

Full Diversity Achieving Space-Time Codes With Linear Receivers

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- 2 System Model and Problem Formulation
- 3 A Criterion to Achieve Full Diversity With ZF or MMSE Receiver
- 4 Overlapped Alamouti Codes
- 5 Simulations
- 6 Conclusion and Future Research

1 Introduction

- Motivation
- Known STBC Achieving Full Diversity With Linear Receivers
- Our Contributions

2 System Model and Problem Formulation

3 A Criterion to Achieve Full Diversity With ZF or MMSE Receiver

4 Overlapped Alamouti Codes

5 Simulations

6 Conclusion and Future Research

Motivation

- Multiple antenna (MIMO) technique has become a key element in the current and future high speed wireless communication standards/systems
 - IEEE 802.11n (WiFi), 802.16 d/e (WiMax), 3GPP, ...
- Space-time coding/modulation plays a key role in an MIMO system to achieve the capacity gain
 - It has been studied over 10 years
 - There have been many space-time codes in the literature
 - Is the problem solved??
 - In my opinion, NOT YET!

Motivation (Contd.)

- Most existing STBC are designed based on the ML decoding that
 - usually computationally complex (except for orthogonal type ones)
 - The prohibitively high decoding complexity may cause many existing space-time codes, such as non-vanishing determinant codes, not practical
 - may not have soft outputs
- For existing STBC such as linear lattice based STBC and BLAST, there are some simplified decoding algorithms, including
 - sphere decoding: achieve ML performance, but still complex and channel dependent
 - soft (and successive) interference cancellation: diversity may not be fully exploited $\rightarrow$ performance losses
- Linear receivers such as ZF and MMSE receivers
 - low computational complexity: linear on the number of information symbols
 - have soft outputs: good when concatenated with outer correcting codes

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- For existing STBC such as linear lattice based STBC and BLAST, there are some simplified decoding algorithms, including
 - sphere decoding: achieve ML performance, but still complex and channel dependent
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- Linear receivers such as ZF and MMSE receivers
 - low computational complexity: linear on the number of information symbols
 - have soft outputs: good when concatenated with outer correcting codes
- **BUT**, almost all the existing STBC do not have diversity gains with linear receivers

Motivation (Contd.)

- Criteria for ML receiver

J.-C. Guey, M. P. Fitz, M. R. Bell, and W.-Y. Kuo, 1999.

V. Tarokh, N. Seshadri, and A. R. Calderbank, 1998.

- full rank/diversity criterion
 - large diversity product (coding gain) criterion
- Among the above two, full diversity criterion is the first one to be satisfied since it governs the exponential in the pairwise error probability decay vs. SNR, i.e.,

$$P_s \leq \bar{c} \cdot (\text{SNR})^{-\text{diversity order}}$$

- **Our Goal:** design STBC that can achieve *full diversity* with linear receivers such as ZF and MMSE

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- full rank/diversity criterion (**more important**)
 - large diversity product (coding gain) criterion
- Among the above two, full diversity criterion is the first one to be satisfied since it governs the exponential in the pairwise error probability decay vs. SNR, i.e.,

$$P_s \leq \bar{c} \cdot (\text{SNR})^{-\text{diversity order}}$$

- **Our Goal:** design STBC that can achieve **full diversity** with linear receivers such as ZF and MMSE

Known STBC having full diversity with linear receivers

OSTBC & Toeplitz Codes

- OSTBC, full diversity with ML receiver:
 - single symbol ML decoding, so ML receiver = linear receiver
 - hence, achieve full diversity with ZF or MMSE receiver
 - however, low symbol rate

H. Wang and X.-G. Xia, "Upper bounds of rates of complex orthogonal space-time block codes," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2788-2796, Oct. 2003.

- $\leq 3/4$ for more than 2 transmit antennas
- conjecture: $(k+1)/(2k)$ for $2k$ or $2k-1$ transmit antennas

- Toeplitz codes (also, delay diversity codes)

- full diversity shown by Zhang-Liu-Wong

J.-K. Zhang, J. Liu, and K. M. Wong, "Linear Toeplitz space time block codes," in *Proc. IEEE Int. Symp. Inform. Theory (ISIT'05)*, Adelaide, Australia, Sept. 4-9, 2005.

J. Liu, J.-K. Zhang, and K. M. Wong, "Properties and applications of the Toeplitz space-time block codes in MISO transmission systems," *IEEE Trans. Inform. Theory*, submitted, Mar. 2006.

- their performance is worse than orthogonal codes

Known STBC having full diversity with linear receivers

Toeplitz Codes (Contd.)

- Toeplitz codes
 - the codeword matrix for M transmit antennas with L information symbols $s_1, s_2, \dots, s_L$ embedded is

$$\begin{pmatrix} s_1 & 0 & \cdots & 0 \\ s_2 & s_1 & \cdots & 0 \\ s_3 & s_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ s_L & s_{L-1} & \cdots & s_1 \\ 0 & s_L & \cdots & s_2 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & s_L \end{pmatrix}$$

that has size of $(L + M - 1) \times M$

- symbol rate

$$\frac{L}{L + M - 1} \rightarrow 1 \text{ as } L \rightarrow \infty$$

for a fixed M

Goals

- Propose a criterion for STBC to achieve full diversity with ZF or MMSE receiver
- Design a novel class of STBC called *overlapped Alamouti codes* that have the following properties:
 - achieve full diversity with ZF or MMSE receiver
 - close to orthogonal
 - symbol rates can approach 1 for any number of transmit antennas
 - significantly outperform Toeplitz codes for all numbers of transmit antennas
 - outperform OSTBC when the number of transmit antennas is above 4

1 Introduction

2 System Model and Problem Formulation

- STBC and MIMO Channel Models
- Equivalent Channel Models
- Problem Formulation

3 A Criterion to Achieve Full Diversity With ZF or MMSE Receiver

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5 Simulations

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Information symbol embedded STBC

- MIMO systems: M transmit and N receive antennas
- Constellation $\mathcal{C}$
- Input symbol sequence $\mathbf{s} = (s_1, s_2, \dots, s_L)^t \in \mathcal{C}^L$ with $s_i \in \mathcal{C}$
- Space-time block codeword matrix $X(\mathbf{s})$: size $T \times M$ with the L symbols s_i in $\mathbf{s}$ embedded
- The STBC $\mathcal{X}$ is the set composed of all the possible $X(\mathbf{s})$, i.e.,

$$\mathcal{X} = \{X(\mathbf{s}), \mathbf{s} = (s_1, s_2, \dots, s_L)^t \in \mathcal{C}^L\}$$

- Symbol rate: $\frac{L}{T}$

MIMO channel model

- Quasi-static Rayleigh flat fading
- Channel model

$$Y = \sqrt{\frac{\rho}{\mu}} X(\mathbf{s}) H + W$$

- $X(\mathbf{s})$: $T \times M$ codeword matrix of STBC
- $H = (h_{m,n})_{1 \leq m \leq M, 1 \leq n \leq N}$: $M \times N$ channel matrix, zero-mean, complex Gaussian distributed with a covariance matrix Σ , unit variance for $h_{m,n}$
- $W = (w_{t,n})_{1 \leq t \leq T, 1 \leq n \leq N}$: $T \times N$ noise matrix with i.i.d. $\mathcal{CN}(0, 1)$ distributed entries
- $Y = (y_{t,n})_{1 \leq t \leq T, 1 \leq n \leq N}$: $T \times N$ received signal matrix
- μ : normalization factor such that ρ is the SNR at each receive antenna

Reformulate the channel model into an equivalent one on which linear decodings are carried

- Extract $\mathbf{s}$ from $X(\mathbf{s})$ in order to decode $\mathbf{s}$ with a linear receiver
- Through some unitary operations, the original channel model $\mathbf{Y} = \sqrt{\frac{\rho}{\mu}} X(\mathbf{s}) H + \mathbf{W}$ is transformed into the following *equivalent channel model*

$$\mathbf{y} = \sqrt{\frac{\rho}{\mu}} \mathcal{H} \mathbf{s} + \mathbf{w}$$

- $\mathcal{H}$: *equivalent channel matrix* that has L columns is from H
- $\mathbf{y}$: *equivalent received signal vector*
- $\mathbf{w}$: *equivalent noise vector* with **i.i.d. $\mathcal{CN}(0, 1)$ distributed entries**
- The existence of an equivalent channel model depends on the structure of the STBC $\mathcal{X}$
 - **special form**: if each row of $X(\mathbf{s})$ has its transmitted symbols all in the form of $\pm s_l$ or, alternatively, all in the form of $\pm s_l^*$, then an equivalent channel model must exist

Examples – Alamouti code for single receive antenna

Example: Alamouti code for $N = 1$

$$X(\mathbf{s}) = \begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{pmatrix},$$

where $\mathbf{s} = (s_1, s_2)^t$. For $N = 1$, channel model

$$\begin{pmatrix} y_{1,1} \\ y_{2,1} \end{pmatrix} = \sqrt{\frac{\rho}{2}} \begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{pmatrix} \begin{pmatrix} h_{1,1} \\ h_{2,1} \end{pmatrix} + \begin{pmatrix} w_{1,1} \\ w_{2,1} \end{pmatrix}$$

is equivalent to

$$\begin{pmatrix} y_{1,1} \\ y_{2,1}^* \end{pmatrix} = \sqrt{\frac{\rho}{2}} \begin{pmatrix} h_{1,1} & h_{2,1} \\ h_{2,1}^* & -h_{1,1}^* \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \begin{pmatrix} w_{1,1} \\ w_{2,1}^* \end{pmatrix}$$

- The $X(\mathbf{s})$ above is of the **special form**, so reformulation is easy
- However, note that the existence of an equivalent channel model is not only restricted to such a special case

Reformulate the channel model into an equivalent one sufficient to derive the equivalent channel model for single receive antenna

- For a fixed $\mathcal{X}$, if an equivalent channel model holds for single receive antenna, i.e., $N = 1$, its corresponding version for multiple receive antennas can be easily derived by stacking the corresponding $\mathbf{y}$, $\mathcal{H}$ and $\mathbf{w}$ for each receive antenna

Example: Alamouti code for $N = 2$

When $N = 2$, the corresponding equivalent channel model is

$$\underbrace{\begin{pmatrix} y_{1,1} \\ y_{2,1}^* \\ y_{1,2} \\ y_{2,2}^* \end{pmatrix}}_{\mathbf{y}} = \sqrt{\frac{\rho}{2}} \underbrace{\begin{pmatrix} h_{1,1} & h_{2,1} \\ h_{2,1}^* & -h_{1,1}^* \\ h_{1,2} & h_{2,2} \\ h_{2,2}^* & -h_{1,2}^* \end{pmatrix}}_{\mathcal{H}} \underbrace{\begin{pmatrix} s_1 \\ s_2 \end{pmatrix}}_{\mathbf{s}} + \underbrace{\begin{pmatrix} w_{1,1} \\ w_{2,1}^* \\ w_{1,2} \\ w_{2,2}^* \end{pmatrix}}_{\mathbf{w}}$$

ZF and MMSE receivers

- From the equivalent channel model (if existed), we have

- ZF receiver

$$\hat{\mathbf{s}}_{ZF} = \sqrt{\frac{\mu}{\rho}} (\mathcal{H}^\dagger \mathcal{H})^{-1} \mathcal{H}^\dagger \mathbf{y}$$

- MMSE receiver

$$\hat{\mathbf{s}}_{MMSE} = \sqrt{\frac{\rho}{\mu}} (I_L + \frac{\rho}{\mu} \mathcal{H}^\dagger \mathcal{H})^{-1} \mathcal{H}^\dagger \mathbf{y}$$

- Well known that MMSE receiver is superior to ZF receiver
- The components of the $\hat{\mathbf{s}}_{ZF}$ or $\hat{\mathbf{s}}_{MMSE}$ can be treated as the soft outputs of the information symbols in $\mathbf{s}$
- **Question:** which property should the $\mathcal{H}$ have in order to achieve full diversity with ZF or MMSE receiver for the STBC $\mathcal{X}$?

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 - Verification of the Criterion
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A criterion (sufficient condition)

for STBC to achieve full diversity with ZF or MMSE receiver

Theorem

For PAM, PSK and square QAM constellations, the STBC $\mathcal{X}$ can achieve full diversity, i.e., the symbol error probability (SEP) $P_e(\hat{s}_l \rightarrow s_l)$ of s_l satisfies

$$P_e(\hat{s}_l \rightarrow s_l) \leq \bar{c} \cdot \rho^{-MN}, \quad l = 1, 2, \dots, L,$$

for some positive constant $\bar{c}$ that is independent of ρ , with ZF or MMSE receiver, if an equivalent channel matrix $\mathcal{H}$ exists and satisfies

$$\text{Criterion} \rightarrow \|\mathcal{H}\| \leq g\|H\| \text{ and } \det(\mathcal{H}^\dagger \mathcal{H}) \geq c\|H\|^{2L}$$

for any realization of the original channel matrix H , where g and c are positive constants independent of H , and L is the number of symbols encoded in each codeword matrix of $\mathcal{X}$, i.e., the length of $\mathbf{s}$.

Maximum symbol rates

if full diversity with ZF or MMSE receiver is desired (criterion is satisfied)

Corollary

The symbol rates of the STBC that satisfy the above criterion and thus achieve full diversity with ZF or MMSE receiver can not be above 1, i.e., they are upper bounded by 1.

- Alamouti code and Toeplitz codes are optimal and asymptotically optimal, respectively, in terms of their symbol rates
- We have shown that any complex OSTBC has an equivalent channel matrix that satisfies the criterion, i.e., achieves full diversity with ZF or MMSE receiver

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 - Examples of Codeword Matrices
 - Equivalent Channel Matrices
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Codeword matrix of overlapped Alamouti codes

- Define matrix $O(\mathbf{v}, p, q)$ of size $(p + q - 1) \times q$ as

$$O(\mathbf{v}, p, q) = \begin{pmatrix} v_1^* & 0 & \cdots & 0 \\ v_2^* & v_1 & \cdots & 0 \\ v_3^* & v_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ v_{p-1}^* & v_{p-2} & \cdots & v_1^* \\ v_p^* & v_{p-1} & \cdots & v_2^* \\ 0 & v_p & \cdots & v_3^* \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & v_p^* \end{pmatrix} \text{ and } \begin{pmatrix} v_1^* & 0 & \cdots & 0 & 0 \\ v_2^* & v_1 & \cdots & 0 & 0 \\ v_3^* & v_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ v_p^* & v_{p-1} & \cdots & v_1^* & 0 \\ 0 & v_p & \cdots & v_2^* & v_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & v_p^* & v_{p-1} \\ 0 & 0 & \cdots & 0 & v_p \end{pmatrix}$$

for odd and even q , respectively, where $\mathbf{v} = (v_1, v_2, \dots, v_p)^t$ is any vector of length p .

Codeword matrix of overlapped Alamouti codes (Contd.)

- Also, define matrix $E(\mathbf{v}, p, q)$ of size $(p + q - 1) \times q$ as

$$E(\mathbf{v}, p, q) = \begin{pmatrix} 0 & \cdots & 0 & v_1 \\ 0 & \cdots & -v_1^* & v_2 \\ 0 & \cdots & -v_2^* & v_3 \\ \vdots & \vdots & \vdots & \vdots \\ v_1 & \cdots & -v_{p-2}^* & v_{p-1} \\ v_2 & \cdots & -v_{p-1}^* & v_p \\ v_3 & \cdots & -v_p^* & 0 \\ \vdots & \vdots & \vdots & \vdots \\ v_p & \cdots & 0 & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & 0 & \cdots & 0 & v_1 \\ 0 & 0 & \cdots & -v_1^* & v_2 \\ 0 & 0 & \cdots & -v_2^* & v_3 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & v_1 & \cdots & -v_{p-1}^* & v_p \\ -v_1^* & v_2 & \cdots & -v_p^* & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -v_{p-1}^* & v_p & \cdots & 0 & 0 \\ -v_p^* & 0 & \cdots & 0 & 0 \end{pmatrix}$$

for odd and even q , respectively,

Codeword matrix of overlapped Alamouti codes (Contd.)

- With respect to different M and L , define

$$\mathbf{s}_o = \begin{cases} (s_1, 0, s_3, 0, \dots, 0, s_{L-1})^t, & M \text{ even}, L \text{ even}; \\ (s_1, 0, s_3, 0, \dots, 0, s_{L-1}, 0)^t, & M \text{ odd}, L \text{ even}; \\ (s_1, 0, s_3, 0, \dots, 0, s_L)^t, & L \text{ odd}, \end{cases}$$

which keeps all the components of $\mathbf{s}$ with odd indexes and

$$\mathbf{s}_e = \begin{cases} (s_2, 0, s_4, 0, \dots, 0, s_L)^t, & M \text{ even}, L \text{ even}; \\ (s_2, 0, s_4, 0, \dots, 0, s_{L-1}, 0, 0)^t, & M \text{ even}, L \text{ odd}; \\ (0, s_2, 0, s_4, 0, \dots, 0, s_L)^t, & M \text{ odd}, L \text{ even}; \\ (0, s_2, 0, s_4, 0, \dots, 0, s_{L-1}, 0)^t, & M \text{ odd}, L \text{ odd}, \end{cases}$$

which instead keeps all the components of $\mathbf{s}$ with even indexes.

- Note that $\mathbf{s}_o$ and $\mathbf{s}_e$ have length $L - 1$ when both M and L are even and length L otherwise

Codeword matrix of overlapped Alamouti codes (Contd.)

- Define the codeword matrix of the overlapped Alamouti code $\mathcal{O}_{M,L}$ for M transmit antennas with L information symbols embedded as

$$X(\mathbf{s}) = \begin{cases} O(\mathbf{s}_o, L, M) + E(\mathbf{s}_e, L, M), & M \text{ odd}; \\ O(\mathbf{s}_o^*, L, M) + E(\mathbf{s}_e, L, M), & M \text{ even, } L \text{ odd}; \\ O(\mathbf{s}_o^*, L-1, M) + E(\mathbf{s}_e, L-1, M), & M \text{ even, } L \text{ even.} \end{cases}$$

- Symbol rate for $\mathcal{O}_{M,L}$

$$\begin{cases} \frac{L}{L+M-2}, & M \text{ \& } L \text{ even}; \\ \frac{L}{L+M-1}, & \text{otherwise.} \end{cases}$$

- Symbol rate approaches 1 as $L \rightarrow \infty$ for a fixed M
- Some equivalent overlapped Alamouti codes exist. For example, substituting $\mathbf{s}_e$ by $-\mathbf{s}_e$ in the definition of $X(\mathbf{s})$ above

Construction examples

codeword matrices of $\mathcal{O}_{2,L}$ for even L

Codeword of $\mathcal{O}_{2,L}$

$$\begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \\ s_3 & s_4 \\ -s_4^* & s_3^* \\ \vdots & \vdots \\ s_{L-3} & s_{L-2} \\ -s_{L-2}^* & s_{L-3}^* \\ s_{L-1} & s_L \\ -s_L^* & s_{L-1}^* \end{pmatrix}$$

- $\frac{L}{2}$ concatenated Alamouti codes
- Full rate 1
- So, overlapped Alamouti codes cover the classical Alamouti code as a special case when $M = L = 2$

Construction examples

codeword matrices of $\mathcal{O}_{3,L}$ and $\mathcal{O}_{4,L}$ for even L

Codeword of $\mathcal{O}_{3,L}$

$$\begin{pmatrix} s_1^* & 0 & 0 \\ 0 & s_1 & s_2 \\ s_3^* & -s_2^* & s_1^* \\ s_2 & s_3 & s_4 \\ s_5^* & -s_4^* & s_3^* \\ s_4 & s_5 & s_6 \\ \vdots & \vdots & \vdots \\ s_{L-4} & s_{L-3} & s_{L-2} \\ s_{L-1}^* & -s_{L-2}^* & s_{L-3}^* \\ s_{L-2} & s_{L-1} & s_L \\ 0 & -s_L^* & s_{L-1}^* \\ s_L & 0 & 0 \end{pmatrix}$$

- Any symbol $\pm s_l$ or $\pm s_l^*$ on the middle column belongs to two Alamouti codes simultaneously
- This is why we use the term “overlapped” in the definition for the novel codes
- Each row has its transmitted symbols either all in the form of $\pm s_l$ or all in the form $\pm s_l^*$
 $\Rightarrow$ easily obtain equivalent channel model

Construction examples

codeword matrices of $\mathcal{O}_{3,L}$ and $\mathcal{O}_{4,L}$ for even L

Codeword of $\mathcal{O}_{3,L}$

$$\begin{pmatrix} s_1^* & 0 & 0 \\ 0 & s_1 & s_2 \\ s_3^* & -s_2^* & s_1^* \\ s_2 & s_3 & s_4 \\ s_5^* & -s_4^* & s_3^* \\ s_4 & s_5 & s_6 \\ \vdots & \vdots & \vdots \\ s_{L-4} & s_{L-3} & s_{L-2} \\ s_{L-1}^* & -s_{L-2}^* & s_{L-3}^* \\ s_{L-2} & s_{L-1} & s_L \\ 0 & -s_L^* & s_{L-1}^* \\ s_L & 0 & 0 \end{pmatrix}$$

Codeword of $\mathcal{O}_{4,L}$

$$\begin{pmatrix} s_1 & 0 & 0 & s_2 \\ 0 & s_1^* & -s_2^* & 0 \\ s_3 & s_2 & s_1 & s_4 \\ -s_2^* & s_3^* & -s_4^* & s_1^* \\ s_5 & s_4 & s_3 & s_6 \\ \vdots & \vdots & \vdots & \vdots \\ -s_{L-4}^* & s_{L-3}^* & -s_{L-2}^* & s_{L-5}^* \\ s_{L-1} & s_{L-2} & s_{L-3} & s_L \\ -s_{L-2}^* & s_{L-1}^* & -s_L^* & s_{L-3}^* \\ 0 & s_L & s_{L-1} & 0 \\ -s_L^* & 0 & 0 & s_{L-1}^* \end{pmatrix}$$

Equivalent channel matrices – Examples

$\mathcal{H}$ for $\mathcal{O}_{3,L}$ with even L when $N = 1$

$M = 3, N = 1, \text{ Even } L$

$$\mathcal{H} = \begin{pmatrix} h_{1,1}^* & 0 & \cdots & 0 & 0 \\ h_{2,1} & h_{3,1} & \cdots & 0 & 0 \\ h_{3,1}^* & -h_{2,1}^* & \cdots & 0 & 0 \\ 0 & h_{1,1} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & h_{1,1}^* & 0 \\ 0 & 0 & \cdots & h_{2,1} & h_{3,1} \\ 0 & 0 & \cdots & h_{3,1}^* & -h_{2,1}^* \\ 0 & 0 & \cdots & 0 & h_{1,1} \end{pmatrix}$$

- All odd columns are orthogonal to all even columns
- Equivalent channel model:

$$\mathbf{y} = \sqrt{\frac{\rho}{\mu}} \mathcal{H} \mathbf{s} + \mathbf{w}$$

Then, the symbols s_l with odd indices are independent of those with even indices

Equivalent channel matrices – Properties

- For the above defined overlapped Alamouti codes, we have shown

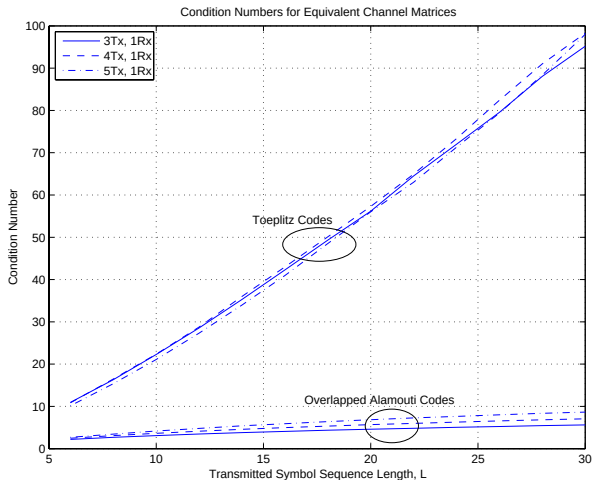
Theorem

For any given positive integers L , M and N , the equivalent channel matrix $\mathcal{H}$ for overlapped Alamouti code $\mathcal{O}_{M,L}$ satisfies $\|\mathcal{H}\| = \sqrt{L}\|H\|$ and $\det(\mathcal{H}^\dagger \mathcal{H}) \geq c\|H\|^{2L}$ for any channel matrix H , where c is a positive constant independent of H , but may depend on L , M and N .

- Therefore, for any L , M and N , the overlapped Alamouti code $\mathcal{O}_{M,L}$ achieves full diversity with ZF or MMSE receiver (will also be observed from the simulations)

Condition number comparison for equivalent channel matrices

Overlapped Alamouti codes vs. Toeplitz Codes



- The equivalent channel matrices for overlapped Alamouti codes are much less singular than those for Toeplitz codes

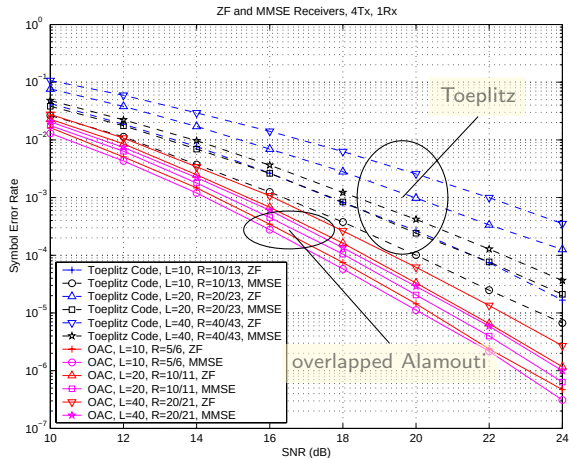
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Simulation setup

- The channel coefficients $h_{m,n}$ for different transmit and receive antenna pair are i.i.d. $\mathcal{CN}(0,1)$ distributed
- Gray mapping for all the constellations

Overlapped Alamouti codes vs. Toeplitz codes

$$M = 4, N = 1$$



- 4-QAM
- The performance loss for overlapped Alamouti codes as L increases is much smaller than that for Toeplitz codes
- The performance improvement of MMSE receiver over ZF receiver for overlapped Alamouti codes is not as significant as that for Toeplitz codes

Overlapped Alamouti codes vs. Juxtaposed Alamouti codes

$$M = 4, N = 2$$

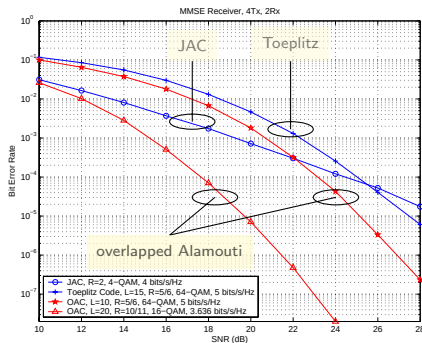
- Juxtaposed Alamouti codes (JAC) proposed in 3GPP for 4 transmit antennas have the codeword matrix in the form of

$$X(\mathbf{s}) = \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \end{pmatrix}.$$

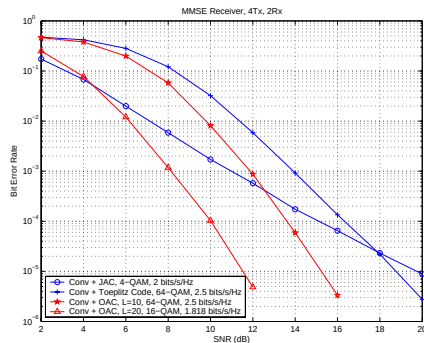
- Symbol rate 2 for JAC
- JAC has diversity 2 for ML receiver, so not full diversity achieving for linear receivers

Overlapped Alamouti codes vs. juxtaposed Alamouti codes (Contd.)

$M = 4, N = 2$



(a). without outer convolutional codes



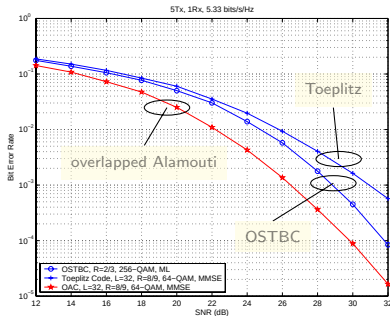
(b). with outer convolutional codes

- Overlapped Alamouti code has significant diversity gains over JAC
- The relative trend among the performance curves keeps the same when there is an outer convolutional code

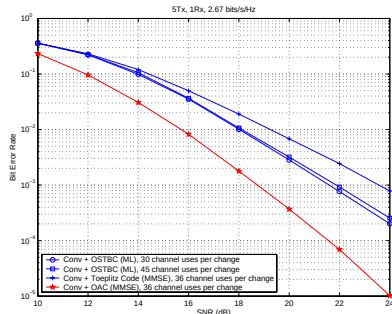
Overlapped Alamouti codes vs. OSTBC

$M = 5, N = 1$

- Overlapped Alamouti codes can have higher rates than OSTBC for $M > 2$, which may lead to performance gains



(a). without outer convolutional codes

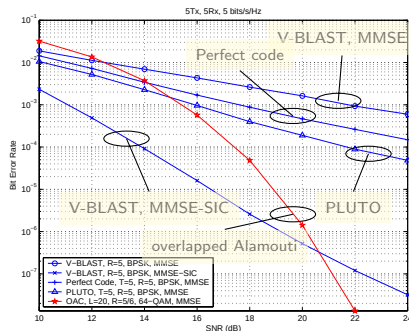


(b). with outer convolutional codes

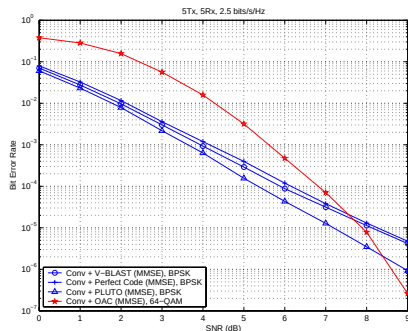
- OSTBC has rate $2/3$ (the highest known rate for $M = 5$) and block length 15
- Similar performance improvements for $M > 5$, where overlapped Alamouti codes have shorter block lengths than OSTBC

Overlapped Alamouti codes vs. full-rate STBC

$M = 5, N = 5$



(a). without outer convolutional codes



(b). with outer convolutional codes

- The PLUTO codes are designed in terms of minimizing BER for MMSE receiver at a very high symbol rate
- The overlapped Alamouti code has significant diversity gains over others
- The overlapped Alamouti code even outperforms V-BLAST with MMSE-SIC (ordered) receiver

- 1 Introduction
- 2 System Model and Problem Formulation
- 3 A Criterion to Achieve Full Diversity With ZF or MMSE Receiver
- 4 Overlapped Alamouti Codes
- 5 Simulations
- 6 Conclusion and Future Research**

Summary

- Proposed a criterion (sufficient condition) for STBC to achieve full diversity with ZF or MMSE receiver in an MIMO system
- Systematically constructed a novel family of STBC named overlapped Alamouti codes that satisfy the proposed criterion and thus achieve full diversity with ZF or MMSE receiver
- Provided simulations to compare the performances of overlapped Alamouti codes with other codes

Future Research

- By thinking back over the computer modem designs in the 90's, there were many standards (for 9.6 kbs/s; 14.4 kbs/s; 19.2 kbs/s; 28.8 kbs/s; 33.6 kbs/s; 56 kbs/s) due to the research progresses in trellis coded modulation (TCM) and the equalization techniques
- We are now building wireless modems and it is just the beginning (WiFi; WiMax etc) and many more standards are coming
- Similar to before, coding and modulation (space-time coding) will also play a key role for generating the future standards
- For all coding/modulation schemes, the complexity and performance tradeoff is always a critical issue
- Space-time codes for low complexity receivers are practically important
 - Shorter space-time code designs with linear receivers
 - Space-time code designs for other low complexity receivers, such as MMSE-SIC (successive interference cancellation) receivers
 - Space-time codes for cooperative systems with low complexity receivers

Papers to Read

- Y. Shang and X.-G. Xia, [Space-Time Block Codes Achieving Full Diversity with Linear Receivers](#), *IEEE Trans. on Information Theory*, Oct. 2008.
- X. Guo and X.-G. Xia, [On Full Diversity Space-Time Block Codes with Partial Interference Cancellation Group Decoding](#), *IEEE Trans. on Information Theory*, Oct. 2009.
- X. Guo and X.-G. Xia, [Correction to ``On Full Diversity Space-Time Block Codes with Partial Interference Cancellation Group Decoding''](#), <http://www.ee.udel.edu/~xxia/Pub.html>, also to appear *IEEE Trans. on Information Theory*, July 2010.