



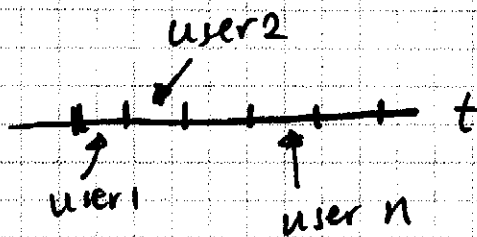
Chapter 8. Code Division Multiple Access

Multiple Access: multiple users share the same channel.

Three common multiple access methods:

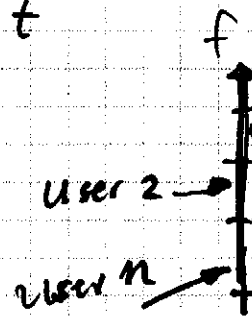
TDMA, FDMA, CDMA

TDMA:



time division multiple access

FDMA:

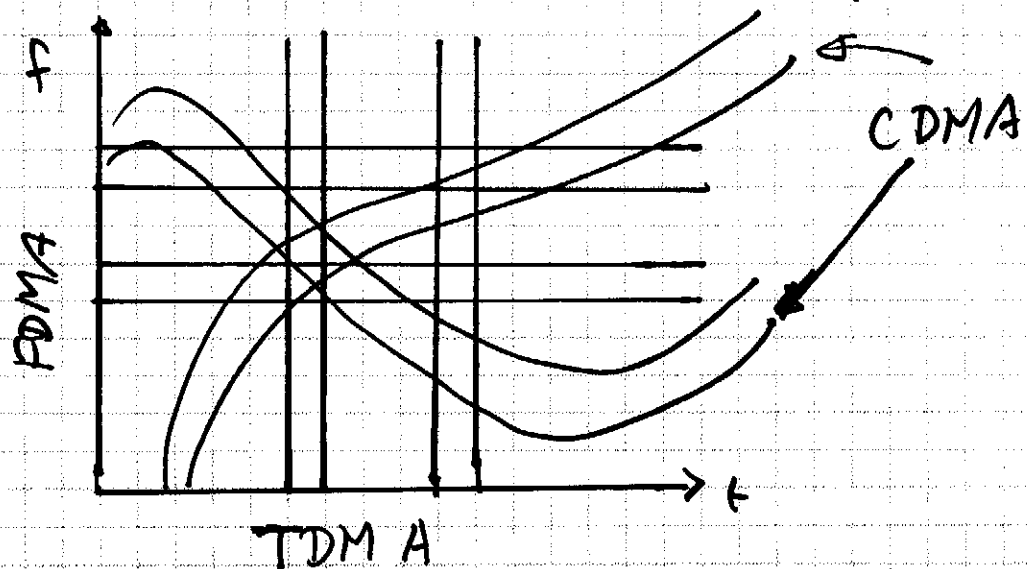


frequency division multiple access

CDMA:

each user use a code sequence.

code division multiple access



TDMA & FDMA

→ narrow band transmission

CDMA

→ wide band transmission

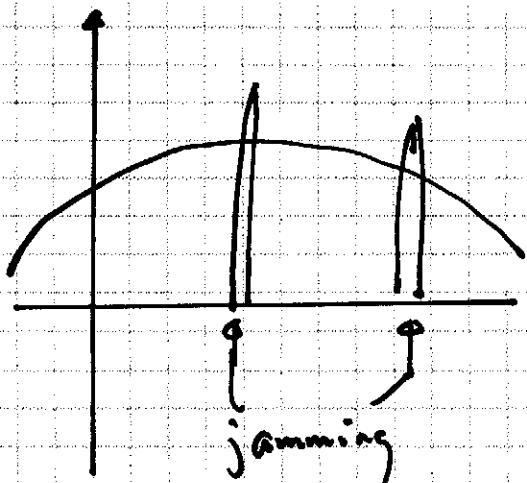


§8-1. Introduction to DS (direct sequence) and FH (frequency hopping) Spread Spectrum.

① DS CDMA

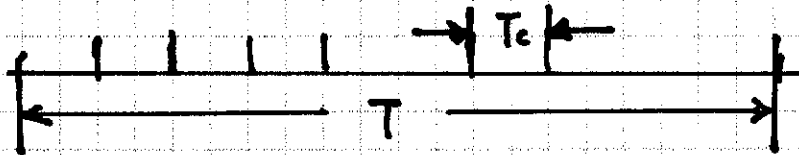
Central idea for CDMA is that each user uses a codeword that has wideband characteristics.

→ Comes from the spread spectrum communication idea in 40's for the anti jamming communication purpose.



Each symbol is spread into several symbols: bandwidth is expanded.

Let T be the symbol duration, T_c be the chip duration



$$G = \frac{T}{T_c}$$

processing gain
spreading gain

Let $a_0, a_1, \dots, a_{G-1}$ be a pseudo-random noise sequence with length G , a_k be the sequence periodically generated from $a_0, a_1, \dots, a_{G-1}$, $a_k \in \{\pm 1\}$.



The spreading waveform

$$a(t) = \sum_k a_k h_a(t - kT_c)$$

where $h_a(t)$ is a real chip amplitude shaping function.

The energy per chip is $E_c = \frac{1}{2} \int_0^{T_c} h_a^2(t) dt$

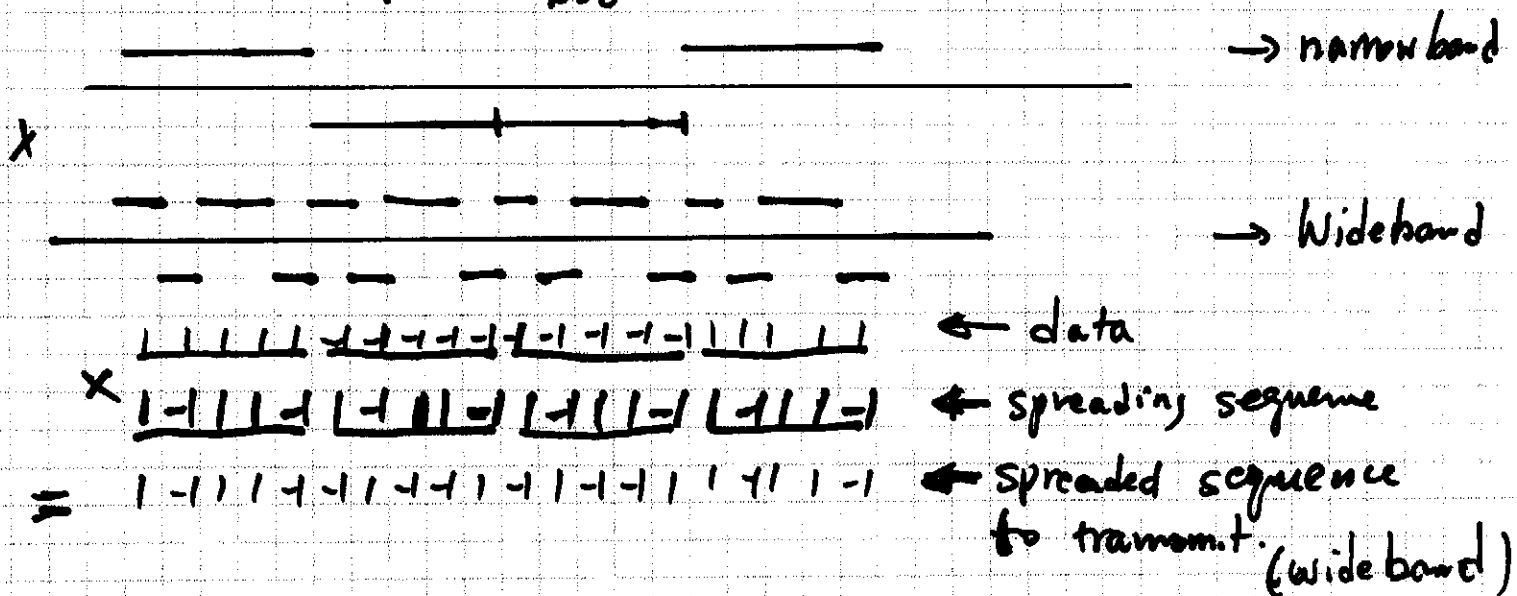
The data sequence can be represented by the waveform

$$x(t) = A \sum_n x_n u_T(t - nT)$$

where $\underline{x} = \{x_n : x_n \in \{\pm 1\}\}$ is the complex source symbol sequence, A is the amplitude.

The transmitted waveform is (single user)

$$\begin{aligned} v(t) &= x(t) \cdot a(t) \\ &= A \sum_n x_n \sum_{k=0}^{G-1} a_k h_a(t - (nG + k)T_c) \end{aligned}$$





Multiple users

$$v_i(t) = x_i(t) a_i(t)$$

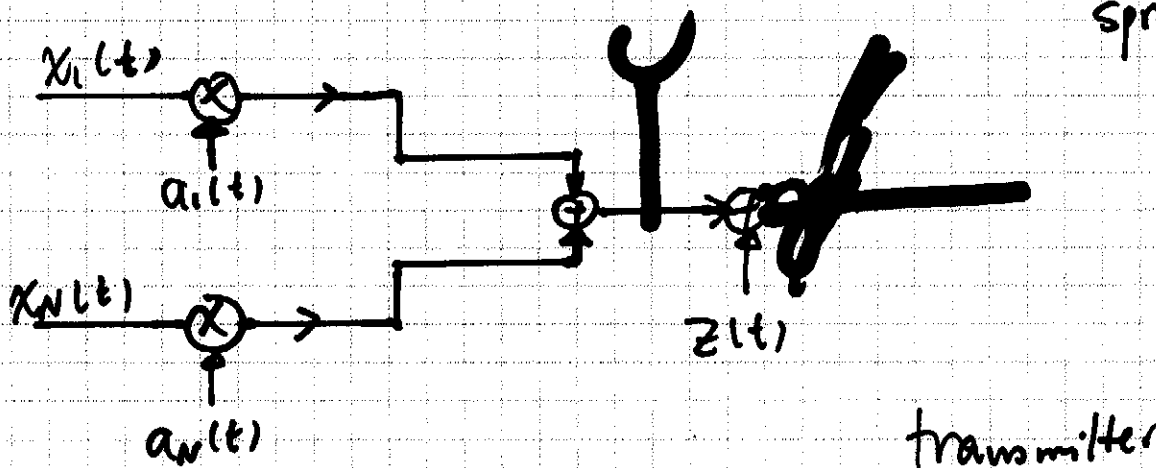
$$= A_i \sum_n x_{i,n} \sum_{k=0}^{G-1} a_{i,k} h_a(t - (nG + k)T_c)$$

The waveform for transmission is

$$\sum_{i=1}^N v_i(t) = \sum_{i=1}^N A_i \sum_n x_{i,n} \sum_{k=0}^{G-1} a_{i,k} h_a(t - (nG + k)T_c)$$

$a_{i,k}$, $0 \leq k \leq G-1$, is the CDMA code for the i th user.

spreading



② FH spread spectrum

* For M-ary frequency shift keying (MFSK),

$$v(t) = A \sum_n e^{j x_n 2\pi \Delta f t} u_T(t - nT)$$

where $x_n \in \{ \pm 1, \pm 3, \dots, \pm M-1 \}$

usually $\Delta f = \frac{1}{2T}$ is chosen so that the waveforms



$V_i(t) = A e^{j\chi_n 2\pi \Delta_f t}$, $0 \leq t \leq T$, are orthogonal.

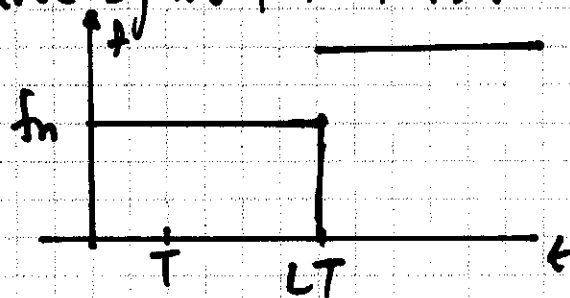
* An FH / MFSK signal is generated by using a PN sequence to select a set of carrier frequency shifts.

* Two types of FH

Slow FH (SFH): One or more source symbols are transmitted per hop:

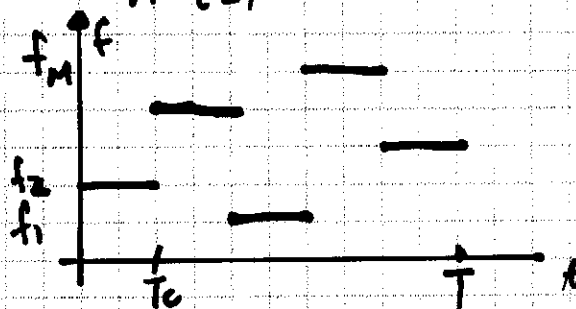
$$V(t) = A \sum_n \sum_{i=1}^L e^{j\chi_{n,i} 2\pi \Delta_f t + 2\pi f_n t} u(t - (nL + i - 1)T)$$

where f_n is the n th hop frequency and $\chi_{n,i}$ is the i th source symbol that is transmitted on the n th hop.



Fast FH (FFH): each symbol uses several hops

$$V(t) = A \sum_n \sum_{i=1}^L e^{j\chi_n 2\pi \Delta_f t + 2\pi f_{n,i} t} u_T(t - (nL + i - 1)T_c)$$



Advantage: It does not need power control.



§8.2. DS CDMA Acquisition and Detection

The signal received at the basestation (BS) (uplink)

$$\begin{aligned}
 r(t) &= \sum_{i=1}^N v_i(t - \tau_i) + \eta(t) \\
 &= \sum_{i=1}^N A_i \sum_n x_{i,n} \sum_{k=0}^{G-1} a_{i,k} h_a(t - \tau_i - (nG + k)T_c) + \eta(t)
 \end{aligned}$$

where τ_i is the time delay for the i th user.

The i th receiver needs to do two things:

- ① acquisition: to detect the time delay τ_i ;
- ② detection: to find $x_{i,n}$, $n \in \mathbb{Z}$.

In order to have the correlation detection work, the acquisition is necessary. For simplicity, we assume $N=2$, i.e., only two users, and the additive noise $\eta(t)=0$.

$$\begin{aligned}
 r(t) &= v_1(t - \tau_1) + v_2(t - \tau_2) \\
 &= A_1 \sum_n x_{1,n} \sum_{k=0}^{G-1} a_{1,k} h_a(t - \tau_1 - (nG + k)T_c) \\
 &\quad + A_2 \sum_n x_{2,n} \sum_{k=0}^{G-1} a_{2,k} h_a(t - \tau_2 - (nG + k)T_c)
 \end{aligned}$$

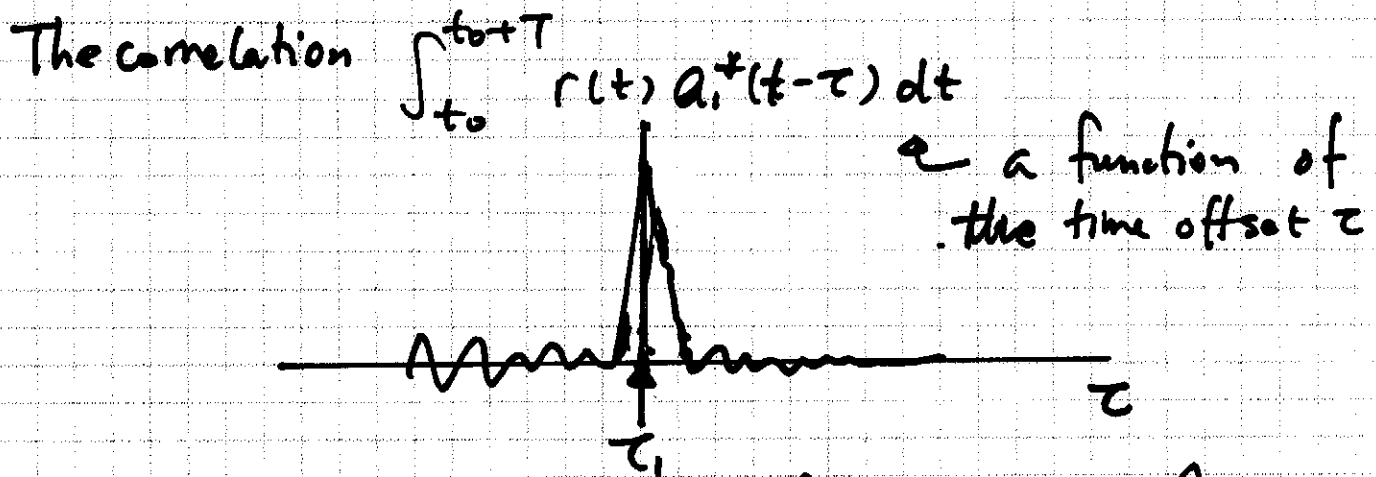
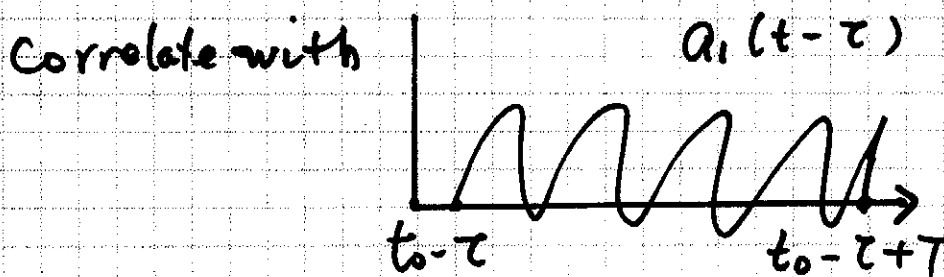
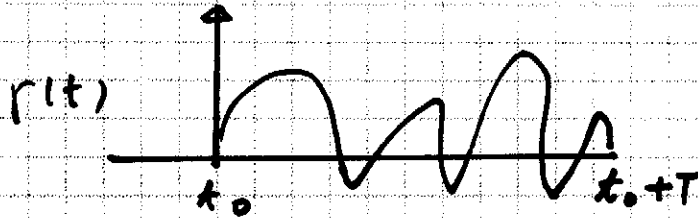
Note that the i th receiver knows the i th CDMA codeword $a_{i,k}$, $0 \leq k \leq G-1$, i.e., $a_i(t)$.



To have a good acquisition and detection, let us see what properties for these codewords $a_{i,k}$ are needed.

Acquisition:

$$r(t) = \chi_1(t - \tau_1) a_1(t - \tau_1) + \chi_2(t - \tau_2) a_2(t - \tau_2)$$



If there is a peak at $\hat{\tau}_1$, then $\tau_1 = \hat{\tau}_1$.

What is the correlation? Let us expand it.



$$I_1(\tau) = \int_{t_0}^{t_0+T} r(t) a_i^*(t-\tau) dt$$

$$= \int_{t_0}^{t_0+T} [\kappa_1(t-\tau_1) a_1(t-\tau_1) a_i^*(t-\tau) + \kappa_2(t-\tau_2) a_2(t-\tau_2) a_i^*(t-\tau)] dt$$

WLOG, we assume $t_0 = 0$ and $0 < \tau_i < T, i=1,2$.

Then,

$$I_1(\tau) = \int_0^T \kappa_1(t-\tau_1) a_1(t-\tau_1) a_i^*(t-\tau) dt$$

$$+ \int_0^T \kappa_2(t-\tau_2) a_2(t-\tau_2) a_i^*(t-\tau) dt$$

$$= \left(\int_0^{\tau_1} + \int_{\tau_1}^T \right) \kappa_1(t-\tau_1) a_1(t-\tau_1) a_i^*(t-\tau) dt$$

$$+ \left(\int_0^{\tau_2} + \int_{\tau_2}^T \right) \kappa_2(t-\tau_2) a_2(t-\tau_2) a_i^*(t-\tau) dt$$

We also assume that $\tau_i = d_i T_c$ for some integers d_i , which is reasonable when T_c is very small. Set $\tau = d T_c$ for integer d . Then

$$\kappa_1(t-\tau_1) = \begin{cases} \kappa_1, -1, & \text{when } 0 < t < \tau_1 \\ \kappa_1, 0, & \text{when } \tau_1 \leq t < T \end{cases}$$

$$\kappa_2(t-\tau_2) = \begin{cases} \kappa_2, -1, & \text{when } 0 < t < \tau_2 \\ \kappa_2, 0, & \text{when } \tau_2 \leq t < T. \end{cases}$$



Thus,

$$I_1(\tau) = \chi_{1,-1} \sum_{k=0}^{d_1-1} a_{1,k-d_1} a_{1,k-d}^* + \chi_{1,0} \sum_{k=d_1}^{G-1} a_{1,k-d_1} a_{1,k-d}^* \\ + \chi_{2,-1} \sum_{k=0}^{d_2-1} a_{2,k-d_2} a_{1,k-d}^* + \chi_{2,0} \sum_{k=d_2}^{G-1} a_{2,k-d_2} a_{1,k-d}^*$$

For acquisition, $\chi_{1,n}$ is unmodulated, i.e., $\chi_{1,n} = 1$ is used.

Thus, $I_1(d) = \sum_{k=0}^{G-1} a_{1,k-d_1} a_{1,k-d}^* + \left(\sum_{k=0}^{d_2-1} + \sum_{k=d_2}^{G-1} \right) a_{2,k-d_2} a_{1,k-d}^*$

Since $\{a_{i,k}\}$ is periodic with period G ,

$$\sum_{k=0}^{G-1} a_{1,k-d_1} a_{1,k-d}^* = R_{a_1,a_1}(d_1-d)$$

symmetric cyclic correlation

$$\left(\sum_{k=0}^{d_2-1} + \sum_{k=d_2}^{G-1} \right) a_{2,k-d_2} a_{1,k-d_1}^* = R_{a_1,a_2}(d_2-d_1)$$

periodic cross correlation

$$\left(\sum_{k=0}^{d_2-1} - \sum_{k=d_2}^{G-1} \right) a_{2,k-d_2} a_{1,k-d_1}^* = \bar{R}_{a_2,a_1}(d_2-d_1)$$

aperiodic cross correlation.

We expect when $d = d_1$, there will be a peak in $I_1(d)$





$$\Rightarrow R_{a_1, a_1}(d) \begin{cases} \text{Small for } d \neq 0 \\ G, \text{ for } d=0 \\ \text{(normalized)} \end{cases} \quad \begin{array}{l} \text{autocorrelation} \\ \text{of } a_1 \text{ at offset} \\ \text{small} \end{array}$$

$$R_{a_2, a_1}(d) \quad \text{small for } a_2 \neq a_1$$

$$\bar{R}_{a_2, a_1}(d) \quad \text{small for } a_2 \neq a_1$$

In this case, we may detect the time offset τ_1 for the 1st user

The same for τ_2 for the second user.

After τ_1 and τ_2 are detected, the next step is to detect the information symbols $X_{i,n}$.

Let us consider $X_{1,0}$.

$$I_1 \triangleq \int_0^T r(t + \tau_1) a_1^*(t) dt \\ = \int_0^T x_1(t) a_1(t) a_1^*(t) dt + \int_0^T x_2(t + \tau_1 - \tau_2) a_2(t + \tau_1 - \tau_2) a_1^*(t) dt$$

$$= x_{1,0} \sum_{k=0}^{G-1} a_{1,k} a_{1,k}^* + \left(\sum_{k=0}^{d_2-d_1-1} \pm \sum_{k=d_2-d_1}^{G-1} \right) a_{2,k+d_1-d_2} a_{1,k}^*$$

$$= x_{1,0} G + R_{a_2, a_1}(d_1 - d_2) \\ \text{or } \bar{R}_{a_2, a_1}(d_1 - d_2)$$



To get $\kappa_{i,0}$, $R_{a_i, a_i}(d)$ to be small for all d .
 $\bar{R}_{a_i, a_i}(d)$

In conclusion: To have a good acquisition and a good detection,

- ① auto correlation of all CDMA codes should be small at all time offsets (off peaks)
- ② all cross correlation of all CDMA codes should be small (period. & aperiodic)

We next want to see how small for the autocorrelation we need to have for DS CDMA codes.

Suppose there are K many users are using the channel simultaneously, i.e.,

$$r(t) = \sum_{i=1}^K \kappa_i(t) a_i(t - \tau_i)$$

Acquisition: $I_1(d) = R_{a_1, a_1}(d - d_1) + R_{a_2, a_2}(d - d_2) + \dots + R_{a_K, a_K}(d - d_K)$

detection: $I_1 = \kappa_{i,0} G + R_{a_2, a_2}(d - d_1) + \dots + R_{a_K, a_K}(d - d_1)$

Next to study this bound $\pm \delta$ $\dots$ $\pm \delta$ active
 $\leq G/\delta$ many users can be supported.



§8.3 Bounds for the Auto/Cross Correlations

Let $\mathcal{C} = \{a_1, \dots, a_K\} \subset \mathbb{C}^G$

each a_k is a CDMA code vector

$$\underline{a}_k = (a_{k,0}, \dots, a_{k,G-1})^T;$$

$$\bar{a}_k = (\dots; a_{k,0}, \dots, a_{k,G-1}, a_{k,0}, \dots, a_{k,G-1}, \dots)$$

is the periodic extension of $\underline{a}_k$

Let $\theta_c = \max \{ |R_{a_i, a_j}(\tau)| \mid a_i, a_j \in \mathcal{C}, \tau \in \mathbb{Z}, i \neq j \}$

maximum value of all the cross correlations.

$$\theta_a = \max \{ |R_{a_i, a_i}(\tau)| \mid a_i \in \mathcal{C}, \tau \neq 0 \}$$

maximum value of autocorrelation at offset times.

Note: We want θ_c and θ_a as small as possible.

Let $R_{a_i, a_i}(0) = G$ for all $a_i \in \mathcal{C}$ (normalized)

Theorem (Sarwate)

$$\frac{\theta_c^2}{G} + \frac{G-1}{G(K-1)} \frac{\theta_a^2}{G} \geq 1,$$

where G is the CDMA code length, K is the number of users (codewords).



Proof: Using

$$\sum_{l=0}^{G-1} |R_{a_i, a_j}(l)|^2 = \sum_{l=0}^{G-1} R_{a_i, a_i}(l) [R_{a_i, a_i}(l)]^*$$

to all codes $a_i, a_j \in \mathcal{C}$, we have

$$\sum_{\substack{a_i \in \mathcal{C} \\ a_j \in \mathcal{C} \\ a_i \neq a_j}} \sum_{l=0}^{G-1} |R_{a_i, a_j}(l)|^2 + \sum_{a_i \in \mathcal{C}} \sum_{l=0}^{G-1} |R_{a_i, a_i}(l)|^2$$

$$= \sum_{a_i \in \mathcal{C}} \sum_{a_j \in \mathcal{C}} \sum_{l=0}^{G-1} |R_{a_i, a_j}(l)|^2$$

$$= \sum_{a_i \in \mathcal{C}} \sum_{a_j \in \mathcal{C}} \sum_{l=0}^{G-1} R_{a_i, a_j}(l) [R_{a_i, a_j}(l)]^*$$

$$= \sum_{l=0}^{G-1} \left| \sum_{a \in \mathcal{C}} R_{a, a}(l) \right|^2$$

$$= \left| \sum_{a \in \mathcal{C}} R_{a, a}(0) \right|^2 + \sum_{l=1}^{G-1} \left| \sum_{a \in \mathcal{C}} R_{a, a}(l) \right|^2$$

$$= K^2 G^2 + \sum_{l=1}^{G-1} \left| \sum_{a \in \mathcal{C}} R_{a, a}(l) \right|^2$$



Look at the left hand side,

$$\sum_{\substack{a_i \in \mathcal{C} \\ a_j \in \mathcal{L} \\ a_i \neq a_j}} \sum_{l=0}^{G-1} |R_{a_i a_j}(l)|^2 + \sum_{a \in \mathcal{C}} |R_{a a}(0)|^2 + \sum_{a \in \mathcal{L}} \sum_{l=1}^{G-1} |R_{a a}(l)|^2$$

$$\leq k(k-1) G \theta_c^2 + k G^2 + k(G-1) \theta_a^2$$

Look at the right hand side

$$k^2 G^2 + \sum_{l=1}^{G-1} \left| \sum_{a \in \mathcal{C}} R_{a a}(l) \right|^2$$

$$\geq k^2 G^2$$

Thus, $k(k-1) G \theta_c^2 + k G^2 + k(G-1) \theta_a^2 \geq k^2 G^2$

$$\Rightarrow k(k-1) G \theta_c^2 + k(G-1) \theta_a^2 \geq k(k-1) G^2$$

$$\Rightarrow \frac{\theta_c^2}{G} + \frac{G-1}{k(k-1)} \frac{\theta_a^2}{G} \geq 1$$

If we let $\theta_{\max} = \max\{\theta_a, \theta_c\}$

$$\Rightarrow \frac{\theta_{\max}^2}{G} = \max \left\{ \frac{\theta_a^2}{G}, \frac{\theta_c^2}{G} \right\} \geq \frac{G(k-1)}{G k-1}$$

Welch bound



11-15

$$\text{When } k \geq 2, \quad \frac{G(k-1)}{6(k-1)} > \frac{1}{2}$$

$$\Rightarrow \frac{\sigma_{\max}^2}{G} > \frac{1}{2}$$

$$\Rightarrow \sigma_{\max} \geq \frac{\sqrt{2}}{2} \sqrt{G}$$

Notice that, for the acquisition:

$$I_1(d) = K a_{1,a_1} (d_1 - d) + \tilde{K} a_{2,a_2} (d_2 - d) + \dots + \tilde{K} a_{K,a_K} (d_K - d)$$

$$\text{When } d_0 = d, \quad \sigma_{\max} < \frac{G}{K}$$

$$\Rightarrow \sigma_{\max} < \frac{G}{K} \quad \text{for acquisition}$$

$$\Rightarrow \frac{\sqrt{2}}{2} \sqrt{G} < \frac{G}{K} \Rightarrow K < \sqrt{2G} \quad (\text{Rough Capacity})$$

$$\text{Active users} \approx \sqrt{G} \quad \text{many}$$