

Space-Time Coding for Cooperative Communications

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Outline

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From Wired Modems to Wireless Modems

- Wired modems have been well developed since 1980's
- Key elements for the progress of wired modems:

< 9.6 kbs/s

equalization (Lucky 60s)

9.6 kbs/s 1984

TCM +equalization (DFE)

14.4 kbs/s

19.20 kbs/s

28.8 kbs/s

33.6 kbs/s

56 kbs/s

TCM

TCM

high dim TCM

high dim TCM

high dim TCM

better

Bandwidth efficient coding
Several bits/s/Hz

+ equalization

Squeeze more
bits to a symbol

Higher SNR

Asymmetric Digital Subscriber Line (ADSL)

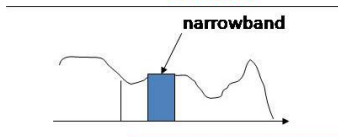
6 Mbs/s

**orthogonal frequency division
multiplexing (OFDM)**

or called **discrete multi-tone (DMT)**

More advanced DSP

Use more bandwidth



What Happens to Wireless Modems

- Fading, interferences, low SNR
 - TCM has not been used successfully in wireless systems
 - OFDM systems are only successful in LAN not WAN yet
- Can we overcome these difficulties?
 - Maybe! To sufficiently use spatial diversities to overcome wireless fading
 - Beginning standards: IEEE 802.11n, 802.16d/e, 3GPP
 - More standards are coming (similar to wired modems)

Exploit Spatial Diversity

- Two ways to exploit spatial diversity:
 - Multi-antenna systems
 - Cooperative communication systems
- Benefits:
 - Potential high capacity gain

Diversity Order and Full Diversity

- If the symbol/bit error rate of a system satisfies

$$P_{\text{error}}(\text{SNR}) \sim c \cdot \text{SNR}^{-d}$$

for some positive constant c , then we say that the system achieves **diversity order** d .

- The larger d is, the smaller the error probability is, for a given channel SNR.
- For an N transmitter (MIMO system or cooperative system) and M receiver wireless system, its highest spatial diversity order is MN that is also the number of freedoms of the system.
- If the maximal diversity order is achieved, we call the system achieves **full diversity**.

Exploit Spatial Diversity: MIMO Systems

Figure: Multi-antenna Communication System



- Multiple antennas are co-located
- Space-time coding plays an important role to achieve the spatial diversity.
- They are (or will be) adopted in the current and future high speed wireless systems.

Space-Time Codes/Modulations

- A space-time code (or modulation) is a collection (set) of matrices (one dimension is space and the other dimension is time).
- A space-time code achieves full diversity when ML receiver is used, if it has the **full rank property**, i.e., the rank of difference matrix of any two distinct matrices in the code is full rank.

- The most well-known space-time code is Alamouti code

$$\begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix},$$

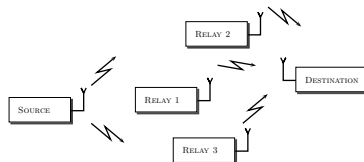
where x_1 and x_2 are two modulated information symbols. It achieves full diversity and its ML receiver is the same as linear receiver.

- Full rank criterion is the weakest condition to achieve the full diversity

MIMO Systems: A Few Words on Space-Time Coding

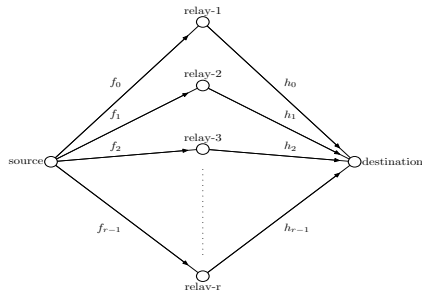
- There have been many studies with many code families in the last decade.
- Almost all the existing space-time codes achieve full spatial diversity (full rank) when the ML receiver is used, since they are designed based on the ML receiver.
- The main challenge in my opinion is still: the tradeoff between *diversity-multiplexing-complexity*, i.e., how to construct high rate space-time codes that achieve full spatial diversity when a low complexity receiver is used.
- We recently obtained design criteria for space-time codes to achieve full diversity when a linear receiver or partial interference cancellation (PIC) group decoding is used and obtained new code families, such as *overlapped Alamouti codes* that satisfy these criteria.

Exploit Spatial Diversity: Cooperative Communication Networks



- Cooperative diversity early studies: Sendonaris-Erkup-Aazhang; Boyer-Falconer-Yanikomeroglu; Laneman-Wornell; Hunter-Nosratinia; Zhao-Valenti;
- The basic idea is to build a *virtual multi-antenna system* to achieve spatial diversity
- Since antennas from different relay nodes are not co-located, synchronization between relay nodes (or antennas) may be a problem

DF and AF Protocols for Relay Networks

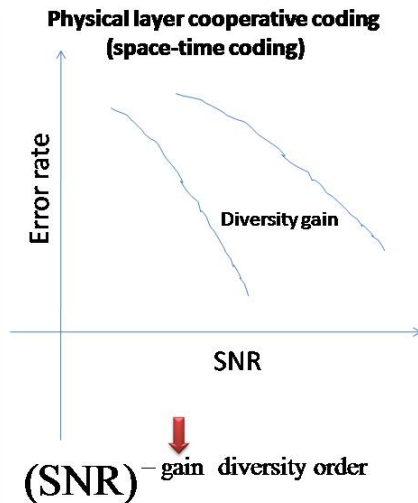
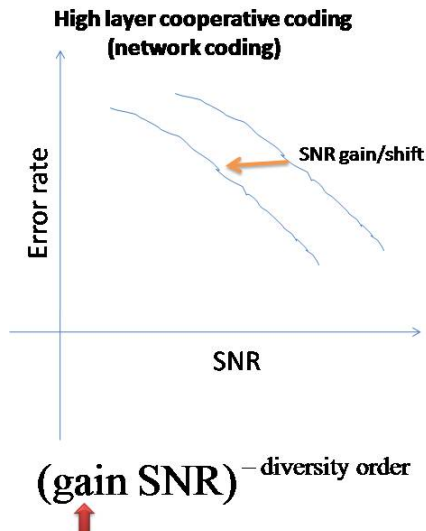


- Two phases: Source $\Rightarrow$ Relay $\Rightarrow$ Destination;
- Decoding-and-forward (DF):
 - Decoding at the relay node;
 - No error is forwarded to the destination;
- Amplify-and-forward (AF):
 - No decoding at the relay;
 - Noise at the first phase is forwarded to the destination;

Two Ways for Cooperative Communications

- Physical layer designs (space-time coding)
 - Complex-valued in general
 - Can be mathematically combined with a complex-valued wireless fading channel 😊
 - May change transmission power 😞
- Higher layer designs (network coding)
 - Binary
 - Hard to be combined with a complex-valued wireless fading channel 😞
 - Does not change transmission power 😊

Cooperative Diversity Comparison



Synchronized Relay Networks

- All the existing full diversity achieving space-time codes for MIMO systems can be directly applied to achieve full cooperative diversity.
 - Laneman-Wornell; Jing-Hassibi;

Asynchronous Relay Network

- Since transmit antennas are not co-located, signals from relay nodes are usually asynchronous in both time and frequency.
- When the relay nodes are asynchronous, the existing space-time codes can not be directly used to achieve the full cooperative diversity.
- Different relays may have different frequency offsets. At the destination, there are multiple frequency offsets that may be difficult to compensate simulatenously.

Asynchronous Relay Network in Time

Source transmits $x_s(1) = x_1, x_s(2) = x_2$

Two relay nodes R_1 and R_2 and both of them have correctly detected the source symbols x_1, x_2

R_1 transmits $x_{r_1}(1) = x_1, x_{r_1}(2) = -x_2^*$
 R_2 transmits $x_{r_2}(1) = x_2, x_{r_2}(2) = x_1^*$ } cooperatively

Synchronous case:

Received matrix is $X_r = \begin{bmatrix} x_{r_1}(1) & x_{r_1}(2) \\ x_{r_2}(1) & x_{r_2}(2) \end{bmatrix} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}$

which is the same as the **Alamouti** scheme and achieve the diversity order 2.

Asynchronous case: Received signal matrix may be

$$\begin{bmatrix} y_1 & -y_2^* & x_1 & -x_2^* & z_1 & -z_2^* \\ y_2 & y_1^* & x_2 & x_1^* & z_2 & z_1^* \end{bmatrix} \rightarrow \begin{bmatrix} -y_2^* & x_1 & -x_2^* \\ x_2 & x_1^* & z_2 \end{bmatrix}$$

When information symbol x_1 takes the same value, the new 2 by 2 matrix has rank 1.

Some Existing Related Works on Timing Offsets

- Time domain techniques:
 - Asynchronous cooperative diversity, Wei-Goeckel-Valenti'04
 - Space-time code without perfect synchronization, X. Li'04
 - Space-time trellis codes achieving full asynchronous cooperative diversity, Y. Li-Xia'05; and Shift-full-rank matrices, Shang-Xia'06
 - Delay tolerant linear STBC, Deman-Hammons'06
 - ...
- OFDM based techniques:
 - DF Alamouti coded system, Mei-Hua-Swami-Daneshrad'05;
 - X. Li-Ng-Han'06;
 - Space-time-frequency coded system, Y. Li-Zhang-Xia'06;
 - AF Alamouti coded system, Z. Li-Xia'07
 - ...

Some Existing Related Works on Frequency Offsets

- Multiple frequency offsets cause intersymbol interferences. Equalizations were proposed by Li-Qu-Zhu'06, Veronesi-Goeckel'06, Tian-Xia-Ching'07, Wang-Xia-Yin'08
- **This is not today's focus.**

Linearly Encoded DF Asynchronous Relay Network

- Relay Node: Distributed linear dispersion coding scheme at the i th relay node
 - Receives information symbol vector: $\mathbf{s}$;
 - Transmits $\mathbf{t}_i = \mathbf{s}\mathbf{A}_i \in \mathbb{C}^m$, $\mathbf{A}_i \in \mathbb{C}^{l \times m}$.
- The codeword from r relay nodes

$$\mathbf{C} = \begin{bmatrix} \mathbf{t}_0 \\ \mathbf{t}_1 \\ \vdots \\ \mathbf{t}_{r-1} \end{bmatrix} \quad (1)$$

is called a *distributed linear dispersion space-time code* (Jing-Hassibi).

Distributed Linearly Convolutional Space-Time Codes

- When $\mathbf{A}_i$ are convolution matrices $\implies \mathbf{C}$ is a *distributed linear convolutional space-time code (DL-STCC)*, whose i th row $\mathbf{t}_i = \mathbf{s}\mathbf{A}_i$:

$$\mathbf{A}_i = \begin{bmatrix} m_{i0} & m_{i1} & m_{i2} & \cdots \\ 0 & m_{i0} & m_{i1} & \cdots \\ 0 & 0 & m_{i0} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

- Generating matrix for linear convolutional space-time code $\mathbf{C}$ is $\mathbf{M} = (m_{ij})$.

Linearly Encoded DF Asynchronous Relay Network

- Delay profile: $\Delta = \{\tau_0, \tau_1, \dots, \tau_{r-1}\}$;
- Maximum delay: $\tau_{\max}$;
- How to combat the delay:
 - Padding t_i (a frame) with at least $\tau_{\max}$ zeros;
 - Prevent the interference between codeword matrices/frames;
- Received signal model at the destination node:

$$\mathbf{y} = \mathbf{h}\mathbf{C}_{\Delta} + \mathbf{n}, \quad (2)$$

where

- $\mathbf{h}$: $1 \times r$ channel vector (from relay nodes to destination node);
- $\mathbf{n}$: $1 \times m$ white Gaussian noise;
- $\mathbf{C}_{\Delta}$: a row shifted version of $\mathbf{C}$ with shift profile Δ ;

The Form of C_{Δ}

$$\begin{bmatrix}
 \overbrace{0 \ 0 \ \dots \ 0}^{\tau_0} \ \star \ \star \ \dots \ \star \ \overbrace{0 \ 0 \ 0 \ 0 \ \dots \ 0}^{\delta_{\max}-\tau_0} \\
 \overbrace{0 \ 0 \ 0 \ \dots \ 0}^{\tau_1} \ \star \ \star \ \dots \ \star \ \overbrace{0 \ 0 \ 0 \ \dots \ 0}^{\delta_{\max}-\tau_1} \\
 \overbrace{0 \ \dots \ 0}^{\tau_2} \ \star \ \star \ \dots \ \star \ \overbrace{0 \ 0 \ 0 \ 0 \ 0 \ \dots \ 0}^{\delta_{\max}-\tau_2} \\
 \dots \ \dots \ \dots \ \dots \ \dots \ \dots \ \dots \ \dots \ \dots \ \dots \\
 \overbrace{0 \ 0 \ 0 \ 0 \ \dots \ 0}^{\tau_{r-1}} \ \star \ \star \ \dots \ \star \ \overbrace{0 \ 0 \ \dots \ 0}^{\delta_{\max}-\tau_{r-1}}
 \end{bmatrix},$$

where $\star$ represents the elements in C .

Design Criterion

- $\mathbf{y}$ can be effectively viewed as the result of sending the code word $\mathbf{C}_\Delta$ (called effective codeword matrix and the corresponding code is called an effective code);
- Assume perfect channel state information (CSI) is known at the destination;
- $\mathbf{C}_\Delta - \tilde{\mathbf{C}}_\Delta$ full rank $\implies$ Full diversity;
- Damen and Hammons studied a class of layered threaded algebraic spacetime (TAST) *block* codes, *IEEE Information Theory*, 2007 — their rates are low.
- **Our goal:** design linear convolutional space-time codes (i.e., their generating matrices $\mathbf{M}$) that can achieve the full diversity under any delay profile.

Definition of Shift-full-rank Matrix (Li-Xia'05, Shang-Xia'06) I

- Let $\mathbf{M} \in \mathbb{C}^{r \times m}$ be

$$\mathbf{M} = \begin{bmatrix} \mathbf{m}_0 \\ \mathbf{m}_1 \\ \vdots \\ \mathbf{m}_{r-1} \end{bmatrix}, \quad (3)$$

where $\mathbf{m}_i \in \mathbb{C}^m$ are the row vectors of $\mathbf{M}$;

- $\mathbf{M}$ is used as the generating matrix for a linear convolutional space-time code;
- Delay profile: $\Delta = \{\tau_0, \tau_1, \dots, \tau_{r-1}\}$;

Definition of Shift-full-rank Matrix (Li-Xia'05, Shang-Xia'06) II

- Define the row shifted version of $\mathbf{M}$ with delay profile Δ as follows:

$$\mathbf{M}_{\Delta} = \begin{bmatrix} \overbrace{0 \ 0 \ \dots \ 0}^{\tau_0} & \mathbf{m}_0 & \overbrace{0 \ 0 \ 0 \ 0 \ \dots \ 0}^{\delta_{\max} - \tau_0} \\ \overbrace{0 \ 0 \ 0 \ \dots \ 0}^{\tau_1} & \mathbf{m}_1 & \overbrace{0 \ 0 \ 0 \ \dots \ 0}^{\delta_{\max} - \tau_1} \\ \dots & \dots & \dots \\ \overbrace{0 \ 0 \ 0 \ 0 \ \dots \ 0}^{\tau_{r-1}} & \mathbf{m}_{r-1} & \overbrace{0 \ 0 \ \dots \ 0}^{\delta_{\max} - \tau_{r-1}} \end{bmatrix}$$

- $\mathbf{M}$ is called *shift-full-rank (SFR)* matrix if $\mathbf{M}_{\Delta}$ is a full row rank matrix for any delay profile Δ .

Shift-full-rank Matrices

An example of SFR matrix:

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Different from the conventional full rank matrices, an SFR (full row rank) adding a column may not be an SFR:

SFR matrix

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

add a column to the left

Not SFR matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

first row right shift 1 symbol

Not full row rank

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Shift-full-rank Property and Full Diversity Property

- $\mathbf{C} - \tilde{\mathbf{C}}$ shift-full rank, $\forall \mathbf{C}, \tilde{\mathbf{C}} \in \mathcal{C}, \mathbf{C} \neq \tilde{\mathbf{C}} \implies$ full diversity property;
 - $\mathbf{M}$ is SFR $\Rightarrow$ its corresponding DL-STCC achieves the **full asynchronous diversity** with the ML receiver.
- Vector $\Leftrightarrow$ Polynomial:

$$\mathbf{v} = [v_0, v_1, \dots, v_{n-1}] \Leftrightarrow P_{\mathbf{v}} = \sum_{i=0}^{n-1} v_i x^i; \quad (4)$$

- Shift $\Leftrightarrow$ multiply by x :

$$\tilde{\mathbf{v}} = [0, v_0, v_1, \dots, v_{n-1}] \Leftrightarrow P_{\tilde{\mathbf{v}}} = x \sum_{i=0}^{n-1} v_i x^i; \quad (5)$$

- Studying the SFR property using the polynomial form is more convenient.

SFR Polynomial Set

- $P = \{p_0(x), \dots, p_{r-1}(x)\}$ be a polynomial set, each polynomial corresponds to a row of a matrix $\mathbf{M}$ (generator polynomials);
- A polynomial set $P = \{p_i(x), 0 \leq i \leq r-1\}$ is *shift-full-rank* (SFR) if the following condition holds: for any non-negative $\tau_i \in \mathbb{Z}, 0 \leq i < r, \sum_{i=0}^{r-1} c_i x^{\tau_i} p_i(x) = 0$ if and only if $c_i = 0, 0 \leq i < r$.
- $\mathbf{M}$ is SFR $\Leftrightarrow P$ is SFR;
- $p_s(x)$ be a non-zero polynomial.
 $\{p_s(x)p_0(x), \dots, p_s(x)p_{r-1}(x)\}$ is an SFR set $\Leftrightarrow$
 $\{p_0(x), \dots, p_{r-1}(x)\}$ is an SFR set.
- Generator polynomial set $\{p_0(x), \dots, p_{r-1}(x)\}$ is an SFR set
 $\Rightarrow$ Its DL-STCC $\mathcal{C}$ has the full diversity property under any delay profile

Construction of an SFR Polynomial Set

Definition

Let $P = \{p_0(x), p_1(x), \dots, p_{r-1}(x)\}$ be a set of generator polynomials. If 0 is not a root of $p_i(x)$, $i = 0, 1, 2, \dots, r-1$, then we call P of standard form.

Theorem (Guo-Xia'08)

Let $P = \{p_0(x), p_1(x)\}$ be a generator polynomial set of standard form. P is an SFR set if and only if $c_0 p_0(x) \neq p_1(x)$ for any $c_0 \in \mathbb{C}$, i.e., $p_0(x)$ and $p_1(x)$ are linearly independent.

Construction of an SFR Polynomial Set for $r = 3$

Theorem (Guo-Xia'08)

Let $P = \{p_0(x), p_1(x), p_2(x)\}$ be a generator polynomial set of standard form and $\{v_0, v_1, v_2\}$ be the corresponding coefficient vector set. Let $c_0 = [v_2]_0/[v_0]_0$, $c_1 = [v_2]_0/[v_1]_0$, $c_2 = [v_1]_0/[v_0]_0$. Let n be the lowest power of $c_0 p_0(x) - p_2(x)$, m be the lowest power of $c_1 p_1(x) - p_2(x)$, and k be the lowest power of $c_2 p_0(x) - p_1(x)$. Then, P is an SFR set if and only if the following four conditions are satisfied:

1. $p_0(x)$, $p_1(x)$ and $p_2(x)$ are linearly independent;
2. $c_0 p_0(x) - p_2(x) \neq \hat{c}_1 x^n p_1(x)$ for any $\hat{c}_1 \in \mathbb{C}$;
3. $c_1 p_1(x) - p_2(x) \neq \hat{c}_0 x^m p_0(x)$ for any $\hat{c}_0 \in \mathbb{C}$;
4. $c_2 p_0(x) - p_1(x) \neq \hat{c}_2 x^k p_2(x)$ for any $\hat{c}_2 \in \mathbb{C}$.

Construction of an SFR Polynomial Set for a General r

A general construction method (**Guo-Xia'08**):

- Step 1.** Choose an initial SFR set with k polynomials,
 $n := k$;
- Step 2.** If $n < r$, goto Step 3; otherwise stop;
- Step 3.** If the GCD of $\{p_0(x), p_1(x), \dots, p_{n-1}(x)\}$ is 1,
then choose two polynomials $a_0(x)$ and $p_n(x)$, so
that $a_0(x) \nmid p_n(x)$, set
 $p_i(x) := p_i(x)a_0(x), i = 0, 1, \dots, n-1$;
if the GCD of $\{p_0(x), p_1(x), \dots, p_{n-1}(x)\}$ is $u(x)$,
choose $p_n(x)$ so that $u(x) \nmid p_n(x)$; set $n := n + 1$,
goto Step 2.

A Design Example (Shang-Xia'06)

$$r=1, \quad e^0 = 1$$

$$r=2, \quad e^1 = 1 \quad 1$$

$$r=3, \quad e^2 = 1 \quad 0 \quad 1$$

$$r=4, \quad e^3 = 1 \quad 1 \quad 1 \quad 1$$

$$r=5, \quad e^4 = 1 \quad 0 \quad 0 \quad 0 \quad 1$$

$$r=6, \quad e^5 = 1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 1$$

$$r=7, \quad e^6 = 1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1$$

$$r=8, \quad e^7 = 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1$$

...

Power Distribution I

- What is the optimal way to distribute power?
- Define the power distribution matrix,

$$\mathbf{A} = \begin{bmatrix} \alpha_0 & 0 & 0 & \cdots & 0 \\ 0 & \alpha_1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \alpha_{r-1} \end{bmatrix}, \quad (6)$$

where $\left| \prod_{k=0}^{r-1} \alpha_k \right| = 1$;

- Define a code $\mathcal{C}_d$: $\mathcal{C}_d = \{ \mathbf{C}_d \mid \mathbf{C}_d = \mathbf{A}\mathbf{C}, \mathbf{C} \in \mathcal{C} \}$, $\mathcal{C}_d$ and $\mathcal{C}$ have the same diversity product;
- Adjust α_i 's so that $\mathcal{C}_d$ has the smallest average power;

Power Distribution II

- The problem we want to solve can be formulated as

$$\min_{\alpha_0, \alpha_1, \dots, \alpha_{r-1}} \sum_{k=0}^{r-1} |\alpha_k|^2 \|\mathbf{v}_k\|^2 \quad (7)$$

subject to the constraint $\left| \prod_{k=0}^{r-1} \alpha_k \right| = 1$;

- One solution for the above equations is

$$\alpha_i = \frac{\left(\prod_{k=0}^{r-1} \|\mathbf{v}_k\|^2 \right)^{\frac{1}{r}}}{\|\mathbf{v}_i\|^2}, i = 0, 1, \dots, r-1; \quad (8)$$

Trace Orthogonality

- General form of linear dispersion code:

$$\mathbf{C} = \sum_{i=0}^{l-1} s_i \mathbf{C}_i; \quad (9)$$

- Trace orthogonality property:

$$\text{tr} \left\{ \mathbf{C}_i \mathbf{C}_j^\dagger \right\} = \begin{cases} 1, & i = j, \\ 0, & i \neq j. \end{cases} \quad (10)$$

- It is used for the linearly encoded system not to loss the capacity called capacity lossless property (Heath-Paulraj'02).

Trace Orthogonality

- Define two mappings $\mathcal{S}_0$ and $\mathcal{S}_1$:

$$\mathcal{S}_0\{\mathbf{v}, j\} = [\mathbf{v}, \overbrace{0, 0, \dots, 0}^{j \text{ 0's}}], \quad (11)$$

$$\mathcal{S}_1\{\mathbf{v}, j\} = [\overbrace{0, 0, \dots, 0}^{j \text{ 0's}}, \mathbf{v}]. \quad (12)$$

Trace Orthogonality

- Let $\Delta = \{\tau_0, \tau_1, \dots, \tau_{r-1}\}$;
- Then the effective codeword (linear dispersion) matrices $\mathbf{C}_{i,\Delta}$ can be written as

$$\mathbf{C}_{i,\Delta} = \begin{bmatrix} \mathcal{S}_0 \{ \mathcal{S}_1 \{ \mathbf{v}_0, i + \tau_0 \}, l_0 - i \} \\ \mathcal{S}_0 \{ \mathcal{S}_1 \{ \mathbf{v}_1, i + \tau_1 \}, l_0 - i \} \\ \vdots \\ \mathcal{S}_0 \{ \mathcal{S}_1 \{ \mathbf{v}_{r-1}, i + \tau_{r-1} \}, l_0 - i \} \end{bmatrix}, \quad i = 0, 1, \dots, l-1. \quad (13)$$

Trace Orthogonality

Theorem (Guo-Xia'08)

The linear dispersion matrices $\mathbf{C}_{i,\Delta}, i = 0, 1, \dots, l$ have the trace orthogonality property under any delay profile Δ if and only if the following condition holds:

$$\sum_{i=0}^{r-1} \mathcal{S}_0\{\mathbf{v}_i, j\} \mathcal{S}_1\{\mathbf{v}_i, j\}^\dagger = \begin{cases} 1, & j = 0; \\ 0, & j = 1, 2, \dots, q, \end{cases} \quad (14)$$

where q is the highest degree of the polynomials $p_i(x)$ in P .

Diversity Property with MMSE-DFE Receiver

- $\mathbf{h}$: the channel matrix,

$$\mathbf{h} = [h_0 \quad h_1 \quad \dots, \quad h_{r-1}], \quad (15)$$

- Write the channel model in the polynomial form,

$$p_{\mathbf{r}}(x) = p_{\mathbf{s}}(x) \left(\sum_{k=0}^{r-1} h_k x^{\tau_k} p_k(x) \right); \quad (16)$$

- ML decoder: Viterbi algorithm (MLSD).

Full Diversity Property with MMSE-DFE Receiver

- Shift-full-rank matrix: number of rows $\leq$ number of columns;
- Time delay $\implies$ extra channel memory;
- Time delay $\uparrow$ or/and number of relay nodes $\uparrow \implies$ the complexity of MLSD becomes higher
- What is the performance of sub-optimal receiver, such as, MMSE-DFE receiver?

Full Diversity Property with MMSE-DFE Receiver

The answer is that our proposed codes can achieve full asynchronous cooperative diversity even with the MMSE-DFE receiver based on the following result.

Theorem (Shang-Xia'07)

Let $\mathbb{H}$ be the equivalent overall channel matrix of a linear dispersion code $\mathcal{C}$. Then $\mathcal{C}$ can achieve full diversity with ZF or MMSE receivers if the following condition holds:

$$\det \left(\mathbb{H}^\dagger \mathbb{H} \right) \geq c_0 \| \mathbf{H} \|_F^{2l}. \quad (17)$$

where c_0 is a positive constant that is independent of the channel matrix $\mathbf{H}$ and determined by the coding scheme $\mathcal{C}$ and l is the number of independent information symbols embedded in $\mathcal{C}$.

An Equivalent Channel Example I

- Consider a two relay system with space-time code

$$\mathcal{C} = \begin{bmatrix} x_1 & x_2 \\ x_2 & x_1 \end{bmatrix}$$

- Original signal model is

$$\begin{bmatrix} y_1 & y_2 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} x_1 & x_2 \\ x_2 & x_1 \end{bmatrix} + \begin{bmatrix} w_1 & w_2 \end{bmatrix}$$

and the original channel $\mathbf{H} = [h_1 \ h_2]$

An Equivalent Channel Example II

- An equivalent signal model is

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2 & h_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

and the equivalent channel

$$\mathbb{H} = \begin{bmatrix} h_1 & h_2 \\ h_2 & h_1 \end{bmatrix}$$

Full Diversity Property with MMSE-DFE Receiver

Theorem (Guo-Xia'08)

Let $\mathcal{C}$ be a DL-STCC and its generator polynomial set be P . If P is an SFR set, then $\mathcal{C}$ can achieve the full spatial diversity when the MMSE-DFE receiver is used no matter how a signal from a relay is delayed, provided that the maximum delay $\tau_{\max}$ is finite.

Examples without Trace Orthogonality Property

■ 2-relay:

$$p_0(x) = 1, \quad p_1(x) = 1 - x \quad (18)$$

■ 3-relay:

$$p_0(x) = 1 + x, \quad (19)$$

$$p_1(x) = 1 - x^2, \quad (20)$$

$$p_2(x) = 1 - x. \quad (21)$$

Examples with Trace-Orthogonality Property

■ 2-relay:

$$p_0(x) = \frac{1}{2}(1+x), \quad p_1(x) = \frac{1}{2}(1-x). \quad (22)$$

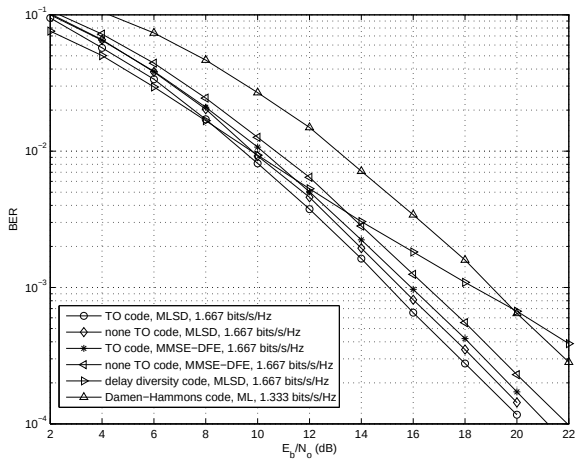
■ 3-relay:

$$\begin{aligned} p_0(x) &= \frac{1}{3}(1+x+x^2), \\ p_1(x) &= \frac{1}{3\sqrt{2}}(1-2x+x^2), \\ p_2(x) &= \frac{1}{\sqrt{6}}(1-x^2). \end{aligned} \quad (23)$$

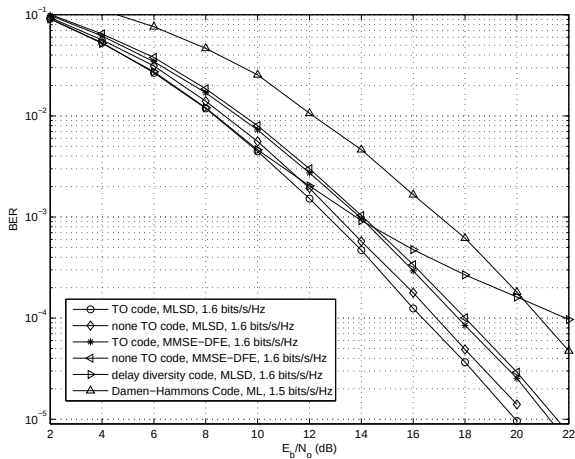
Simulation Parameters

- The channel between any two nodes is quasi-static Rayleigh flat fading channel;
- Timing error is uniformly distributed in $[0, \tau_{\max}]$;
- The length of the symbol sequence is 20;
- The maximum delay is $\tau_{\max} = 3$, the padding length $\delta_{\max} = 3$.
- MMSE-DFE tap length is 8.

Simulation Results: Two Relay Nodes



Simulation Results: Three Relay Nodes



Conclusion and Some Future Research Problems

- Presented a systematic design of distributed linear convolutional space-time codes achieving full diversity with MMSE-DFE receiver without synchronization requirement;
- We have recently developed efficient equalization methods for this coding scheme when there are also multiple frequency offsets (Wang-Xia et al, Proc. ISIT 2008)
- Some research problems:
 - Full diversity achieving codes with other low complexity receivers, such as SIC etc.
 - Higher rate code designs
 - Combination of space-time coding and network coding to achieve both SNR and diversity gains
 - Codes robust to both timing errors and frequency offsets
 - Complexity-Diversity-Multiplexing tradeoff for both MIMO and cooperative systems
- Wireless modem era is still in the beginning stage

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THANK YOU !!

Some Papers to Read

- Yabo Li and X.-G. Xia, [A Family of Distributed Space-Time Trellis Codes with Asynchronous Cooperative Diversity](#), *IEEE Trans. on Communications*, April 2007.
- Yue Shang and X.-G. Xia, [Shift Full Rank Matrices and Applications in Space-Time Trellis Codes for Relay Networks with Asynchronous Cooperative Diversity](#), *IEEE Trans. on Information Theory*, July 2006.
- X. Guo and X.-G. Xia, A Distributed Space-Time Coding in Asynchronous Wireless Relay Networks, *IEEE Trans. on Wireless Communications*, May 2008.
- Zheng Li and X.-G. Xia, A Simple Alamouti Space-Time Transmission Scheme for Asynchronous Cooperative Systems, *IEEE Signal Processing Letters*, Nov. 2007.
- Y. Li, W. Zhang, and X.-G. Xia, Distributive High-Rate Space-Frequency Codes Achieving Full Cooperative and Multipath Diversities for Asynchronous Cooperative Communications, *IEEE Trans. on Vehicular Technology*, Jan. 2009.
- Zheng Li, X.-G. Xia, and B. Li, Achieving Full Diversity and Fast ML Decoding via Simple Analog Network Coding for Asynchronous Two-Way Relay Networks, *IEEE Trans. on Communications*, Dec. 2009.
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- Zheng Li, X.-G. Xia, and M. H. Lee, [A Simple Orthogonal Space-Time Coding Scheme for Asynchronous Cooperative Systems for Frequency Selective Fading Channels](#), *IEEE Transactions on Communications*, August 2010. [A correction](#), <http://www.ee.udel.edu/~xxia/Pub.html>