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Rethink delay Doppler channels and time-frequency coding

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ABSTRACT

Keywords: Orthogonal frequency division multiplexing (OFDM) Vector orthogonal frequency-division multiplexing (VOFDM) Orthogonal time frequency space (OTFS) Delay Doppler channel Space/time/frequency coding In this paper, we rethink delay Doppler channels (also called doubly selective channels). We prove that no modulation schemes (including the current active VOFDM/OTFS) can compensate a nontrivial Doppler spread well. We then discuss some of the existing methods to deal with timevarying channels, in particular time-frequency (TF) coding in an OFDM system. TF coding is equivalent to space-time coding in the math part. We also summarize state of the art on space-time coding that was an active research topic over a decade ago.

1. No modulation scheme can compensate non-trivial Doppler shifts

A doubly selective channel, i.e., it has both time spread and Doppler spread, has re-attracted significant attention lately due to the recent Starlink success. In fact, such a channel was studied in the 1990's, see for example [1,2]. A douly selective channel is also called a delay Doppler channel [3]. A recent active topic is orthogonal time frequency space (OTFS) modulation [4,5] that has been shown identical to vector OFDM (VOFDM) [6–8] in Refs. [9–12]. OTFS has been claimed to be able to deal with a delay Doppler channel well, which, we think, is mis-leading and no modulation scheme can well compensate a non-trivial Doppler spread, as we shall see in details below.

A delay Doppler channel can be described as follows [1–3]. At time delay τ , let

$$h(\tau, t) = g(\tau) e^{-j\Omega(\tau)t}$$
(1)

be its channel response with Doppler shift $\Omega(\tau)$ that is a function of time delay τ . It means that the path $h(\tau, t)$ of time delay τ has Doppler shift $\Omega(\tau)$ and in general, different paths at different time delays may have different Doppler shifts.

Let s(t) be a transmitted signal. Then, the received signal y(t) at time t is

$$y(t) = \int h(\tau, t) s(t - \tau) d\tau + w(t) = \int g(\tau) s(t - \tau) e^{-j\Omega(\tau)t} d\tau + w(t),$$
(2)

where w(t) is the additive noise.

When the Doppler shift function $\Omega(\tau)$ in equation (2) is a constant Ω that does not depend on τ , which is called trivial Doppler spread case, it means that all the channel responses at all the time delays have the same Doppler shift Ω . In this case, it is easy to see that this Doppler shift can be compensated at either transmitter or receiver and the compensated channel then becomes a time spread only

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channel. This argument applies to the channels on any finite time interval where the Doppler shift function $\Omega(\tau)$ is approximately constant, i.e., it can be approximated by a constant independent of time delay variable τ .

Otherwise, different multipaths have different Doppler shifts and it is called non-trivial Doppler spread case. In this case, the received signal in equation (2) becomes

$$\mathbf{y}(t) = \int g(t-\tau)s(\tau)\mathrm{e}^{-j\Omega(t-\tau)t}d\tau + w(t).$$
(3)

For example, when the Doppler shift function $\Omega(\tau)$ is linear in terms of τ , i.e., $\Omega(\tau) = \Omega \tau$ for some non-zero constant Ω , then the received signal in equation (3) is

$$y(t) = \int g(t-\tau)s(\tau)e^{j\Omega t t}d\tau e^{-j\Omega t^2} + w(t).$$
(4)

From the above signal model, the non-trivial Doppler spread part is also a function of variable t, while the transmitted signal $s(\tau)$ only depends on τ and no matter what $s(\tau)$ is, it has nothing to do with the variable t in the Doppler spread, thus any transmit signal s(t) cannot compensate the Doppler spread, no matter what modulation scheme is used. This conclusion holds for both continuous and discrete time signal models, and also for signal models on any time interval that could be short. With the above result, neither VOFDM/OTFS nor GFDM [13–15] can compensate a non-trivial Doppler spread as also briefly mentioned in Ref. [16] from a joint time-frequency analysis point of view.

A delay Doppler channel is a time-varying channel. Although there is no modulation that can compensate the non-trivial Doppler spread well, there were methods to deal with general time-varying channels to have improved performance over 20 years ago. The basic idea is to use a block of time slots together in demodulation or decoding. These methods include bit-interleaved coded modulation (BICM) [17] and signal space diversity for narrow band systems [18,19], which can be applied along the frequency components in broadband OFDM systems, and time-frequency (TF) coding for broadband OFDM systems. Mathematically TF coding is equivalent to space-time (ST) coding and both of them are the two special cases of space-time-frequency (STF) coding. Ref. [20] is a tutorial paper about ST/SF/STF coding, where one can find the related original works on this topic.

Note that for TF coding, multiple OFDM symbols may be coded and decoded together. If only one OFDM symbol is considered, information symbols in one OFDM symbol can be coded/decoded together by using signal space diversity along the frequency index as mentioned earlier. The main problem for signal space diversity and TF coding is the demodulation/decoding complexity, since all the information symbols in one or multiple OFDM symbols are decoded/demodulated together.

If one OFDM symbol is demodulated together, firstly we think that it is againist the motivation to emply OFDM with simple demodulation complexty, and secondly, to deal with time-varying channels, one could randomly generate an $N \times N$ unitary matrix and apply it as a precoding before the OFDM of *N* subcarriers. This may not perform as good as the signal space diversity technique in the frequency domain that can achieve full multipath diversity in theory, but may perform as good as any existing linearly modified OFDM systems (XFDM, XYDM, XYZM), all of which may have too high demodulation complexities.

Although VOFDM/OTFS cannot compensate a non-trivial Doppler spread, it does have improved performance for time-varying channels over OFDM. This is because for VOFDM/OTFS, a vector of information symbols are demodulated together [7,8,11,21,22]. For example, the frequency domain equalizer along a vector of symbols can be used. However, this performance improvement is not because of the particular delay Doppler channel but applies to any time-varying channel.

In addition to its good performance for time-varying channels, VOFDM is the most general modulation in terms of dealing with intersymbol interference (ISI) channels. Assume M is the vector size of VOFDM and N is the number of subcarriers as in OFDM. For an ISI channel H(z), at the receiver, after the cyclic prefix removal, the received signal becomes, see Ref. [7]:

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{w}_k,\tag{5}$$

for $k = 0, 1, \dots, N - 1$, and \mathbf{x}_k and \mathbf{y}_k are the $M \times 1$ information symbol vector and the $M \times 1$ received signal vector (after the component-wise *N*-point FFT of the received signal vectors), respectively, and \mathbf{w}_k is the additive noise vector, at the *k*th frequency component. And \mathbf{H}_k is the $M \times M$ pseudo-circulant channel matrix $\mathbf{H}(z)$ evaluated at the *k*th frequency of $z = \exp(j2\pi k/N)$. The pseudo-circulant matrix $\mathbf{H}(z)$ is obtained from the polyphase components of the original ISI channel function H(z) and its more details are referred to Ref. [7]. The received signal model in equation (5) was mathematically new in the literature for an ISI channel, which is the generalization of the OFDM received signal model in terms of ISI level. When the vector size M = 1, VOFDM returns to the conventional OFDM and the received signal model (5) is the same as the conventional one of the OFDM. When *M* is not smaller than the ISI size and N = 1, it returns to the conventional single carrier frequency domain equalizer (SF-FDE) [22]. Thus, VOFDM is also a bridge between OFDM and SC-FDE. For a general vector size *M* and each *k*, there are at most *M* symbols interfering each other inside each vector subchannel (5) but there is no ISI across vector subchannels, i.e., across *k*. Thus, for vector size $M = 1, 2, 3, \dots$, VOFDM is the most general modullation scheme in terms of ISI level that converts an ISI channel to multiple independent subchannels each of which may have no ISI, M = 2 symbols in ISI, M = 3 symbols in ISI, \dots , respectively.

It is important to note that one can obviously add pulses to VOFDM in transmission, the same as adding pulses to OFDM in transmission. The key to determine whether a modulation is the same as VOFDM is to see whether it leads to the same receive signal model (5) at the receiver for any fixed ISI channel, when the rectangular pulse is used.

Recall that dealing with ISI channels has been always the most important physical layer task in digital communications in the past, no matter whether in wired or wireless systems. In fact, this is also the most important in radar applications with broadband waveforms.

Although in radar applications there is no concept of ISI, it is called inter-range cell interference (IRCI) [23–25]. The reason is simple and it is because both of them use EM waves to transmit and receive. Thus, in our opinion, as the most general modulation to deal with an ISI channel, VOFDM/OTFS plays a more important role over an ISI channel than over a time-varying channel (or, in particular, a delay Doppler channel).

2. Time-frequency coding

TF coding is an old concept that can be done by utilizing the signal space diversity technique [18,19] along the frequency index in one OFDM symbol as mentioned before or two dimensional coding across multiple OFDM symbols similar to ST coding that can be thought of a special case of space-time-frequency (STF) coding and more details can be found in Ref. [20]. Also, a signal space diversity design is equivalent to a diagonal space-time block code design [26].

As mentioned earlier, the major problem for these approaches using TF/STF coding is the high demodulation/decoding complexity. Otherwise, no modulations (XFDM, XYDM, XYZM) can perform better than frequency domain signal space diversity or TF/STF coding over time-varying channels including delay Doppler channels. A key for these techniques is ST code design that can be briefly described below.

An ST code is a collection of the same size matrices, which information bits are mapped to, to transmit. One dimension of a matrix in an ST code corresponds to time and the other dimension corresponds to space, i.e., transmit antennas. Without loss of generality, let us assume all the matrices are squared, i.e., the dimensions of time and space are the same. If the maximum-likelihood (ML) decoding is used at the receiver, an ST code achieves full diversity if any difference matrix of any two distinct matrices in the ST code has full rank. The minimum of the absolute determinant value of all the difference matrices of two distinct matrices in the ST code corresponds to the coding gain (or called diversity product), whose maximum is desired in a design of ST codes, called optimal ST codes. Such an optimal 2×2 unitary code (i.e., each matrix in the code is unitary) of size 6 is obtained in Ref. [27] and a best known 2×2 unitary code of size 16 is obtained in Ref. [28].

The most well-known (also one of the earliest) ST block code is the Alamouti code [29] for two transmit antennas, which corresponds to complex numbers for real information symbols and quaternions for complex information symbols. It has been generalized to orthogonal space-time block codes (OTSBC) for a general number of transmit antennas [30]. The orthogonality of an OSTBC provides the fastest ML decoding (symbol-wise decoding) and the full diversity. Unfortunately, this orthogonality is too strong so that the symbol rate (symbols per channel use) of an OTSBC is upper bounded by 3/4 for more than 2 transmit antennas and is conjectured to be upper bounded by (k + 1)/(2k) for n = 2k - 1 or 2k transmit antennas [31]. This conjecture is true if no complex linear combinations of two or more information symbols is allowed in an OSTBC [32]. Systematic designs of OSTBCs achieving the conjectured rate upper bound for an arbitrary number of transmit antennas are presented in Refs. [32–34]. The designs in Ref. [34] are inductive and have closed forms, while the other two in Refs. [32,33] are human-assisted or computer-assisted, and do not have closed forms.

Decoding algorithm	Complexity	Rate	Full diversity criterion
Maximum-likelihood (ML)	Highest	Highest (n)	Weakest: full rank criterion Linear independence of equivalent channel column vectors over signal constellation
Conditional PIC group decoding	Î	Ť	
PIC group decoding		Group size(K)	
Linear receiver (ZF/MMSE)	Lowest	Lowest (1)	Strongest Linear independence of equivalent channel column vectors
Orthogonal codes	Unnecessary	1/2	Orthogonal

n transmit antennas

Fig. 1. Space-time coding tradeoff between decoding complexity, rate, and performance.

The above generally mentioned TF/STF/ST coding with full diversity is based on ML demodulation/decoding. Although OSTBC has fast ML decoding, their rates are low and approach 1/2 when the number of transmit antennas goes large. However, for a low decoding complexity and having full diversity in the meantime, the orthogonality in an OSTBC is not necessary. Later we started to design ST codes based on other low complexity demodulation algorithms, such as linear receiver [35], partial interference cancellation (PIC) group decoding [36,37], and conditional PIC group decoding [38]. In summary, achieving a low complexity decoding and full diversity as well, the sacrifice is the code rate. For example, the maximal code rate for linear receiver is 1 [35] and codes with rates approaching 1 and achieving full diversity with linear receiver are designed in Ref. [35] as well. The maximal code rate for PIC group decoding is *K* that is the group size [36]. One of the latest designs in this direction is [39]. Fig. 1 illustrates the tradeoff between decoding complexity, code rate, and full diversity (i.e., performance). For more details, we would like to refer the reader to Ref. [40].

As a final note, ST coding has been combined with VOFDM in Refs. [7,41,42] to achieve an improved performance over fading channels.

3. Conclusion

In this paper, we first showed that no modulation schemes can compensate a non-trivial Doppler spread. The true reason for all the claimed modulation schemes, such as VOFDM/OTFS, are good for delay Doppler channels is due to their block-wise/vector-wise demodulation in time or/and in frequency, which is good for general time-varying channels the same reason as BICM, signal space diversity, and time-frequency coding. It is not because they can specially treat/compensate Doppler spread, and it is mis-leading in the community and is clarified in this paper.

We also summarized the main features of VOFDM and ST codes. Since this paper is not about a comprehensive tutorial on ST codes, a lot interesting works on ST/STF codes over a decade ago were not mentioned.

Declaration of competing interest

The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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