

A Design of High-Rate Space-Frequency Codes for MIMO-OFDM Systems

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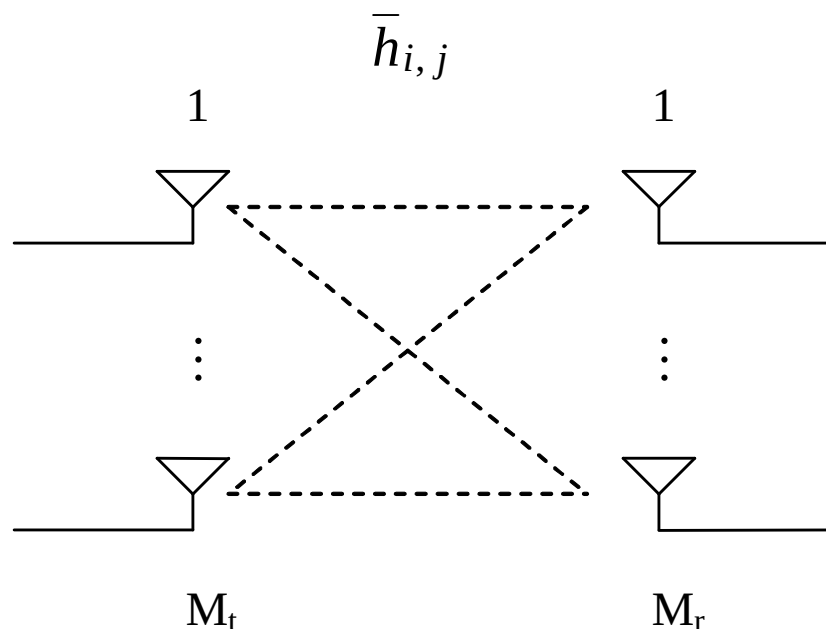
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Outline

- Motivation
- Space-frequency coding
- High-rate full-diversity SFC design
- Summary and future work

Multiple Antennas System



● $\bar{h}_{i,j}, \bar{\tau}_{i,j}$: are the multipath gain and delay from the j th Tx to the i th Rx, respectively ($j = 1, \dots, M_t; i = 1, \dots, M_r$)

● $\bar{h}_{i,j} = [h_{i,j}(0) \quad h_{i,j}(1) \quad \cdots \quad h_{i,j}(L-1)]$

● $\bar{\tau}_{i,j} = [\tau_{i,j}(0) \quad \tau_{i,j}(1) \quad \cdots \quad \tau_{i,j}(L-1)]$

Advantages of MIMO System

- MIMO increases capacity of communication
 - MIMO uses independent channel fading due to multipath propagation to increase capacity
 - No extra bandwidth is required
- MIMO gives reliable communication
 - Multiple independent samples of the same signal at the receiver give rise to diversity

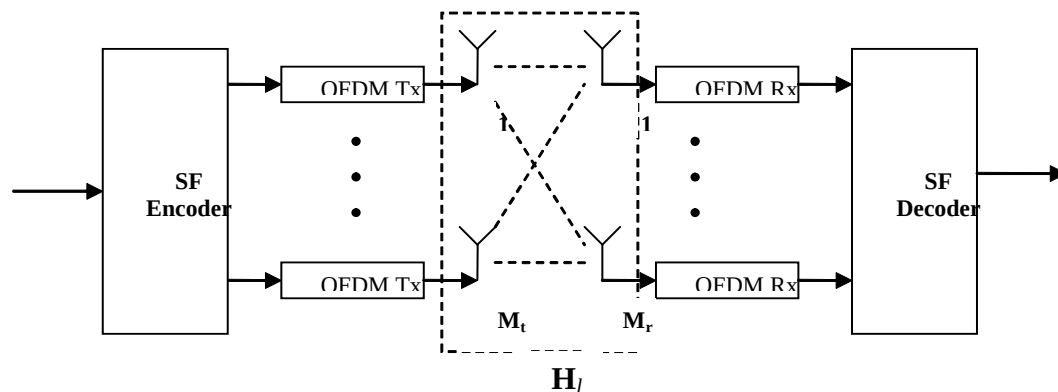
Multiple Antennas Coding

- Space-time coding (STC)
 - achieves the diversity gain $M_t M_r$
 - is effective in flat-fading channels only
- ST-OFDM
 - is used for broadband communication where MIMO channel exhibits FSK induced by multipath fading.
 - Its diversity order is $M_t M_r$, rather than $M_t M_r L$ (full).
- SF-OFDM
 - It encodes the information symbols onto different Tx (space) and OFDM subchannels (frequency bins).
 - It can achieve the potential full diversity $M_t M_r L$.

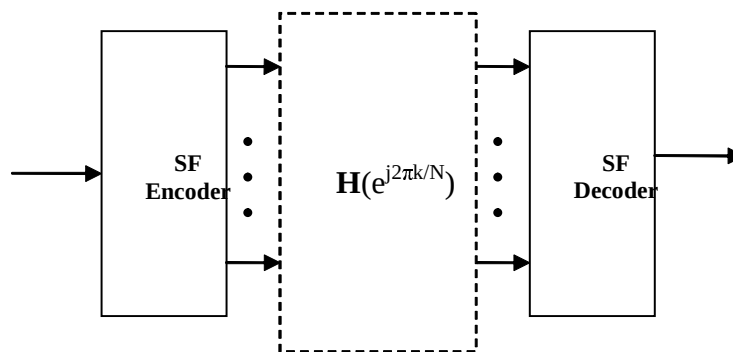
Advances in SFC Design

- Lee-Williams (2000), Gong-Letaief (2002), Hong-Hughes (2002) used STC directly as SFC, but without guarantees of full-diversity
- Bölcskei (2001) organized the columns of FFT matrix into full-diversity SFC, but the symbol rate is only $\frac{1}{M_t L}$
- Su-Safar-Olfat-Liu (2003) improved the rate of full-diversity SFC to $\frac{1}{L}$ by repetition mapping from STC
- Su-Safar-Liu (2004) achieved the full-rate (rate 1) and full-diversity SFC
- Our focus is on the design of high-rate ($R > 1$) full-diversity SFC.

SF-OFDM System Model



(a) Typical MIMO-OFDM system



(b) Equivalent MIMO system

- Assume a vector $\mathbf{S}$, of which the elements are from the constellation $\mathcal{A}$ (QPSK, or QAM), is encoded into $\mathbf{C}$ via

$$f : \mathbf{S} \longrightarrow \mathbf{C}$$

where

$$\mathbf{S} = [S_0 \quad S_1 \quad \cdots \quad S_{N_s}]^T,$$

$$\mathbf{C} = \begin{pmatrix} c_0^{(1)} & c_1^{(1)} & \cdots & c_{N-1}^{(1)} \\ c_0^{(2)} & c_1^{(2)} & \cdots & c_{N-1}^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ c_0^{(M_t)} & c_1^{(M_t)} & \cdots & c_{N-1}^{(M_t)} \end{pmatrix}.$$

Signal Model (cont.)

- At the i th receive antenna, after FFT the signal at the k th subchannel, denoted by $r_k^{(i)}$, is given by

$$r_k^{(i)} = \sum_{j=1}^{M_t} H_{i,j}(k) c_k^{(j)} + n_k^{(i)}$$

where $H_{i,j}(k) = \sum_{l=0}^{L-1} h_{i,j}(l) e^{-j2\pi k \tau_{i,j}(l)/T_s}$ and $c_k^{(j)}$ is the symbol transmitted from the k th subchannel at the j th Tx.

- The ML detection at the receiver is given by

$$\hat{\mathbf{C}} = \arg \min_{\mathbf{C}} \sum_{i=1}^{M_r} \sum_{k=0}^{N-1} \left\| r_k^{(i)} - \sum_{j=1}^{M_t} H_{i,j}(k) c_k^{(j)} \right\|^2$$

Pair-wise Error Probability

- Suppose that the ML detector decides in favor of $\mathbf{E}$ wrongly when $\mathbf{C}$ is in fact transmitted, the pairwise error probability can be upper bounded by [Bölcskei-Paulraj, 2000]

$$P(\mathbf{C} \rightarrow \mathbf{E}) \leq \left(\prod_{i=1}^m \lambda_i \right)^{-M_r} \left(\frac{E_s}{4\sigma_n^2} \right)^{-mM_r}$$

where m is the rank of $\mathbf{R}(\mathbf{C}, \mathbf{E})[\mathbf{R}(\mathbf{C}, \mathbf{E})]^\mathcal{H}$ and $\{\lambda_1, \dots, \lambda_m\}$ are the nonzero eigenvalues of $\mathbf{R}(\mathbf{C}, \mathbf{E})[\mathbf{R}(\mathbf{C}, \mathbf{E})]^\mathcal{H}$.

$$\mathbf{R}(\mathbf{C}, \mathbf{E}) = [\mathbf{D}_0(\mathbf{C} - \mathbf{E})^\mathcal{T}, \mathbf{D}_1(\mathbf{C} - \mathbf{E})^\mathcal{T}, \dots, \mathbf{D}_{L-1}(\mathbf{C} - \mathbf{E})^\mathcal{T}],$$

$$\text{and } \mathbf{D}_l = \text{diag}\{h(l)e^{-j2\pi k\tau(l)/T_s}\}_{k=0}^{N-1}.$$

- In the high-SNR region, the average error probability for communication over a fading channel usually behave as:

$$P \sim G_c \cdot \text{SNR}^{-G_d}$$

where G_c is referred to as the **coding advantage** and G_d is called the **diversity order**.

- Here, $G_c = (\prod_{i=1}^m \lambda_i)^{-M_r}$ and $G_d = m M_r$. The SF code design criteria can be summarized as
 - **(rank criteria)** Maximize the minimum rank m of $\mathbf{R}(\mathbf{C}, \mathbf{E})$
 - **(determinant criteria)** Maximize the minimum product, $\prod_{i=1}^m \lambda_i$ of $\mathbf{R}(\mathbf{C}, \mathbf{E})[\mathbf{R}(\mathbf{C}, \mathbf{E})]^\mathcal{H}$
- for all distinct pairs $\mathbf{C}$ and $\mathbf{E}$.

Full-Diversity SFC Design

- Assume $N \geq M_t L$, an SFC constructed by

$$f : \mathbf{S}^{N_s \times 1} \longrightarrow \mathbf{C}^{M_t \times N}$$

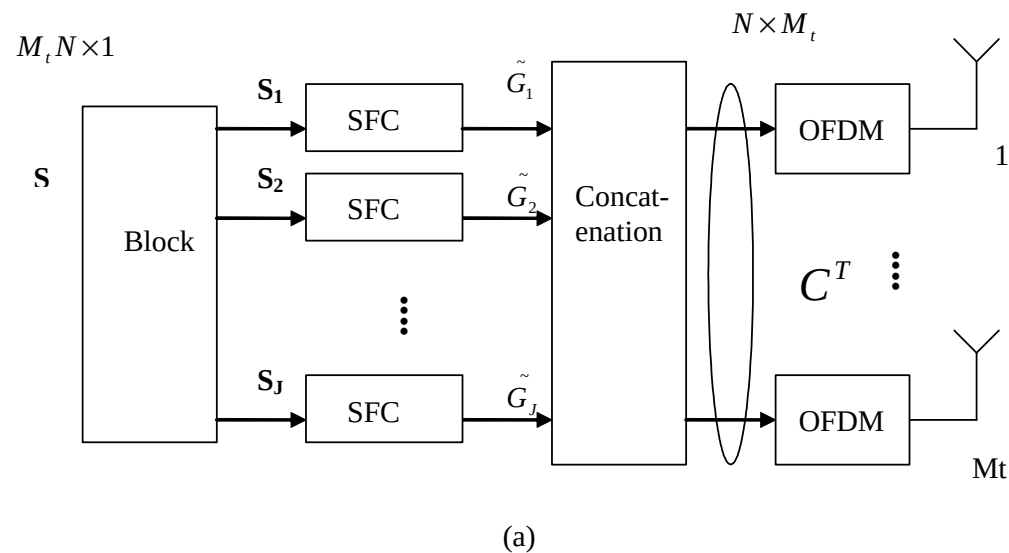
achieves full diversity if and only if $\mathbf{R}(\mathbf{C}, \mathbf{E})$ has full rank $M_t L$ for all $\mathbf{C} \neq \mathbf{E}$, where

$$\mathbf{R}(\mathbf{C}, \mathbf{E}) = [\mathbf{D}_0(\mathbf{C} - \mathbf{E})^T, \mathbf{D}_1(\mathbf{C} - \mathbf{E})^T, \dots, \mathbf{D}_{L-1}(\mathbf{C} - \mathbf{E})^T]$$

Equivalently,

- $(\mathbf{C} - \mathbf{E})^T$ has full rank M_t
- $\mathbf{B}_i$ are linearly independent with $\mathbf{B}_j$ for $i \neq j$
($i, j = 0, 1, \dots, L - 1$), where $\mathbf{B}_i = \mathbf{D}_i(\mathbf{C} - \mathbf{E})^T$

High-rate SFC Design

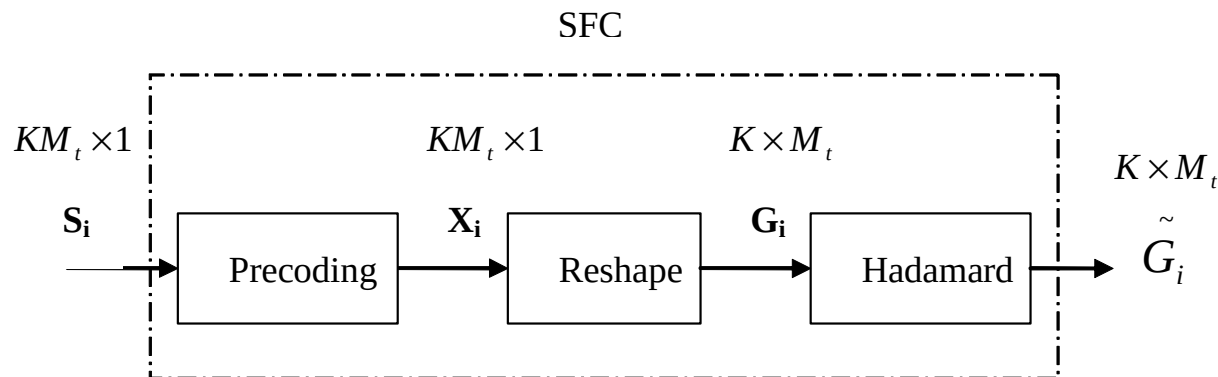


- S : Input block of size $N M_t \times 1$.
- S_i : Sub-block of size $K M_t \times 1$ for $i = 1, \dots, J$, where $J = N/K$ and $K = 2^{\lceil \log_2(M_t L) \rceil}$.
- $\tilde{G}_i$: Encoded matrix of size $K \times M_t$ for $i = 1, \dots, J$.
- $\mathbf{C}^T$: SFC given by $\mathbf{C}^T = [\tilde{\mathbf{G}}_1^T \quad \tilde{\mathbf{G}}_2^T \quad \dots \quad \tilde{\mathbf{G}}_J^T]^T$.

High-rate SFC Design (cont.)

- The rate of the proposed SFC is $R = \frac{N_s}{N} = M_t$
- Why do we divide the input block in that way?
 - It facilitates the low-complexity decoding at receiver;
 - J is always an integer since K is power of 2;
 - $K \geq M_t L$, so we move to pursue the full rank of $K \times M_t L$ matrix $\mathbf{R}(\tilde{\mathbf{G}}_i, \tilde{\mathbf{E}}_i)$ rather than $\mathbf{R}(\mathbf{C}, \mathbf{E})$
- Our focus next is to make at least one matrix $\tilde{\mathbf{G}}_i$, ($1 \leq i \leq J$) so that $\mathbf{R}(\tilde{\mathbf{G}}_i, \tilde{\mathbf{E}}_i)$ achieve full rank when $\mathbf{S}_i - \hat{\mathbf{S}}_i \neq \mathbf{0}$. Equivalently, full-diversity SFC is desired.

High-rate SFC Design (cont.)



(b)

- (precoding) $\mathbf{X}_i = \Theta \mathbf{S}_i$, where $\Theta = \text{vander}(\theta_1 \ \theta_2 \ \dots \ \theta_{KM_t})$ and $\theta_k = e^{j \frac{\pi(4k-3)}{2KM_t}}$
- reshaping $KM_t \times 1$ vector $\mathbf{X}_i$ into $K \times M_t$ matrix $\mathbf{G}_i$
- (Hadamard) $\tilde{\mathbf{G}}_i = \mathbf{G}_i \circ (\bar{\mathbf{H}}_{a \times M_t} \otimes \mathbf{1}_{b \times 1})$, where $\bar{\mathbf{H}}_{a \times M_t}$ is the first M_t columns of the $a \times a$ Hadamard matrix. $a = 2^{\lceil \log_2 M_t \rceil}$ and $b = \frac{K}{a}$.

Example of High-rate SFC

For $M_t = 2$, $L = 2$, and $N = 4$, the design of rate-2 SFC is shown as the following steps,

- get $K = 4$, $J = 1$, $a = 2$ and $b = 2$
- construct the 8×8 precoding matrix Θ
- precoding the data vector $\mathbf{S}^{8 \times 1}$ with Θ into $\mathbf{X} = [x_1 \ x_2 \ \cdots \ x_8]^T$
- after reshaping and Hadamard, we finally get

$$\mathbf{C}^T = \begin{pmatrix} x_1 & x_5 \\ x_2 & x_6 \\ x_3 & -x_7 \\ x_4 & -x_8 \end{pmatrix}$$

- An SFC denoted by $\mathbf{C}$ is a full-diversity if $\mathbf{R}(\mathbf{C}, \mathbf{E})$ has full rank, i.e.

$$(1) \quad \det |(\mathbf{R}(\mathbf{C}, \mathbf{E}))^{\mathcal{H}} \mathbf{R}(\mathbf{C}, \mathbf{E})| \neq 0$$

for all distinct pairs $\mathbf{C}$ and $\mathbf{E}$.

- By exhaustive search in space of $(\mathbf{C} - \mathbf{E})$, we can numerically prove the full rank of (1) for BPSK, QPSK and 4-PAM, ($M_t = 2$ and $L = 2$).

Dimension of search space for specific modulation

	BPSK	QPSK	4-PAM
$S - \tilde{S}$	3	9	7
$\mathbf{C} - \mathbf{E}$	3^8	9^8	7^8

Simulation Results

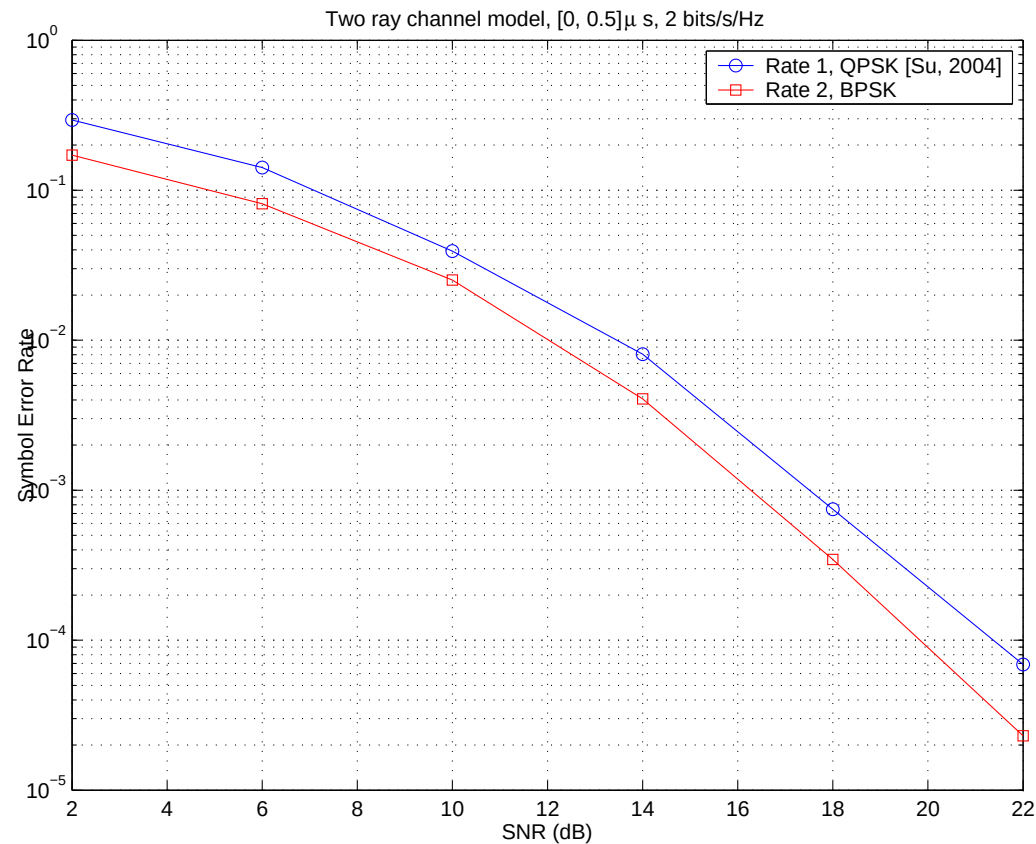


Figure 1: Symbol Error Rate, 2 bits/s/Hz, $\tau = [0, 0.5]\mu s$

Simulation Results (cont.)

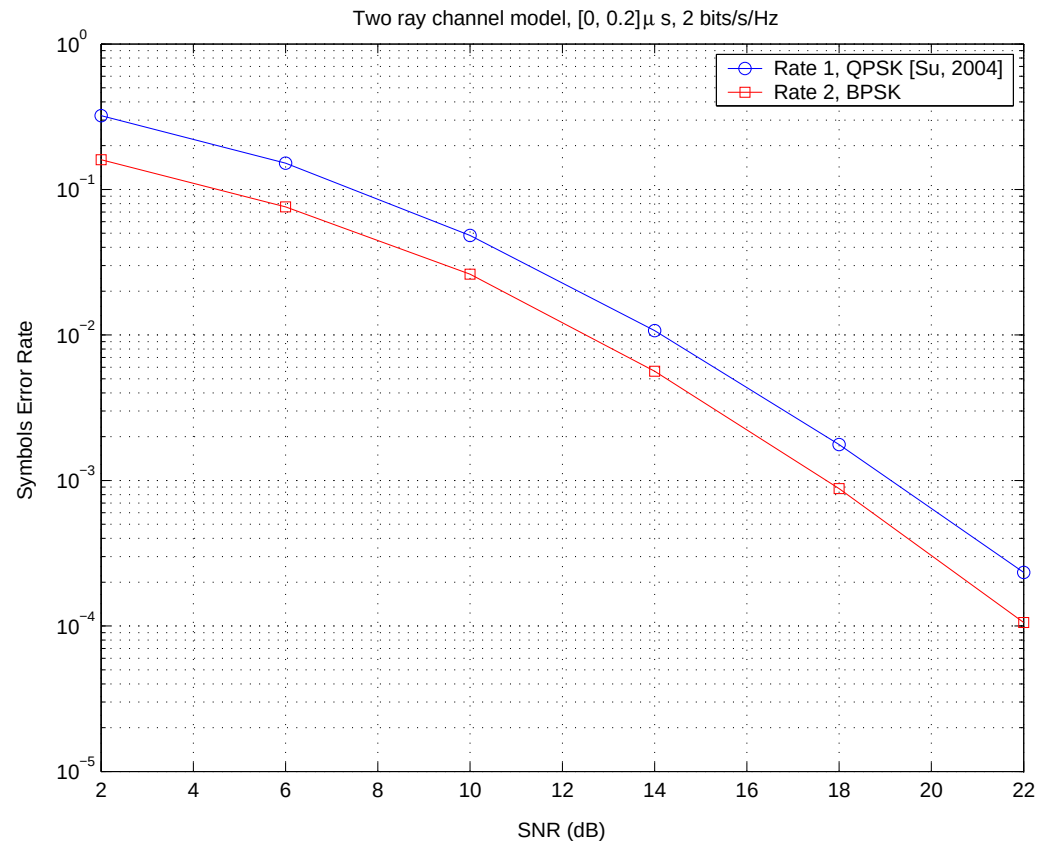


Figure 2: Symbol Error Rate, 2 bits/s/Hz, $\tau = [0, 0.2] \mu s$

Summary and Future Work

- Proposed a design of high-rate (rate M_t) SFC
- The proposed SFC was validated to achieve the full-diversity by numerical and simulation results
- General theory of high-rate full-diversity SFC design are needed (for any number of Tx and Rx, any channel length, and any constellations)
- Low-complexity decoding (sphere decoding)

Thank you!