \\ \mathbf{H}_1(z) & \mathbf{H}_0(z) & \cdots & z^{-1}\mathbf{H}_2(z) \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{H}_{M-2}(z) & \mathbf{H}_{M-3}(z) & \cdots & z^{-1}\mathbf{H}_{M-1}(z) \\ \mathbf{H}_{M-1}(z) & \mathbf{H}_{M-2}(z) & \cdots & \mathbf{H}_0(z) \end{bmatrix}. \quad \text{(II.27)}$$

Here, $\mathbf{H}_l(z)$ is the lth polyphase component of the matrix $\mathbf{H}(z)$ as follows:

$$\mathbf{H}_l(z) = \sum_n H(Ln+l)z^{-n}, \qquad 0 \le l \le L-1$$

where $H(m)$ are the $M \times N$ constant matrices from $\mathbf{H}(z) = \sum_m H(m)z^{-m}$. Matrix $[\mathbf{H}(z)]_L$ is block pseudo-circulant. $[Y]_L(n)$ and $[\eta]_L(n)$ with size $NL \times 1$ in Fig. 6 are the blocked forms of the vector sequences $Y(n)$ and $\eta(n)$, respectively. Correspondingly, The minimum rate-increase precoder $\mathbf{G}(z)$ has size $NL \times (NL-1)$. Therefore, if the blocked channel polynomial matrix $[\mathbf{H}(z)]_L$ in Fig. 6 is still irreducible, then the system in Fig. 6 is reduced to the one in Fig. 1(b).

Before proving the irreducibility of the matrix $[\mathbf{H}(z)]_L$, let us investigate the effects of the blocking operations above. Notice that the overall data rate expansion in Fig. 6 is $1/((M-1)L)$ by choosing $N = M-1$ and $K = NL-1$, which approaches zero when the block size L is large. The advantage is that the data rate expansion at the transmitter can be reduced by employing the above blocking procedure, even when the number of antennas is fixed.

We now need to prove that the blocked version $[\mathbf{H}(z)]_L$ of $\mathbf{H}(z)$ is irreducible when $\mathbf{H}(z)$ itself is irreducible. Since $[\mathbf{H}(z)]_L$ is block pseudo-circulant, by permuting its rows and columns, it can be converted into the block matrix with MN blocks and each of the blocks $B_{m,n}(z)$ is an $L \times L$ pseudo-circulant matrix

$$[\mathbf{H}(z)]_L = P_l(B_{m,n}(z))_{M\times N}P_r$$

where P_l and P_r are the row and column block permutation matrices. Similar to (II.21) and (II.22) the $L \times L$ pseudo-circulant matrix $B_{m,n}(z^L)$ can be diagonalized as

$$B_{m,n}(z^L) = (\mathbf{W}_L^* \mathbf{\Lambda}_L(z))^{-1} \mathrm{diag}(H_{m,n}(z), H_{m,n}(zW_L), \cdots, H_{m,n}(zW_L^{L-1})) \cdot \mathbf{W}_L^* \mathbf{\Lambda}_L(z)$$

where $H_{m,n}(z)$ come from matrix $\mathbf{H}(z) = (H_{m,n}(z))_{M \times N}$. Therefore

$$[\mathbf{H}(z^L)]_L = P_l [\mathbf{W}]_L^{-1} (\mathrm{diag}(H_{m,n}(z), H_{m,n}(zW_L), \cdots, H_{m,n}(zW_L^{L-1})))_{M \times N} \cdot [\mathbf{W}]_L P_r$$

where

$$[\mathbf{W}]_L = \mathrm{diag}(\mathbf{W}_L^* \mathbf{\Lambda}_L(z), \cdots, \mathbf{W}_L^* \mathbf{\Lambda}_L(z)).$$

By implementing the same permutations

$$[\mathbf{H}(z^L)]_L = P_l [\mathbf{W}]_L^{-1} P_l \mathrm{diag}(\mathbf{H}(z), \mathbf{H}(zW_L), \cdots, \mathbf{H}(zW_L^{L-1})) \cdot P_r [\mathbf{W}]_L P_r.$$

Since matrices $P_l[\mathbf{W}]_L^{-1} P_l$ and $P_r[\mathbf{W}]_L P_r$ are irreducible, matrix $[\mathbf{H}(z)]_L$ is irreducible if and only is $\mathbf{H}(z)$ is irreducible. This proves the following lemma.

Lemma 1: The blocked version $[\mathbf{H}(z)]_L$ in (II.27) of $\mathbf{H}(z)$ is irreducible if and only if $\mathbf{H}(z)$ is irreducible.

This lemma and the previous discussion on data rate expansion in the transmitter lead to the following result.

Theorem 5: For any $\epsilon > 0$, there exists a positive integer N for the precoder $\mathbf{G}(z)$ in (II.3) such that the data rate expansion at the transmitter for the antenna array system in Fig. 5 is less than ϵ and at the same time, the input signal $\mathbf{X}(z)$ can be blindly identified from the undersampled outputs $y_l(n)$, $1 \le l \le M$, of the M antennas with the undersampling factor $N = M - 1$ using the closed-form algorithm in Section II-B-2

It should be noticed that, although blind identifiability in the above two theorems holds theoretically for an arbitrary small amount of data (or bandwidth) expansion, the implementation of the closed-form algorithm in Section II-B-2 may become prohibitive when the sizes of the precoders get larger. We want to emphasize that the focus of this paper is on feasibility studies rather than algorithm development. There is an evident need for more sophisticated precoding-based blind identification algorithms which are of practical importance.

Another remark we want to make here is the following observation. When the order Q_h of the ISI channel $\mathbf{H}(z)$ is large, the size of the linear system (II.17) is also large due to the large number of unknowns in $\mathcal{F}$ in (II.13) for $\mathbf{H}^{-1}(z)$. In this case, it might be better to use the current MIMO blind identification methods to reduce the large order ISI channel $\mathbf{H}(z)$ into a nonsingular constant matrix, i.e., a zero-order ISI channel T. Then, the technique developed in [3] and [51], or zeroth-order polynomial matrix ambiguity resistant precoders in this paper can be used to blindly identify the input signal and the constant ambiguity matrix T. The tradeoff between these two approaches is under our current investigation.

Last but not the least, we want to point out that the precoders proposed in (II.3) have some interesting features which are essential to applications. For example, assuming that the input data to the precoders are modulated complex values, such as $e^{j2\pi k/4}$, $k = 0, 1, 2, 3$, in QPSK modulation, since the precoder in (II.3) only sums the current sample $X(n)$ and the past $X(n-r-1)$ as $X(n) + X(n-r-1)$, the output data $Z(n)$ from the precoder, which are to be transmitted after a pulse shaping filter, preserves the modulation symbol patterns except some occasional 0 symbols. This implies that the precoding in Fig. 2 and Fig. 4 can be implemented without introducing undue complexity.

III. Characterization on Polynomial Ambiguity Resistant Precoders

In this section, we want to present some new and known properties and characterizations of PARP, which are useful in the PARP construction. It was proved in [4] that for a PARP $N \times K$ polynomial matrix we have $N > K$. Therefore, in what follows, we always assume $N > K$ unless otherwise specified. It was also shown in [3], [51], and [4] that any constant precoders cannot be PARP of any order when $K > 1$. When $K = 1$, the precoded system is equivalent to the fractionally spaced equalizer system studied in [18] and [19], which is blindly identifiable. In what follows, we always assume $K > 1$.

A. PARP-Equivalence and Canonical Forms

Let us first see an equivalence for PARP, which is first introduced in [5] for the ambiguity resistant precoder canonical forms. Let $\mathcal{M}_{N \times K}(z)$ denote the set of all $N \times K$ polynomial matrices.

Definition 3: The transformation $T_{\mathbf{P},\mathbf{Q}}$ of $\mathcal{M}_{N \times K}(z)$ defined by

$$T_{\mathbf{P},\mathbf{Q}}(\mathbf{A}(z)) = \mathbf{P}\mathbf{A}(z)\mathbf{Q}(z), \qquad \forall\, \mathbf{A}(z) \in \mathcal{M}_{N \times K}(z)$$

where $\mathbf{P}$ is an $N \times N$ nonsingular constant matrix and $\mathbf{Q}(z)$ is a $K \times K$ unimodular polynomial matrix, is called a PARP-equivalence transformation, and $T_{\mathbf{P},\mathbf{Q}}(\mathbf{A}(z))$ and $\mathbf{A}(z)$ are called PARP-equivalent.

One can see that a PARP-equivalence transformation includes all three row elementary operations with constant multipliers and all three column elementary operations where an operation of multiplying a nonzero degree polynomial to a column is not included. From the PARP-equivalence definition, we have the following result.

Theorem 6: A PARP-equivalence transformation preserves the (strong) rth PARP property, i.e., an $N \times K$ polynomial matrix $\mathbf{G}(z)$ is (strong) rth PARP if and only if $\mathbf{P}\mathbf{G}(z)\mathbf{Q}(z)$ is (strong) rth PARP for any $N \times N$ nonsingular constant matrix $\mathbf{P}$ and any unimodular polynomial matrix $\mathbf{Q}(z)$.

Proof: Consider equation

$$\mathbf{E}(z)\mathbf{P}\mathbf{G}(z)\mathbf{Q}(z) = \mathbf{P}\mathbf{G}(z)\mathbf{Q}(z)\mathbf{V}(z).$$

Then

$$\mathbf{P}^{-1}\mathbf{E}(z)\mathbf{P} \cdot \mathbf{G}(z) = \mathbf{G}(z) \cdot \mathbf{Q}(z)\mathbf{V}(z)\mathbf{Q}(z)^{-1}.$$

If $\mathbf{G}(z)$ is (strong) rth PARP, then we have $(\mathbf{P}^{-1}\mathbf{E}(z)\mathbf{P} = \alpha(z) I_N)$, $\mathbf{Q}(z)\mathbf{V}(z)\mathbf{Q}(z)^{-1} = \alpha(z) I_K$ for some polynomial $\alpha(z)$ of order at most r, i.e., $\mathbf{P}\mathbf{G}(z)\mathbf{Q}(z)$ is (strong) rth PARP.

On the other hand, if $\mathbf{PG}(z)\mathbf{Q}(z)$ is (strong) rth PARP, then from $\mathbf{E}(z)\mathbf{G}(z) = \mathbf{G}(z)\mathbf{V}(z)$ we have

$$\mathbf{PE}(z)\mathbf{P}^{-1} \cdot \mathbf{PG}(z)\mathbf{Q}(z) = \mathbf{PG}(z)\mathbf{Q}(z) \cdot \mathbf{Q}(z)^{-1}\mathbf{V}(z)\mathbf{Q}(z).$$

So, we have $(\mathbf{PE}(z)\mathbf{P}^{-1} = \alpha(z)I_N)$, $\mathbf{Q}(z)^{-1}\mathbf{V}(z)\mathbf{Q}(z) = \alpha(z)I_K$ for some polynomial $\alpha(z)$ of order at most r, i.e., $\mathbf{G}(z)$ is (strong) rth PARP. **q.e.d.**

This theorem tells us that a PARP-equivalent transformation maintains the PARP property. In other words, as soon as a PARP is constructed, its all PARP-equivalent transformations are PARP too. By noticing from Definition 3 that $\mathbf{P}$ and $\mathbf{Q}(z)$ are arbitrarily nonsingular and unimodular matrices, respectively, PARP-equivalent transformations easily provide a rich family of PARP from a single PARP.

The following canonical form under the PARP-equivalence transformation was obtained in [5].

Theorem 7: Any irreducible matrix in $\mathcal{M}_{N\times K}(z)$ is PARP-equivalent to a polynomial matrix of the following form:

$$\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ g_{K1}(z) & g_{K2}(z) & g_{K3}(z) & \cdots & g_{K(K-1)}(z) & g_{KK}(z) \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ g_{N1}(z) & g_{N2}(z) & g_{N3}(z) & \cdots & g_{N(K-1)}(z) & g_{NK}(z) \end{pmatrix} \quad \text{(III.1)}$$

with $\gcd(g_{KK}, g_{(K+1)K}, \cdots, g_{NK}) = 1$ and $\deg g_{KK} \le \deg g_{(K+1)K} \le \cdots \le \deg g_{NK}$. Furthermore, $g_{kl}(z)$ can be either zero or a nonconstant polynomial (i.e., $\deg g_{kl} \ge 1$) for $K \le k \le N$ and $1 \le l \le K-1$, and $g_{N1}(z) = \cdots = g_{N(L-1)}(z) = 0$, $1 \le \deg g_{NL} < \cdots < \deg g_{NK}$ for some L with $1 \le L \le K$.

With the above canonical form, to consider a PARP we only need to consider a PARP of the form (III.1). The following sufficient condition for strong zeroth PARP with $N = K+1$ was obtained in [5].

Theorem 8: Let $\mathbf{G}(z)$ have the canonical form (III.1) with $N = K+1$. If $g_{N1}(z) = \cdots = g_{N(L-1)}(z) = 0$ and $1 \le \deg g_{NL} < \cdots < \deg g_{NK}$ for some L, $1 \le L \le K$, and if

$$\{1, g_{K1}, g_{K2}, \cdots, g_{K(L-1)}, g_{NL}, \cdots, g_{N(K-1)}\}$$

are linearly independent over the complex filed, and $W_1 \cap W_2 = \{0\}$, where

$$W_1 = \text{span}\{g_{NK}, g_{NK}g_{K1}, \cdots, g_{NK}g_{K(K-1)}\}$$
$$W_2 = \text{span}\{g_{KK}, g_{KK}g_{K1}, \cdots, g_{KK}g_{K(L-1)}, g_{KK}g_{NL}, \cdots, g_{KK}g_{N(K-1)}\}$$

where span means the set of all linear combinations with constant coefficients, then $\mathbf{G}(z)$ is strong zeroth order PARP.

It was claimed in [5] that in Theorem 8 the two conditions: i) $\{1, g_{K1}, g_{K2}, \cdots, g_{K(L-1)}, g_{NL}, \cdots, g_{N(K-1)}\}$ are linearly independent and ii) $W_1 \cap W_2 = \{0\}$, are also necessary. However, they are not necessary from the following counterexample:

$$\mathbf{G}(z) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ z^{-1} & z^{-2} & 1+z^{-3} \\ 0 & z^{-1} & z^{-5} \end{pmatrix}. \quad \text{(III.2)}$$

Clearly, $\{1, g_{31}, g_{42}\}$ are not linearly independent, but $\mathbf{G}(z)$ is actually strong zeroth PAR as we shall see later.

To conclude this subsection, we generalize the linear independence as follows.

Definition 4: A set of polynomials $\{g_i(z)\}_{1\le i\le n}$ is said *rth-order linearly independent* (rth LID) if

$$\sum_{i=1}^{n} e_i(z)g_i(z) = 0 \Longleftrightarrow e_i(z) = 0, \qquad i = 1, 2, \cdots, n$$

where $\{e_i(z)\}_{1\le i\le n}$ are polynomials of orders at most r.

In the above definition, when $r=0$, it reduces to the conventional linear independence of polynomials. To give an intuition on the above rth LID, if $\deg(g_i) < \deg(g_{i+1}) - r$, then $g_i(z)$ are rth LID. For example, 1, z^{-r-1}, and z^{-2r-2} are rth LID.

B. (Strong) rth PARP with $N > K > 1$

In this subsection, we want to present a new relationship between rth PARP and strong rth PARP. We also derive a new sufficient condition for the strong rth PARP with $N > K$.

Theorem 9: Let $\mathbf{G}(z)$ be of the canonical form (III.1). If $\mathbf{G}(z)$ is rth PARP, then

$$\max_{K\le i\le N}(\deg g_{iK}) > r.$$

Proof: If $\deg g_{iK} \le r$ for any i with $K \le i \le N$, let

$$\mathbf{E}(z) = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 & 0 & \cdots & 0 \\ g_{KK}(z) & g_{KK}(z) & \cdots & g_{KK}(z) & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ g_{NK}(z) & g_{NK}(z) & \cdots & g_{NK}(z) & 0 & \cdots & 1 \end{pmatrix}$$

and

$$\mathbf{V}(z) = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 & 0 \\ 1 & 1 & \cdots & 1 & 1 \end{pmatrix}.$$

Then we can check that $\mathbf{E}(z)\mathbf{G}(z) = \mathbf{G}(z)\mathbf{V}(z)$, and $\mathbf{E}(z)$ is an $N \times N$ polynomial matrix of order at most r. This is contradictory to the rth PARP of $\mathbf{G}(z)$. **q.e.d.**

This theorem provides a necessary condition for a precoder to be rth PARP, and from this theorem one can clearly see that any constant matrix cannot be PARP of any order. If we have a rth PARP $\mathbf{G}(z)$ of size $N \times K$, it is easy to construct rth PARP of size $M \times K$ with $M > N$ from the following result.

Theorem 10: If an $M \times K$ polynomial matrix $\mathbf{A}(z)$ is PARP-equivalent to $\binom{\mathbf{G}(z)}{\mathbf{0}_{(M-N)\times K}}$, and $\mathbf{G}(z)$ is rth PARP, then $\mathbf{A}(z)$ is also rth PARP. However, $\mathbf{A}(z)$ must not be strong rth PARP, even when $\mathbf{G}(z)$ is strong rth PARP.

Proof: From equation

$$\begin{pmatrix} \mathbf{E}_{11}(z) & \mathbf{E}_{12}(z) \\ \mathbf{E}_{21}(z) & \mathbf{E}_{22}(z) \end{pmatrix} \begin{pmatrix} \mathbf{G}(z) \\ \mathbf{0}_{(M-N)\times K} \end{pmatrix} = \begin{pmatrix} \mathbf{G}(z) \\ \mathbf{0}_{(M-N)\times K} \end{pmatrix} \mathbf{V}(z)$$

we get $\mathbf{E}_{11}(z)\mathbf{G}(z) = \mathbf{G}(z)\mathbf{V}(z)$. Since $\mathbf{G}(z)$ is rth PARP, so $\mathbf{V}(z) = \alpha(z)I_K$ for some polynomial $\alpha(z)$ of order at most r, i.e., $\binom{\mathbf{G}(z)}{\mathbf{0}_{(M-N)\times K}}$ is rth PARP.

However, since

$$\begin{pmatrix} I_N & \mathbf{0} \\ \mathbf{0} & 2I_{M-N} \end{pmatrix} \begin{pmatrix} \mathbf{G}(z) \\ \mathbf{0}_{(M-N)\times K} \end{pmatrix} = \begin{pmatrix} \mathbf{G}(z) \\ \mathbf{0}_{(M-N)\times K} \end{pmatrix} I_K$$

where $\mathbf{E}(z)$ is not equal to any $\alpha(z)I_N$, i.e., $\binom{\mathbf{G}(z)}{\mathbf{0}_{(M-N)\times K}}$ is not strong zeroth PARP. So it is not strong rth PARP either. **q.e.d.**

We now see a new connection between strong PARP and PARP.

Theorem 11: Suppose that $\mathbf{G}(z)$ has the form (III.1) with $N > K$. If $\mathbf{G}(z)$ is rth PARP, and $\{g_{KK}, \cdots, g_{NK}\}$ are rth order linearly independent, then $\mathbf{G}(z)$ is also strong rth PARP.

Proof: $\mathbf{G}(z)$ can be written as

$$\mathbf{G}(z) = \begin{pmatrix} I_{K-1} & \mathbf{0} \\ \mathbf{G}_{11}(z) & \mathbf{G}_{12}(z) \end{pmatrix}$$

where

$$\mathbf{G}_{11}(z) = \begin{pmatrix} g_{K1}(z) & \cdots & g_{K(K-1)}(z) \\ \cdots & \cdots & \cdots \\ g_{N1}(z) & \cdots & g_{N(K-1)}(z) \end{pmatrix}$$

and

$$\mathbf{G}_{12}(z) = \begin{pmatrix} g_{KK}(z) \\ \vdots \\ g_{NK}(z) \end{pmatrix}.$$

Consider equation

$$\begin{pmatrix} \mathbf{E}_{11}(z) & \mathbf{E}_{12}(z) \\ \mathbf{E}_{21}(z) & \mathbf{E}_{22}(z) \end{pmatrix} \begin{pmatrix} I_{K-1} & \mathbf{0} \\ \mathbf{G}_{11}(z) & \mathbf{G}_{12}(z) \end{pmatrix} = \begin{pmatrix} I_{K-1} & \mathbf{0} \\ \mathbf{G}_{11}(z) & \mathbf{G}_{12}(z) \end{pmatrix} \mathbf{V}(z)$$

where $\mathbf{E}_{11}(z)$, $\mathbf{E}_{12}(z)$, $\mathbf{E}_{21}(z)$ and $\mathbf{E}_{22}(z)$ are polynomial matrices of orders at most r. If $\mathbf{G}(z)$ is rth PARP, then $\mathbf{V}(z) = \alpha(z)I_K$ for some polynomial $\alpha(z)$ of order at most r. Therefore

$$\mathbf{E}_{11}(z) + \mathbf{E}_{12}(z)\mathbf{G}_{11}(z) = \alpha(z)I_{K-1} \quad \text{(III.3)}$$

$$\mathbf{E}_{21}(z) + \mathbf{E}_{22}(z)\mathbf{G}_{11}(z) = \alpha(z)\mathbf{G}_{11}(z) \quad \text{(III.4)}$$

$$\mathbf{E}_{12}(z)\mathbf{G}_{12}(z) = \mathbf{0} \quad \text{(III.5)}$$

$$\mathbf{E}_{22}(z)\mathbf{G}_{12}(z) = \alpha(z)\mathbf{G}_{12}(z). \quad \text{(III.6)}$$

Since $\{g_{KK}, \cdots, g_{NK}\}$ are rth order linearly independent, from (III.5) and (III.6) we have $\mathbf{E}_{12}(z) = \mathbf{0}$ and $\mathbf{E}_{22}(z) = \alpha(z)I_{N-K+1}$. Substituting $\mathbf{E}_{12}(z)$ and $\mathbf{E}_{22}(z)$ into (III.3) and (III.4), we obtain $\mathbf{E}_{11}(z) = \alpha(z)I_{K-1}$ and $\mathbf{E}_{21}(z) = \mathbf{0}$. So $\mathbf{G}(z)$ is strong rth PARP. **q.e.d.**

Theorem 12: Suppose that $\mathbf{G}(z)$ has the form (III.1) with $N > K$. If

$$\begin{aligned} &A_{i^{(1)}, i^{(2)}} \\ &\triangleq \Big\{ g_{mK}g_{i^{(1)}K},\, g_{nK}g_{i^{(2)}K}, \\ &\quad g_{mK}(g_{i^{(1)}l}g_{i^{(2)}K} - g_{i^{(2)}l}g_{i^{(1)}K}), \\ &\quad 1 \le l \le K-1,\, K \le m, n \le N, \text{ and } n \ne i^{(1)} \Big\} \end{aligned}$$

are rth order linearly independent for some $i^{(1)}$ and $i^{(2)}$ with $K \le i^{(1)} < i^{(2)} \le N$, then $\mathbf{G}(z)$ is strong rth PARP.

Proof: Consider equation $\mathbf{E}(z)\mathbf{G}(z) = \mathbf{G}(z)\mathbf{V}(z)$, where $\mathbf{E}(z)$ is an $N \times N$ nonzero polynomial matrix of order at most r and $\mathbf{V}(z)$ is a $K \times K$ polynomial matrix. Denote $\mathbf{E}(z) = (e_{ij})_{N\times N}$ and $\mathbf{V}(z) = (v_{ij})_{K\times K}$, then we have the following equations:

$$e_{ij} + \sum_{m=K}^{N} e_{im}g_{mj} = v_{ij}, \qquad 1 \le i, \quad j \le K-1 \quad \text{(III.7)}$$

$$e_{ij} + \sum_{m=K}^{N} e_{im}g_{mj} = \sum_{n=1}^{K} v_{nj}g_{in}, \quad K \le i \le N, \quad 1 \le j \le K-1 \quad \text{(III.8)}$$

$$\sum_{m=K}^{N} e_{im}g_{mK} = v_{iK}, \qquad 1 \le i \le K-1 \quad \text{(III.9)}$$

$$\sum_{m=K}^{N} e_{im}g_{mK} = \sum_{n=1}^{K} v_{nK}g_{in}, \qquad K \le i \le N. \quad \text{(III.10)}$$

From (III.10) we have

$$\left(\sum_{m=K}^{N} e_{i^{(1)}m}g_{mK} - \sum_{n=1}^{K-1} v_{nK}g_{i^{(1)}n} \right) g_{i^{(2)}K} = \left(\sum_{m=K}^{N} e_{i^{(2)}m}g_{mK} - \sum_{n=1}^{K-1} v_{nK}g_{i^{(2)}n} \right) g_{i^{(1)}K}. \quad \text{(III.11)}$$

Substituting (III.9) to (III.11), we get

$$\sum_{m=K}^{N} e_{i^{(1)}m}g_{mK}g_{i^{(2)}K} - \sum_{n=1}^{K-1}\sum_{m=K}^{N} e_{nm}g_{mK}g_{i^{(1)}n}g_{i^{(2)}K} = \sum_{m=K}^{N} e_{i^{(2)}m}g_{mK}g_{i^{(1)}K} - \sum_{n=1}^{K-1}\sum_{m=K}^{N} e_{nm}g_{mK}g_{i^{(2)}n}g_{i^{(1)}K}$$

i.e.,

$$(e_{i^{(1)}i^{(1)}} - e_{i^{(2)}i^{(2)}})g_{i^{(1)}K}g_{i^{(2)}K} + \sum_{\substack{m=K \\ m\ne i^{(1)}}}^{N} e_{i^{(1)}m}g_{mK}g_{i^{(2)}K} - \sum_{\substack{m=K \\ m\ne i^{(2)}}}^{N} e_{i^{(2)}m}g_{mK}g_{i^{(1)}K} - \sum_{n=1}^{K-1}\sum_{m=K}^{N} e_{nm}g_{mK}(g_{i^{(1)}n}g_{i^{(2)}K} - g_{i^{(2)}n}g_{i^{(1)}K}) = 0.$$

Since $A_{i^{(1)}, i^{(2)}}$ are rth LID, so $e_{i^{(1)}i^{(1)}} = e_{i^{(2)}i^{(2)}}$, $e_{i^{(1)}m} = 0$ and $e_{i^{(2)}n} = 0$ for $K \le m$, $n \le N$ with $m \ne i^{(1)}$ and $n \ne i^{(2)}$, and $e_{ij} = 0$ for $1 \le i \le K-1$ and $K \le j \le N$. Also we can obtain $v_{KK} = e_{i^{(1)}i^{(1)}} = e_{i^{(2)}i^{(2)}}$ and $v_{iK} = 0$ for $1 \le i \le K-1$.

Now from (III.7) and (III.8) we have

$$e_{ij} = v_{ij}, \qquad 1 \le i, \quad j \le K-1 \quad \text{(III.12)}$$

$$e_{i^{(1)}j} + e_{i^{(1)}i^{(1)}}g_{i^{(1)}j} = \sum_{n=1}^{K} v_{nj}g_{i^{(1)}n}, \qquad 1 \le j \le K-1 \quad \text{(III.13)}$$

$$e_{i^{(2)}j} + e_{i^{(2)}i^{(2)}}g_{i^{(2)}j} = \sum_{n=1}^{K} v_{nj}g_{i^{(2)}n}, \qquad 1 \le j \le K-1. \quad \text{(III.14)}$$

From (III.13) and (III.14) we get

$$\left(e_{i^{(1)}j} + e_{i^{(1)}i^{(1)}}g_{i^{(1)}j} - \sum_{n=1}^{K-1} v_{nj}g_{i^{(1)}n}\right) g_{i^{(2)}K} = \left(e_{i^{(2)}j} + e_{i^{(2)}i^{(2)}}g_{i^{(2)}j} - \sum_{n=1}^{K-1} v_{nj}g_{i^{(2)}n}\right) g_{i^{(1)}K}$$

for any j with $1 \le j \le K-1$, i.e.,

$$\begin{aligned} &e_{i^{(1)}j}g_{i^{(2)}K} - e_{i^{(2)}j}g_{i^{(1)}k} \\ &\quad + (e_{i^{(1)}i^{(1)}} - e_{jj})(g_{i^{(1)}j}g_{i^{(2)}K} - g_{i^{(2)}j}g_{i^{(1)}K}) \\ &\quad - \sum_{\substack{n=1\\n\neq j}}^{K-1} e_{nj}(g_{i^{(1)}n}g_{i^{(2)}K} - g_{i^{(2)}n}g_{i^{(1)}K}) = 0, \\ &\quad 1 \le j \le K-1. \end{aligned} \tag{III.15}$$

The rth LID of $A_{i^{(1)},\, i^{(2)}}$ implies that $\{g_{i^{(2)}K}g_{i^{(1)}K},\, g_{i^{(2)}K}^2,\, g_{i^{(2)}K}(g_{i^{(1)}l}g_{i^{(2)}K} - g_{i^{(2)}l}g_{i^{(1)}K}),\, 1 \le l \le K-1\}$ are rth LID, i.e., $\{g_{i^{(1)}K},\, g_{i^{(2)}K},\, g_{i^{(1)}l}g_{i^{(2)}K} - g_{i^{(2)}l}g_{i^{(1)}K},\, 1 \le l \le K-1\}$ are rth LID. So from (III.15) we have $e_{i^{(1)}i^{(1)}} = e_{jj}$, $e_{i^{(1)}j} = e_{i^{(2)}j} = 0$ and $e_{nj} = 0$ for $1 \le j \ne n \le K-1$. Thus we get $\mathbf{V}(z) = e_{i^{(1)}i^{(1)}}I_K$, i.e., $\mathbf{G}(z)$ is rth PARP.

From the rth LID of $A_{i^{(1)},\, i^{(2)}}$ again, we know that $\{g_{mK}g_{i^{(1)}K},\, K \le m \le N\}$ are rth LID, i.e., $\{g_{KK},\, g_{(K+1)K},\, \cdots,\, g_{NK}\}$ are rth LID. According to Theorem 11, $\mathbf{G}(z)$ is also strong rth PARP. **q.e.d.**

The above theorem provides us a new and more general sufficient condition for constructing $N \times K$ strong rth PARP and, therefore, also rth PARP for a general N with $N > K$.

C. (Strong) rth PARP with $N = K+1$

In this subsection, we discuss polynomial matrices only with $N = K+1$ and achieve some simplified results. This case is interesting in practice since, for a given K, the case of $N = K+1$ corresponds to the minimal data rate expansion case in the precoding.

Theorem 13: Let $\mathbf{G}(z)$ has the form (III.1) with $N = K+1$. Then $\mathbf{G}(z)$ is rth PARP if and only if $\mathbf{G}(z)$ is strong rth PARP.

Proof: The sufficiency is obvious. Now we prove the necessity. If $\mathbf{G}(z)$ is rth PARP, according to Theorem 11, we only need to prove that $\{g_{KK},\, g_{NK}\}$ are rth-order linearly independent (rth LID).

If there exist polynomials $e_1(z)$ and $e_2(z)$ of orders at most r such that

$$e_1(z)g_{KK}(z) + e_2(z)g_{NK}(z) = 0$$

since $\gcd(g_{KK},\, g_{NK}) = 1$ from the canonical form, we have $g_{NK}(z)|e_1(z)$ and $g_{KK}(z)|e_2(z)$. According to Theorem 9, $\max(\deg g_{KK},\, \deg g_{NK}) > r$. So $e_1(z) = 0$ and $e_2(z) = 0$, i.e., $\{g_{KK},\, g_{NK}\}$ are rth LID. This proves the necessity. **q.e.d.**

The following theorem can be derived from Theorem 12 directly.

Theorem 14: Suppose that $\mathbf{G}(z)$ has the form (III.1) with $N = K+1$. If

$$\begin{aligned} &A_{K,\, N} \\ &\triangleq \{g_{KK}g_{NK},\, g_{KK}^2,\, g_{NK}^2,\, g_{KK}(g_{NK}g_{Kn} - g_{KK}g_{Nn}), \\ &\quad g_{NK}(g_{NK}g_{Kn} - g_{KK}g_{Nn}),\, 1 \le n \le K-1\} \end{aligned}$$

are rth-order linearly independent, then $\mathbf{G}(z)$ is strong rth PARP.

As a remark, the condition in Theorem 14 is sharper than the conditions in Theorem 8. Let us see the example in (III.2). We know $\mathbf{G}(z)$ does not satisfy the conditions in Theorem 8. But $A_{3,\,4} \triangleq \{z^{-8} + z^{-5},\, z^{-6} + 2z^{-3} + 1,\, z^{-10},\, z^{-9} + z^{-6},\, z^{-10} - 2z^{-4} - z^{-1},\, z^{-11},\, z^{-12} - z^{-9} - z^{-5}\}$ are actually zeroth LID, i.e., $\mathbf{G}(z)$ is strong zeroth PARP by Theorem 14. *Moreover*, we can show that $A_{K,\, N}$ are zeroth LID if the conditions in Theorem 8 are true. In fact, if there exist constants a, b, c, d_n and e_n $(1 \le n \le K-1)$ such that

$$\begin{aligned} &ag_{KK}g_{NK} + bg_{KK}^2 + cg_{NK}^2 \\ &\quad + \sum_{n=1}^{K-1} d_n g_{KK}(g_{NK}g_{Kn} - g_{KK}g_{Nn}) \\ &\quad + \sum_{n=1}^{K-1} e_n g_{NK}(g_{NK}g_{Kn} - g_{KK}g_{Nn}) = 0 \end{aligned}$$

then

$$g_{NK} \left| \left(bg_{KK}^2 - \sum_{n=1}^{K-1} d_n g_{KK}^2 g_{Nn}\right)\right. .$$

From the assumption in Theorem 8, we get $b = 0$, $d_n = 0$ for $L \le n \le K-1$, and

$$\begin{aligned} &ag_{KK} + cg_{NK} + \sum_{n=1}^{L-1} d_n\, g_{KK}g_{Kn} \\ &\quad + \sum_{n=1}^{K-1} e_n(g_{NK}g_{Kn} - g_{KK}g_{Nn}) = 0 \end{aligned}$$

i.e.,

$$\left(c + \sum_{n=1}^{K-1} e_n g_{Kn}\right) g_{NK} = g_{KK}\left(-a - \sum_{n=1}^{L-1} d_n g_{Kn} + \sum_{n+L}^{K-1} e_n g_{KK}g_{Nn}\right).$$

Since $W_1 \cap W_2 = \{0\}$ and $\{1,\, g_{K1},\, \cdots,\, g_{K(L-1)},\, g_{NL},\, \cdots,\, g_{N(K-1)}\}$ are zeroth LID, we get $c = 0$, $a = 0$, $d_n = 0$ and $e_n = 0$ $(1 \le n \le K-1)$. So $A_{K,\, N}$ are zeroth LID.

The rth LID of $A_{K,\, N}$ implies that $\{g_{KK},\, g_{NK},\, g_{NK}g_{Kn} - g_{KK}g_{Nn},\, 1 \le n \le K-1\}$ are rth LID. In fact, the rth LID of $\{g_{KK}, g_{NK}, g_{NK}g_{Kn} - g_{KK}g_{Nn}, 1 \le n \le K-1\}$ is also necessary for the rth PARP of $\mathbf{G}(z)$, and is certainly necessary for the strong rth PARP of $\mathbf{G}(z)$, as we see from the following result.

Theorem 15: If $\mathbf{G}(z)$ has the form (III.1) with $N = K + 1$, and $\mathbf{G}(z)$ is rth PARP, then $\{g_{KK}, g_{NK}, g_{NK}g_{Kn} - g_{KK}g_{Nn}, 1 \le n \le K-1\}$ are rth-order linearly independent.

Proof: Assume that $\{g_{KK}, g_{NK}, g_{NK}g_{Kn} - g_{KK}g_{Nn}, 1 \le n \le K-1\}$ are not rth LID, then there exist polynomials a, b and c_n, $1 \le n \le K-1$, of order at most r such that

$$a g_{KK} + b g_{NK} + \sum_{n=1}^{K-1} c_n (g_{NK}g_{Kn} - g_{KK}g_{Nn}) = 0$$

i.e.,

$$\left(b + \sum_{n=1}^{K-1} c_n g_{Kn}\right) g_{NK} = g_{KK}\left(-a + \sum_{n=1}^{K-1} c_n g_{Nn}\right).$$

So

$$g_{KK}\left|\left(b + \sum_{n=1}^{K-1} c_n g_{Kn}\right)\right. \quad \text{and} \quad g_{NK}\left|\left(-a + \sum_{n=1}^{K-1} c_n g_{Nn}\right)\right. .$$

Now let $\mathbf{E}(z) = (e_{ij})_{N\times N}$, where $e_{NN} = e_{KK}$, $e_{NK} = 0$, $e_{KN} = 0$, $e_{nK} = 0$ and $e_{nN} = 0$ for $1 \le n \le K-1$, and for any j with $1 \le j \le K-1$, $e_{jj} = e_{KK} - c_j$, $e_{Kj} = b$, $e_{Nj} = -a$, and $e_{nj} = -c_n$, $1 \le n \le K-1$ for $n \ne j$. Let $\mathbf{V}(z) = (v_{ij})_{K\times K}$, where $v_{KK} = e_{NN}$, $v_{ij} = e_{ij}$ for $1 \le i, j \le K-1$, and for any j with $1 \le j \le K-1$

$$v_{Kj} = \left(b + \sum_{n=1}^{K-1} c_n g_{Kn}\right)\Big/ g_{KK}$$

or

$$\left(-a + \sum_{n=1}^{K-1} c_n g_{Nn}\right)\Big/ g_{NK}.$$

We can check that $\mathbf{E}(z)\mathbf{G}(z) = \mathbf{G}(z)\mathbf{V}(z)$. It is obvious that $\mathbf{V}(z) \ne \alpha(z) I_K$. This is contradictory to that $\mathbf{G}(z)$ is rth PARP. **q.e.d.**

Combining Theorems 14 and 15, we have the following corollary for a complete characterization of a systematic (strong) PARP, which also provides a construction method for rth strong PARP by separating the degrees of $g_{(K+1)n}$, $1 \le n \le K$, by $r+1$ from one to another.

Corollary 1: If $\mathbf{G}(z)$ has the *systematic form*, as shown at the bottom of the page, then $\mathbf{G}(z)$ is strong rth PARP if and only if $\{1, g_{(K+1)1}, g_{(K+1)2}, \cdots, g_{(K+1)K}\}$ are rth-order linearly independent.

Proof: The necessity comes from Theorem 15 immediately. Now we prove the sufficiency. According to Theorem 14, we need to prove that $\{1, g_{(K+1)n}, g_{(K+1)K}g_{(K+1)n}, 1 \le n \le K\}$ are rth LID.

With PARP-equivalence transformations, we can assume $\deg g_{(K+1)n} < \deg g_{(K+1)K}$ for $1 \le n \le K-1$. If there exist polynomials a, b_n, and c_n, $1 \le n \le K$, of orders at most r such thata

$$a + \sum_{n=1}^{K} b_n g_{(K+1)n} + \sum_{n=1}^{K} c_n g_{(K+1)K} g_{(K+1)n} = 0 \quad \text{(III.16)}$$

then

$$g_{(K+1)K}\left|\left(a + \sum_{n=1}^{K-1} b_n g_{(K+1)n}\right)\right. .$$

From the rth LID of $\{1, g_{(K+1)1}, g_{(K+1)2}, \cdots, g_{(K+1)K}\}$, we have $a = 0$ and $b_n = 0$ for $1 \le n \le K-1$. Using (III.16) again, we have $b_K = 0$ and $c_n = 0$ for $1 \le n \le K$. So $\{1, g_{(K+1)1}, g_{(K+1)2}, \cdots, g_{(K+1)K}\}$ are rth LID. **q.e.d.**

The special case when $r = 0$ in the above corollary has been obtained in [5]. From Theorem 13, the result in Corollary 1 also holds for rth PARP. The following corollary is not hard to see from Theorem 14, which provides a convenient way to construct nonsystematic strong rth PARP with $N = K+1$.

Corollary 2: Suppose that $\mathbf{G}(z)$ has the form (III.1) with $N = K+1$. If $g_{N1}(z) = \cdots = g_{N(L-1)}(z) = 0$ for some L $(1 \le L \le K)$, and

$$\begin{aligned} \deg g_{Kn} &< \deg g_{K(n+1)} - r, \qquad 1 \le n < L-1 \\ \deg g_{Nn} &< \deg g_{N(n+1)} - r, \qquad L \le n < K \end{aligned}$$

and $\deg g_{K(L-1)} < \deg g_{NL} - r$, then $\mathbf{G}(z)$ is strong rth PARP.

To see the above result, let us consider the case when $L = 1$. In this case, when the degrees of the polynomials $g_{Nn}(z)$, $1 \le n \le K$, in $\mathbf{G}(z)$ are at least $r+1$ differ from their adjacent ones, the precoder is then strong rth PARP.

IV. Numerical Examples

In this section, we want to present two numerical examples to verify the theory/algorithm developed in Section II. Simulated outputs from a baud-rate sampled single-receiver system and an undersampled antenna array system are used for blind identification. The results presented here are to illustrate the feasibility rather than efficiency of the proposed precoding and blind

$$\mathbf{G}(z) = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ g_{(K+1)1}(z) & g_{(K+1)2}(z) & g_{(K+1)3}(z) & \cdots & g_{(K+1)K}(z) \end{pmatrix}$$

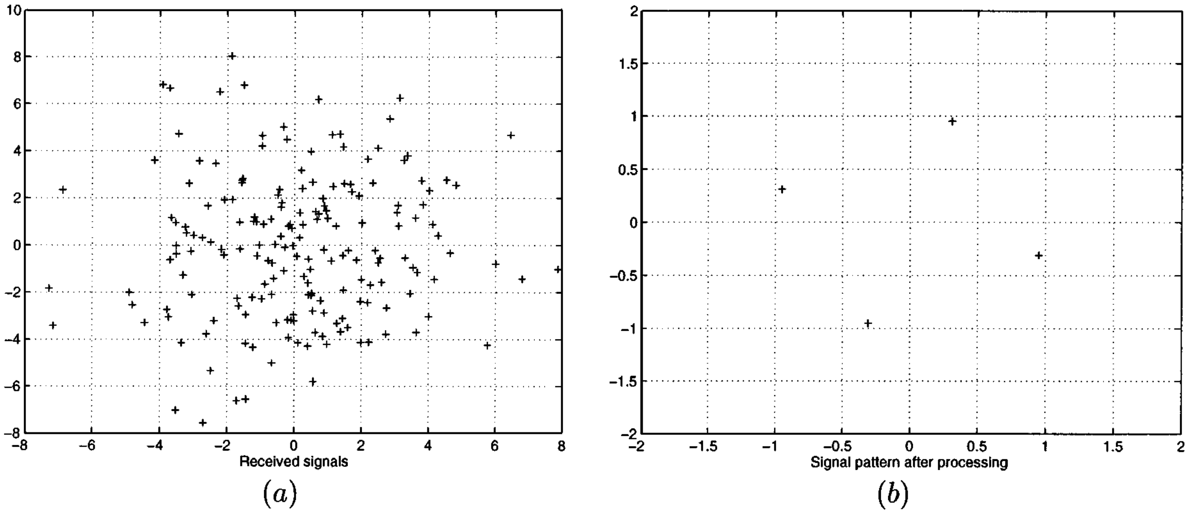

Fig. 7. (a) ISI channel outputs with baud sampling; (b) recovered signal after blind identification using precoding techniques.

identification techniques, although some robustness in handling noisy data is demonstrated by the proposed algorithm.

A. Single Antenna Receiver with Baud Sampling Rate

In this example, we set the order of the baud-rate sampled ISI channel to be 4. The ISI channel is randomly selected, which in this example is

$$H(z) = 0.9275 - 0.5174z^{-1} + 0.2343z^{-2} + 0.7955z^{-3} + 0.1551z^{-4}.$$

The parameters in Fig. 2 and Fig. 3 and (II.23) and (II.24) are $K = 2$, $N = 3$, $M = 4$. In this case, the channel matrix $\mathbf{H}(z)$ in (II.24) is shown in the matrix at the bottom of the page. The order of $\mathbf{H}(z)$, Q_h, is thus 1. Based on (II.25), it is adequate to use $r = 3$ for the precoder $\mathbf{G}(z)$ in (II.3). The order of $\mathbf{G}(z)$ is $r + 1 = 4$. $\mathbf{G}(z)$ is capable of resisting any third-order polynomial matrix ambiguity.

QPSK signals are used as the input signal in this example. The received data without identification is shown in Fig. 7(a). The processed data after applying the proposed blind technique is shown in Fig. 7(b). In this particular example, we use noise-free observations to demonstrate that the proposed techniques can provide closed-form solution with a finite number of data samples.

B. Undersampled Antenna Array Receivers

In this example, we use four antennas and undersample the received signals by factor 3, i.e., $M = 4$ and $N = 3$ in Figs. 4 and 5. Four ISI channels $H_l(z)$, $l = 1, 2, 3, 4$, are randomly chosen, which in this example are

$$\begin{aligned} H_1(z) = &(0.3323 + 0.3446j) + (-0.2337 + 0.7782j)z^{-1} \\ &+ 0.2511j)z^{-2} + (-0.5945 + 1.1582j)z^{-3} \\ &+ (-0.5398 - 1.2997j)z^{-4} \\ &+ (-1.5044 - 2.7960j)z^{-5}; \\ H_2(z) = &(0.5589 - 0.7233j) + (1.4499 + 2.1805j)z^{-1} \\ &+ (-0.9646 - 0.3105j)z^{-2} \\ &+ (0.1302 + 0.8625j)z^{-3} \\ &+ (1.8800 + 0.3066j)z^{-4} \\ &+ (-0.0954 + 0.6967j)z^{-5}; \\ H_3(z) = &(0.8999 + 1.2682j) + (1.8361 + 0.4378j)z^{-1} \\ &+ (0.0388 - 0.9230j)z^{-2} \\ &\cdot (0.0350 - 1.0347j)z^{-3} \\ &+ (-1.0038 + 0.9690j)z^{-4} \\ &+ (0.3967 + 3.2069j)z^{-5}; \\ H_4(z) = &(-0.2009 - 0.0312j) + (-0.3829 + 1.3333j)z^{-1} \\ &+ (0.7655 - 0.3848j)z^{-2} \\ &\cdot (-0.6247 - 0.1927j)z^{-3} \\ &+ (-0.4974 - 0.7473j)z^{-4} \\ &+ (-0.5271 + 0.5360j)z^{-5}. \end{aligned}$$

In this case, the channel matrix $\mathbf{H}(z)$ in Fig. 5 is of order $Q_h = 1$. Similar to the previous example, the parameter r in

$$\mathbf{H}(z) = \begin{bmatrix} 0.9275 + 0.1551z^{-1} & 0.7955z^{-1} & 0.2343z^{-1} \\ -0.5174 & 0.9275 + 0.1551z^{-1} & 0.7955z^{-1} \\ 0.2343 & -0.5174 & 0.9275 + 0.1551z^{-1} \\ 0.7955 & 0.2343 & -0.5174 \end{bmatrix}$$

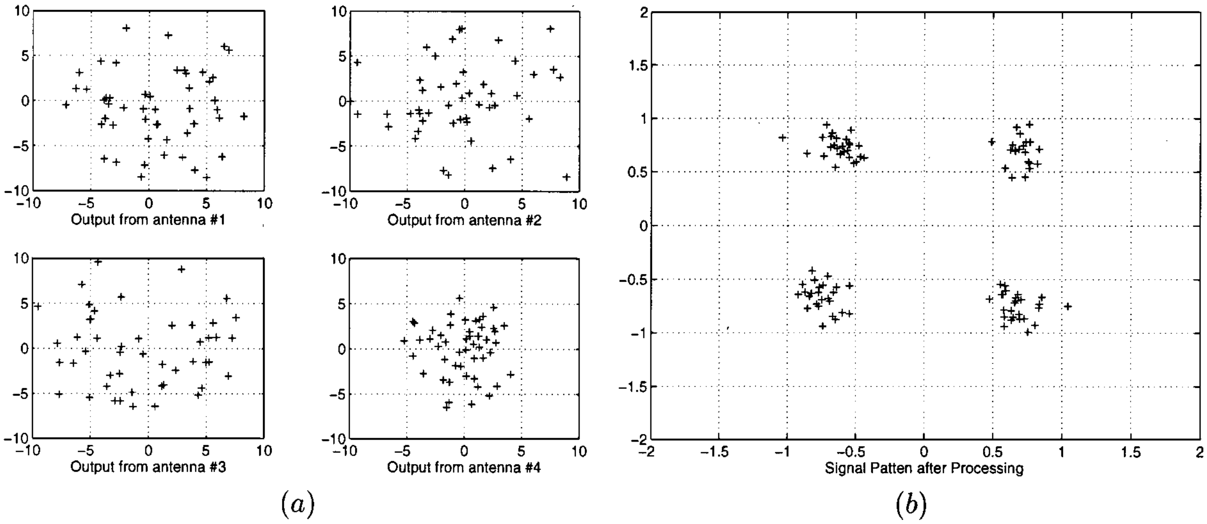

Fig. 8. (a) Undersampled antenna outputs before blind identification. (b) Recovered signal after blind identification using precoding techniques.

(II.3) is set to be 3, which enables the precoder $\mathbf{G}(z)$ to resist a third-order polynomial matrix ambiguity.

Instead of noise-free data, we apply the proposed blind identification algorithm to a minimum amount of output vectors, 50, under 30 dB SNR. Fig. 8(a) and (b) compare the signal patterns before and after the identification.

V. Conclusions

In this paper, we studied the following two questions of a precoded MIMO system: Let

$$\mathbf{Y}(z) = \mathbf{H}(z)\mathbf{G}(z)\mathbf{X}(z)$$

and what is the condition on $\mathbf{G}(z)$ such that $\mathbf{X}(z)$ and/or $\mathbf{H}^{-1}(z)$ can be recovered from $\mathbf{Y}(z)$ and $\mathbf{G}(z)$? How to construct such $\mathbf{G}(z)$? By answering these questions, we have reviewed some results obtained in [3], [51], [4], and [5] and also presented some new results on filterbank precoding for blind channel equalization, namely PARP. With PARP, the transmitter or the receiver does not need to know an ISI channel for the recovery of the input signal. There are two kinds of PARP: PARP and strong PARP, where PARP is for the input signal recovery while the strong PARP is for both the input signal and the channel inverse recovery. We have shown that a filterbank precoded system has the blind identifiability if and only if the filterbank precoder is PAPR. We have also shown that a filterbank precoder of size $(K+1) \times K$ is PARP if and only if it is also strong PARP. Some new characterizations and constructions of (strong) PARP have been also presented. A main difference between the study in this paper and others on equalization and precoding is that the approach in this paper is deterministic while the others are mostly stochastic that may need a long segment of a received data at the receiver; furthermore this paper provides a systematic study of the questions raised above, which are two natural questions about the filterbank precoding for the blind equalization.

It is observed that, at least in theory, more memory in the precoders provides stronger ambiguity resistance and more powerful equalizers: zeroth-order block precoders without any memory in [1] only allow ideal FIR equalization at the receiver when the channel is known; first-order precoders in [3], [51] allow ideal FIR blind equalizers at the receiver when the ambiguity is a constant matrix; while rth-order precoders in [4] allow ideal FIR blind equalization at the receiver when the ambiguity is a $(r-1)$th-order polynomial matrix. For an optimality on PARP in terms of the robustness of the channel additive noise, we refer the reader to [6]. Practical applications of PAPR in wireless communication systems are under current investigation.

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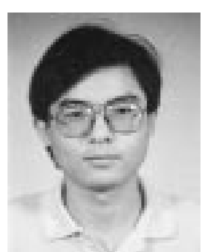

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