



Mean-Square-Error Equalizer (MSE):

Define the output of the Channel

$$\underline{r}_k = (r_k, r_{k-1}, \dots, r_{k-N+1})^T,$$

the output of the equalizer can be written as

$$\tilde{x}_k = \underline{c}^T \underline{r}_k = \underline{r}_k^T \underline{c}.$$

An MSE equalizer adjusts the tap coefficient to minimize the MSE

$$J \triangleq E|\kappa_{k-d} - \tilde{x}_k|^2$$

$$= E\{\underline{c}^T \underline{r}_k \underline{r}_k^H \underline{c}^* - 2\operatorname{Re}\{\underline{r}_k^H \underline{c}^* \kappa_{k-d}\} + |\kappa_{k-d}|^2\}$$

$$\nabla_{\underline{c}} J = 0 \Rightarrow \underline{c}_{\text{opt}} = (\underline{M}_r)^{-1} \underline{r}_x^*,$$

where $\underline{M}_r = E[\underline{r}_k \underline{r}_k^H]$

$$\underline{r}_x = E[\underline{r}_k \kappa_{k-d}]$$

$$[J = \underline{c}^T \underline{M}_r \underline{c}^* - 2\operatorname{Re}\{\underline{r}_x^H \underline{c}^*\} + 1, \text{ Assuming } E|\kappa_{k-d}|^2 = 1,$$

$$\nabla_{\underline{c}} J = \left(\frac{\partial J}{\partial c_0}, \dots, \frac{\partial J}{\partial c_M} \right) = 2\underline{c}^T \underline{M}_r - 2\underline{r}_x^H]$$

$$\Rightarrow J_{\min} = \underline{c}_{\text{opt}}^T \underline{M}_r \underline{c}_{\text{opt}}^* - 2\operatorname{Re}\{\underline{r}_x^H \underline{c}_{\text{opt}}^*\} + 1$$

$$= 1 - \underline{r}_x^H \underline{M}_r^{-1} \underline{r}_x = \|\mathbf{g} - \hat{\mathbf{g}}\|^2 + N_b \|\underline{c}\|^2.$$

$$\left(\begin{array}{l} \text{and } \underline{M}_r = E[\underline{r}_k \underline{r}_k^H] \\ \underline{I}_x = E[\underline{x}_{n-d} \underline{x}_n^H] \end{array} \right)$$



Taking the z -transform at both sides:

$$C(z) (G(z) G^*(1/z^*) + N_0) = G^*(1/z^*)$$

$$\Rightarrow C(z) = \frac{G^*(1/z^*)}{G(z) G^*(1/z^*) + N_0}$$

The equivalent MSE equaliser including the noise whitening filter $1/G^*(1/z^*)$ is

$$C'(z) = \frac{1}{G(z) G^*(1/z^*) + N_0} = \frac{1}{F(z) + N_0}$$

(Note the $C'(z)$ for zF is $\frac{1}{F(z)}$)

$\Rightarrow zF$ & MSE are the same when there is no noise.

The MMSE in this case is

$$\begin{aligned} J_{\min} &= 1 - \frac{1}{T} \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \frac{F_{\Sigma}(f)}{F_{\Sigma}(f) + N_0} df \\ &= T \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \frac{N_0}{F_{\Sigma}(f) + N_0} df \end{aligned}$$

where $F_{\Sigma}(f)$ is the folded spectrum:

$$F_{\Sigma}(f) = F_s \sum_k F((\omega + 2\pi k) F_s) \quad \text{with } F_s = \frac{1}{T_s}.$$

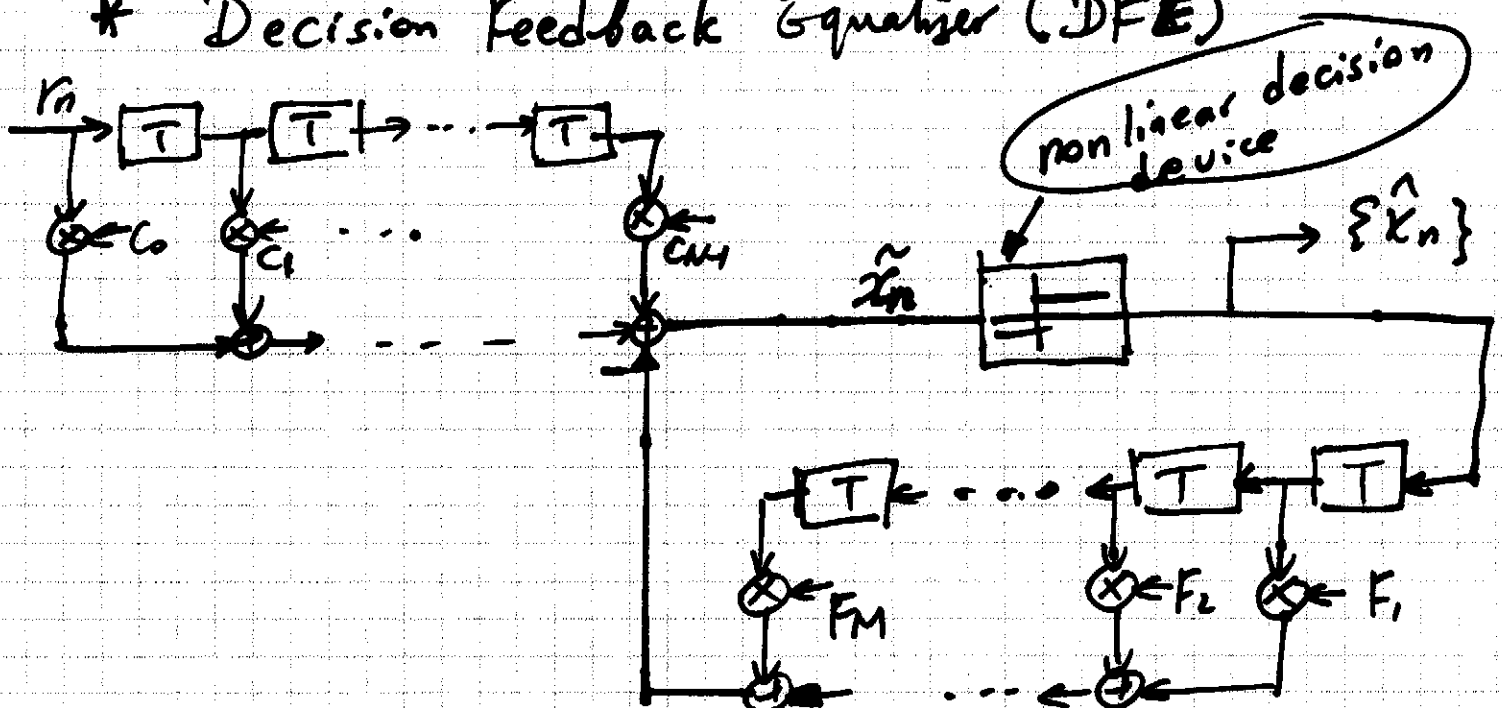


Another useful measure for the effectiveness of linear equalization techniques is the signal-noise-plus-interference ratio (SNIR):

$$SNIR = \frac{|g_d|^2}{\sum_{\substack{j=0 \\ j \neq d}}^{M-1} |g_j|^2 + N_0 \sum_{j=0}^{M-1} |g_j|^2}$$

All linear equalizers may enhance the noise when the channel (ISI) $g(z)$ has spectral null, i.e., $G(e^{j\omega}) \approx 0$ for some frequency ω_0 . Next we introduce a non-linear equalizer.

* Decision Feedback Equalizer (DFE)



It is used in Modem designs

→ TCM + DFE

80-90%



DFE includes two parts:

- ① a feed forward section → Identical to the linear forward equalizer to reduce ISI
- ② a feedback section → estimate the ISI contributed by the symbols

It is nonlinear due to the nonlinear decision device

The output of the DFE is

$$\hat{x}_n = \sum_{i=0}^{N-1} c_{N-1-i} r_{n-i} - \sum_{i=1}^M F_i \hat{x}_{n-i}$$

↑
feed forward
↑
feedback
← previously detected symbols

Note that $\underline{g} = \underline{c} * \underline{g}$ (equalized channel)

$$\begin{aligned} \tilde{x}_n &= \sum_{i=-L}^{N-1} g_{N-1-i} x_{n+i} - \sum_{i=1}^M F_i \tilde{x}_{n-i} + n_n \\ &= x_n g_{N-1} + \sum_{i=1}^M (x_{n-i} - \tilde{x}_{n-i}) g_{N-1+i} + \sum_{i=1}^M (g_{N-1+i} - F_i) \tilde{x}_{n-i} \\ &\quad + \sum_{i=1}^{N-1} g_{N-1+i} x_{n+i} + \sum_{i=M+1}^L g_{N-1+i} x_{n-i} + n_n \end{aligned}$$

(due to quantized noise)

If we choose $F_i = g_{N-1+i} = \underline{c}^T \underline{g}_{N-1+i}$, $i=1, 2, \dots, M$, and correct decisions are made, i.e., $x_{n-i} - \tilde{x}_{n-i} = 0$, then

$$\hat{x}_n = x_n g_{N-1} + \sum_{i=1}^{N-1} g_{N-1+i} x_{n+i} + \sum_{i=M+1}^L g_{N-1+i} x_{n-i} + n_n$$

↑
residue for the forward filters
↑
residue for the backward filters. (feedback)



Equalizer Tap Solution: (to find $\underline{\epsilon}$ and $\underline{F}$)

Let $\underline{\epsilon} = (\epsilon_0, \epsilon_1, \dots, \epsilon_{N-1})^T$

$\underline{r}_n = (r_{n+N-1}, r_{n+N-2}, \dots, r_n)^T$

$\underline{\hat{x}}_n = (\hat{x}_{n-1}, \hat{x}_{n-2}, \dots, \hat{x}_{n-M})^T$

$\underline{F} = (F_1, F_2, \dots, F_M)^T$

and $\underline{\tilde{\epsilon}} = (\underline{\epsilon}^T, \underline{F}^T)^T$

$\underline{\tilde{r}}_n = (\underline{r}_n^T, \underline{\hat{x}}_n^T)^T$

Then, the MSE can be expressed as

$$J = E |x_n - \hat{x}_n|^2$$

$$= E \{ \underline{\tilde{\epsilon}}^H \underline{\tilde{r}}_n \underline{\tilde{r}}_n^H \underline{\tilde{\epsilon}} - 2 \text{Re} \{ \underline{\tilde{r}}_n^H \underline{\tilde{\epsilon}}^* x_n \} + |x_n|^2 \}$$

Let $\underline{\tilde{M}}_r \triangleq E \{ \underline{\tilde{r}}_n \underline{\tilde{r}}_n^H \}$

$\underline{\tilde{r}}_x^H = E \{ \underline{\tilde{r}}_n^H x_n \}$

$$\Rightarrow \underline{\tilde{\epsilon}}_{\text{op}} = (\underline{\tilde{M}}_r)^{-1} \underline{\tilde{r}}_x^H$$

$\underline{\tilde{\epsilon}}$ includes both $\underline{\epsilon}$ and $\underline{F}$

Adaptive solution:

$$C_j^{n+1} = C_j^n + \alpha \epsilon_n \hat{r}_{n+j}^*, \quad j=0, 1, \dots, N-1$$

$$F_j^{n+1} = F_j^n + \alpha \epsilon_n \hat{x}_{n-j}^*, \quad j=1, 2, \dots, M$$

α, ϵ_k are the same as before.



Performance of the DFE

J. Salz '1973 proved:

$$J_{min} = \exp \left\{ T \int_{-1/4T}^{1/4T} \ln \left[\frac{N_0}{|H e^{j\omega T}|^2 + N_0} \right] df \right\}$$

* Sequence Estimation

$$r_n = \sum_{i=0}^L g_i x_{n-i} + \eta_n$$

This ISZ is also a finite state machine.
It has a trellis $\Rightarrow$ you may employ

Viterbi algorithm to estimate $\{x_n\}$

$\rightarrow$ called maximum-likelihood sequence estimation (MLSE). $\rightarrow$ optimal

but complex since L may be large.

* Comparisons of Equalizers

Three channels:

GA 11 taps typical data-quality twisted copper pair telephone channel

(0, 0.0485 + j0.0194, 0.0573 + j0.0253, 0.0786 + j0.0282, 0.0874 + j0.0447, 0.9222 + j0.3031, 0.1427 + j0.0349, 0.0835 + j0.0157, 0.0621 + j0.0078, 0.0359 + j0.0049, 0.0284 + j0.0019),

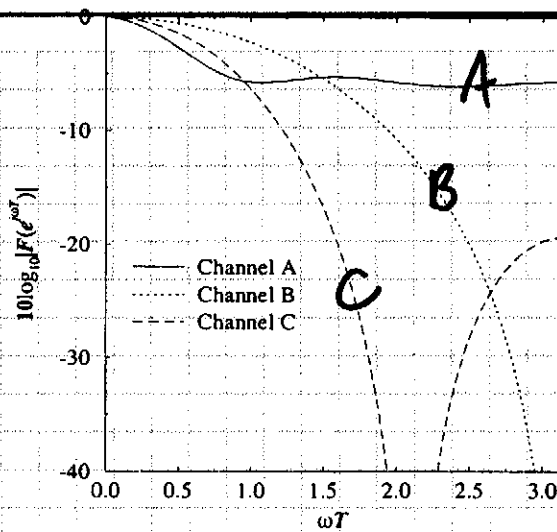


Figure 7.10. Amplitude spectrum for several static ISI channels.

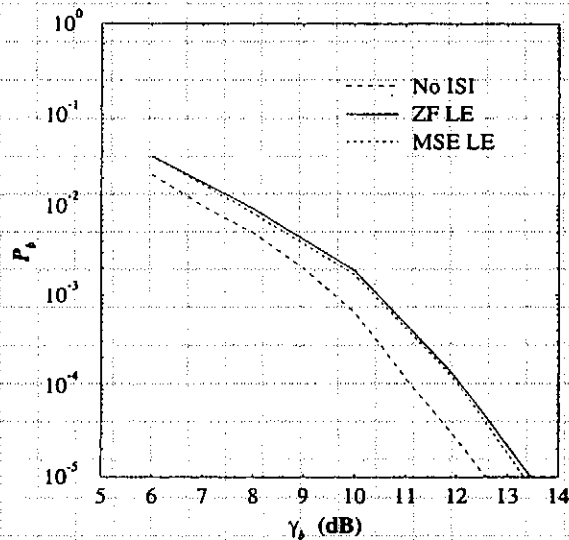


Figure 7.11. Performance of 21-tap ZF and MSE linear equalizers for 4-PSK signaling Channel A in Fig. 7.10.

T -spaced and have $(L + 1)$ -taps, as shown in Fig. 7.5. From Fig. 7.5, it can be seen that the channel has a finite number of states. If the size of the signal

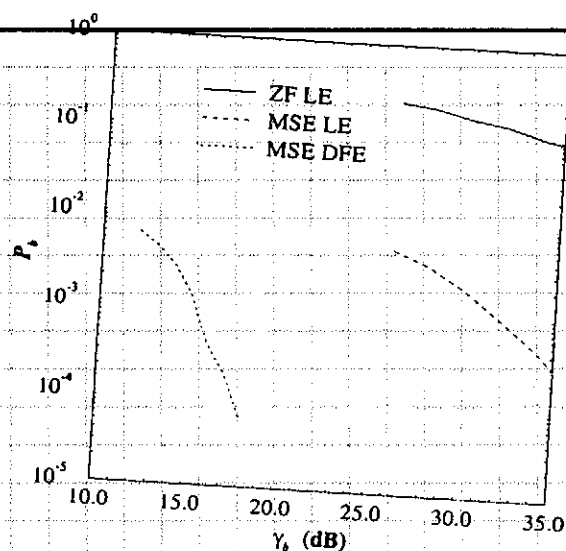


Figure 7.12. Performance of the 21-tap ZF and MSE linear equalizers and a 21-tap DFE equalizers for 4-PSK signaling on Channel B in Fig. 7.10.

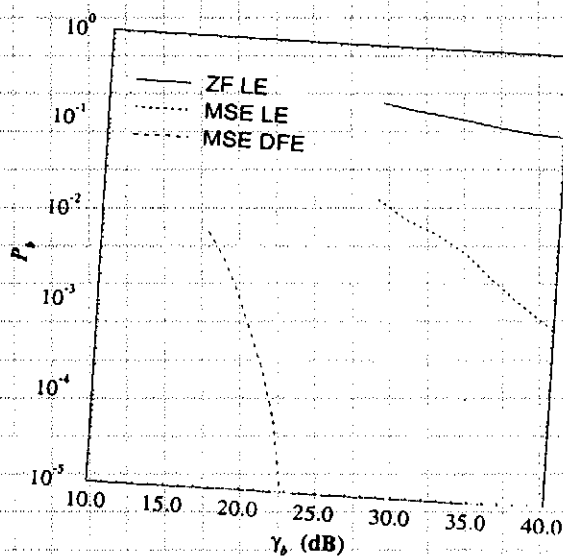


Figure 7.13. Performance of the 21-tap ZF and MSE linear equalizers and a 21-tap DFE equalizers for 4-PSK signaling on Channel C in Fig. 7.10.



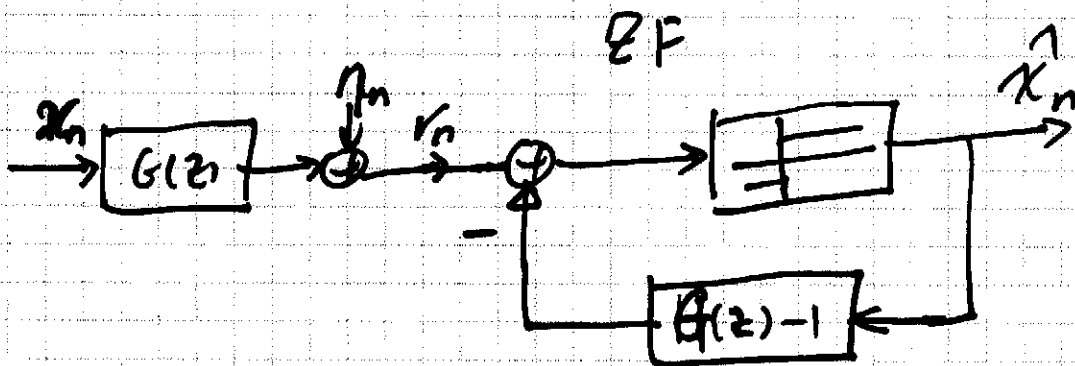
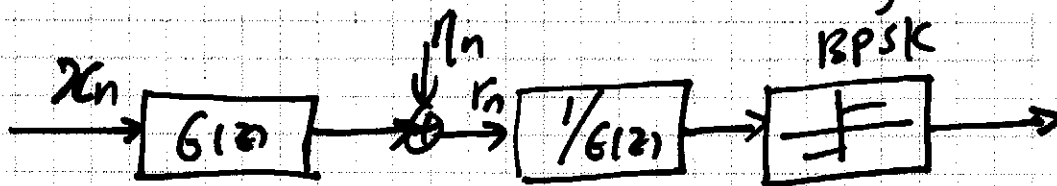
$$\underline{g}_B = (0.407, 0.815, 0.407)$$

$$\underline{g}_C = (0.227, 0.460, 0.688, 0.460, 0.227)$$

Channels B and C have severe ISI.

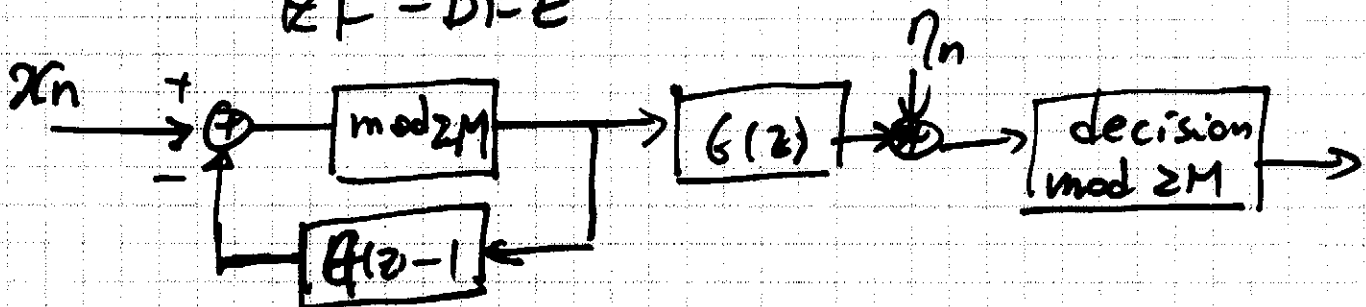
Channel A has less frequency-selective.

Tomlinson-Harashina Precoding (TH Precoding)



ZF decision-feedback equalizer

$$ZF - D = E$$



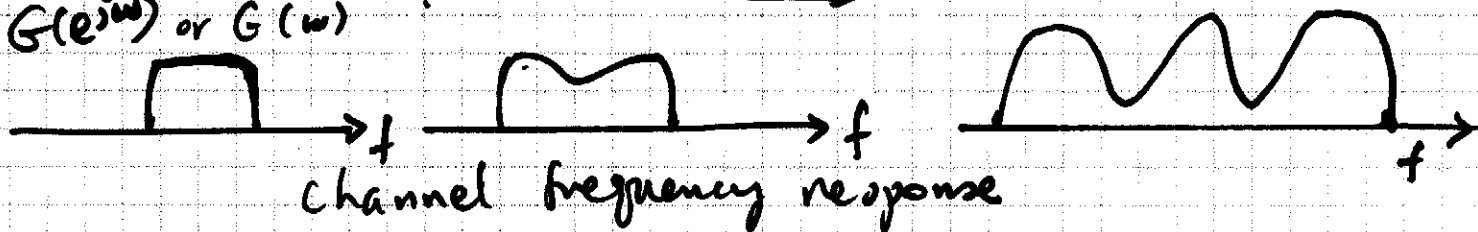
TH precoding

Assume x_n is 2M-PAM

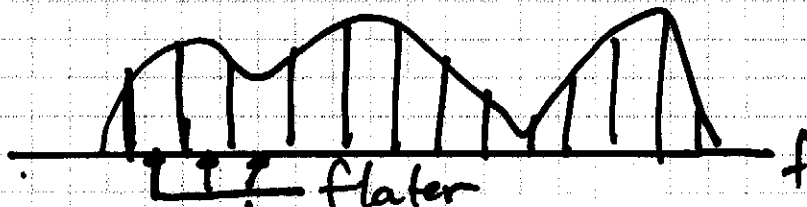


§5. Orthogonal Frequency Division Multiplexing (OFDM)

When signal bandwidth (data rate) becomes large,
Channel becomes less ideal become wider
 $G(e^{j\omega})$ or $G(\omega)$



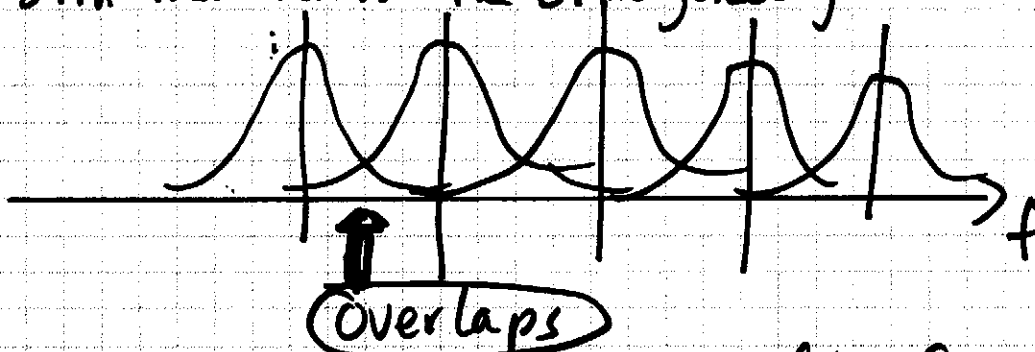
main idea is to split a wider band
Channel into smaller band channels and each



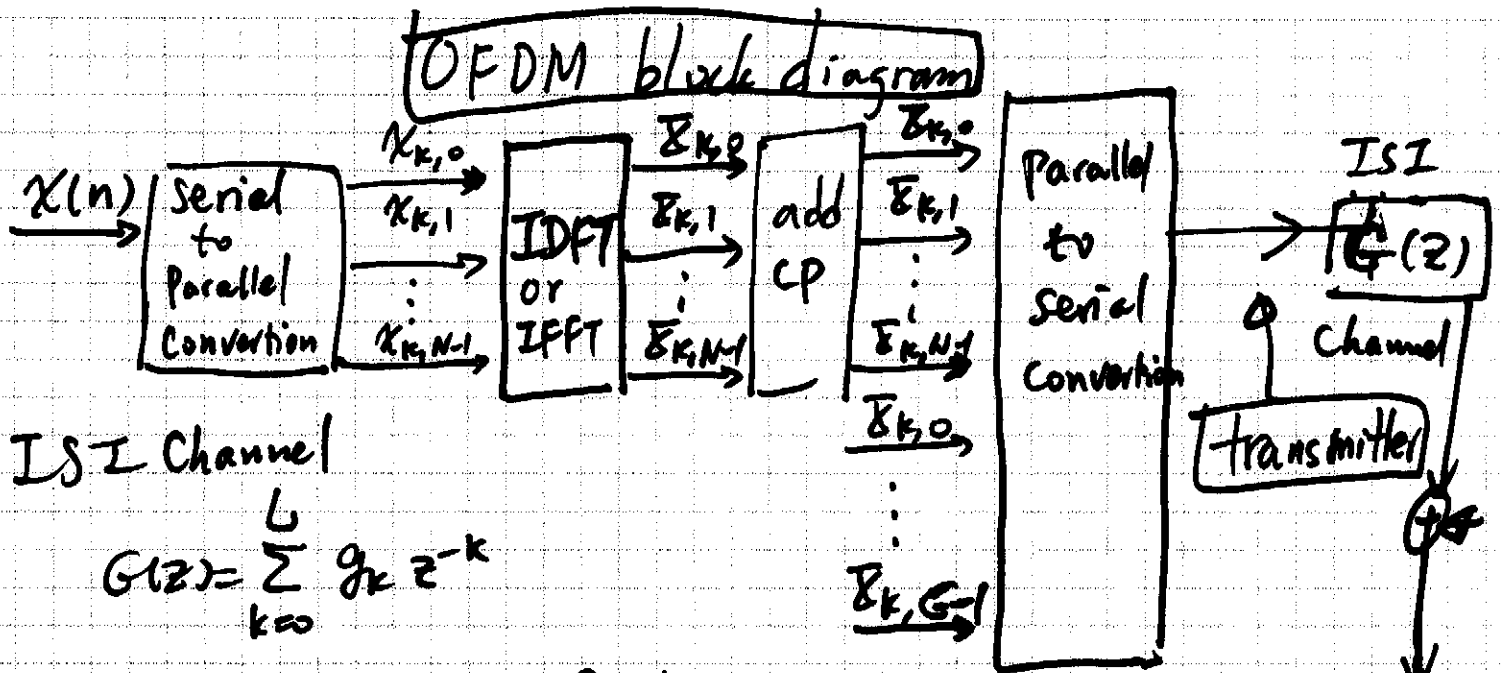
smaller subchannels are closer idea (flat).

If all these subchannels are well operated, it
is not bandwidth efficient

OFDM allows overlaps between these subchannels
and still maintains the orthogonality



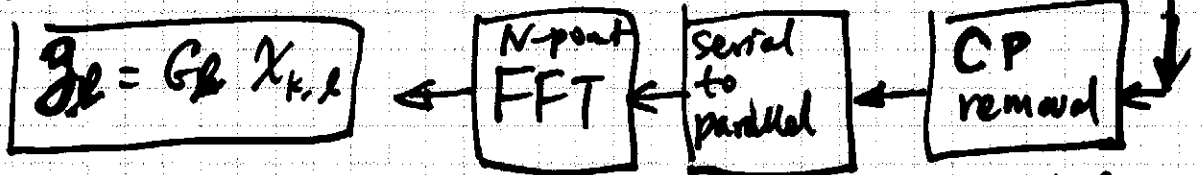
The trick is to add cyclic prefix (CP).



$G \geq L$
 CP length (Guard interval length)

The l th subcarrier

ISI free



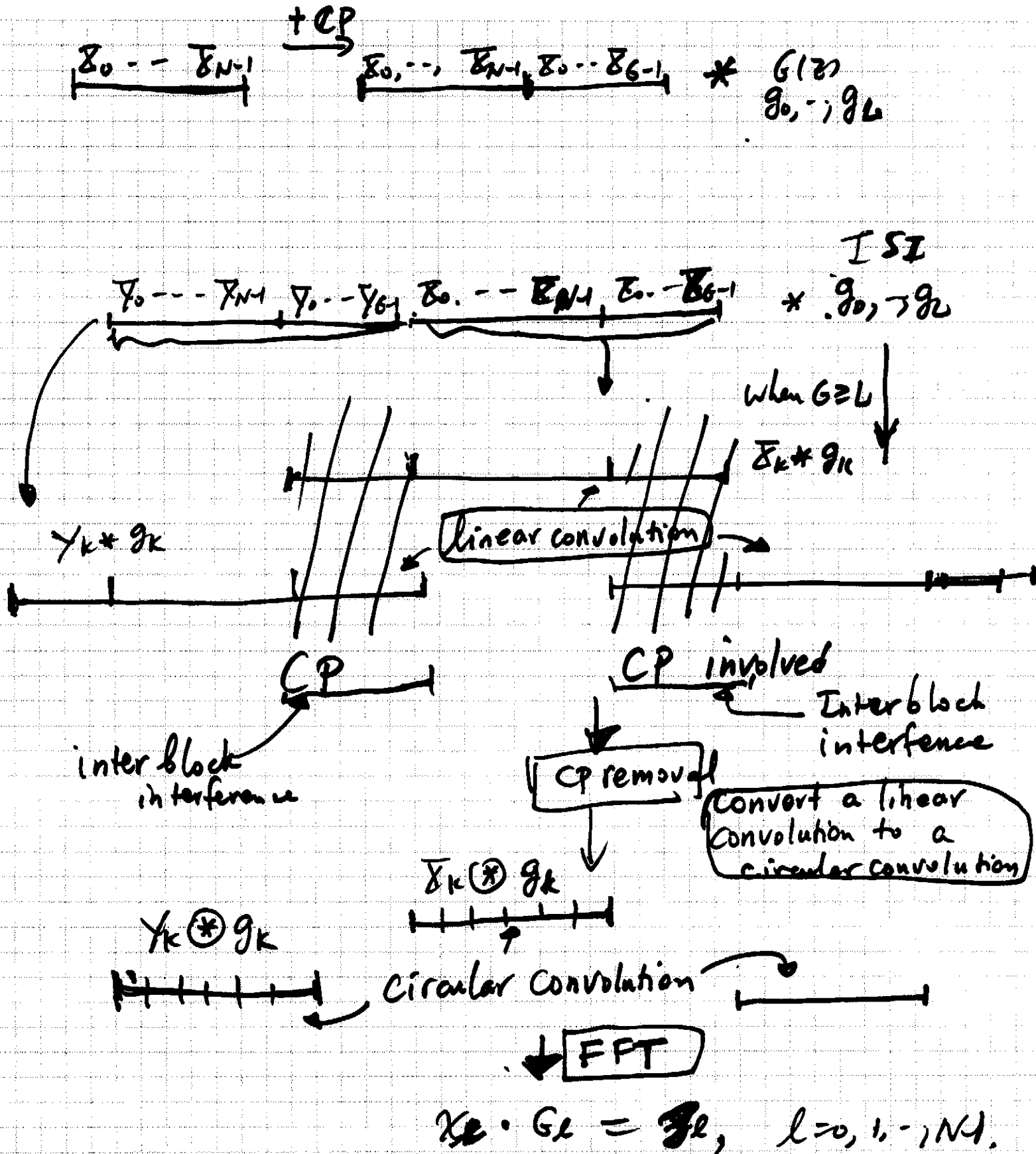
$l=0, \dots, N-1$

N is the # of subcarriers

$$G_l = G(\omega_l) = G\left(\frac{l2\pi}{N}\right)$$

Every $N+G$ symbols are a block and delete the first (or the last) G symbols

An ISI Channel $G(z)$ is converted to N parallel ISI-free subchannels $G_l, l=0, 1, \dots, N$.





* Peak to Average Power Ratio (PAPR)

Let us say that the original data is QPSK.
Without using OFDM (no IDFT is used before transmission), the data has the same magnitude, i.e., the power is constant. After the N -point IDFT

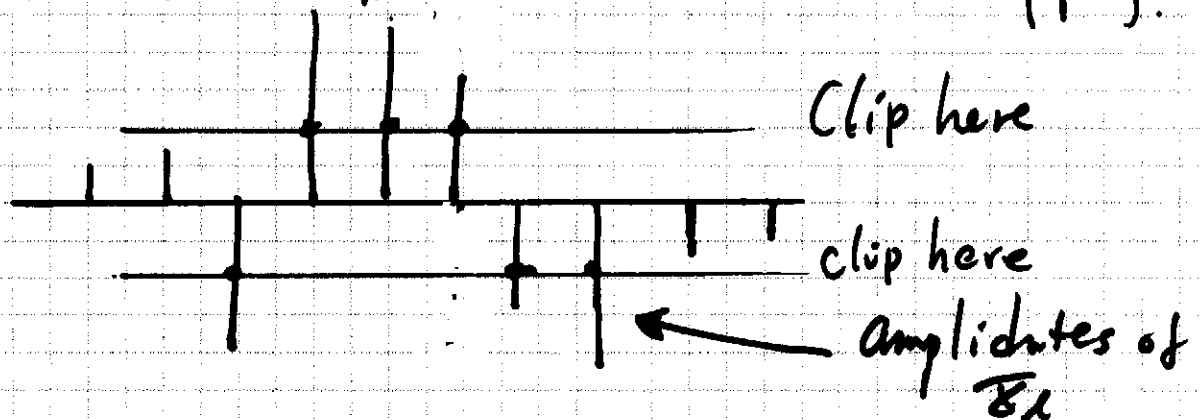
$$\frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \underset{\substack{\uparrow \\ \text{QPSK}}}{X_k} e^{\frac{j2\pi kl}{N}} = X_l$$

When all X_k are the same, $|X_l|^2 = N$,
the average power of X_l is $E|X_l|^2 = |X_k|^2 = 1$ (normalized)

$$\text{PAPR} = \frac{N}{1} = N \rightarrow \text{high}$$

There are many methods to reduce PAPR.

One of the most effective methods is clipping.





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Vector OFDM Single Transmit Antenna Systems

Xiang-Gen Xia

Dept of ECE

University of Delaware

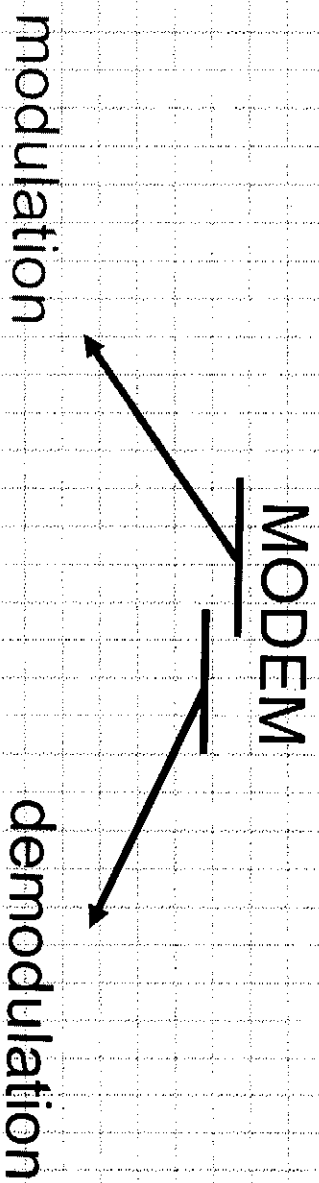
X.-G. Xia, “Precoded and Vector OFDM Robust to Channel Spectral Nulls and with Reduced Cyclic Prefix Length in Single Transmit Antenna Systems,” *IEEE Trans. on Communications*, Aug. 2001.





Outline

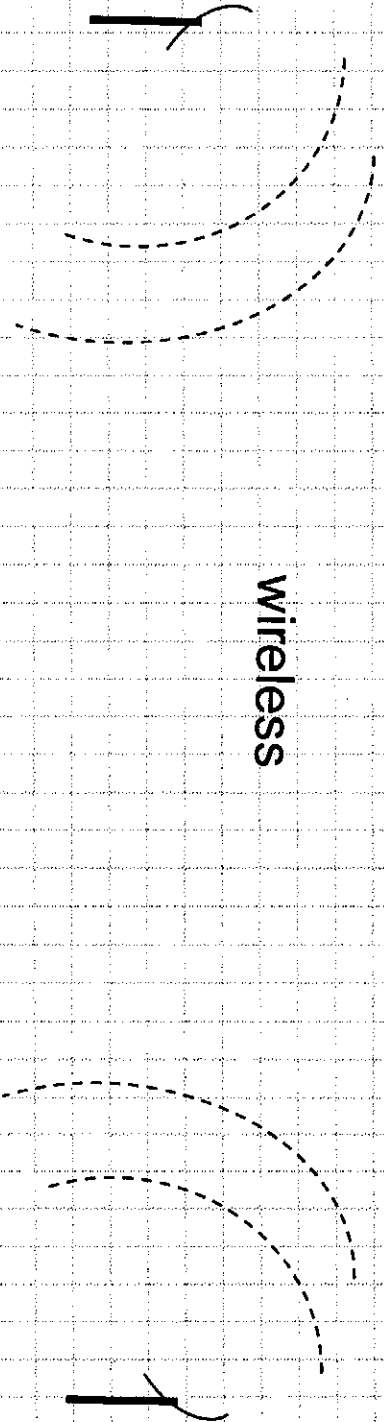
- Background
- OFDM Overview
- Vector OFDM
- VOFDM with Matrix Modulation
- Simulation
- Conclusion



Today's focus is physical layer modem.

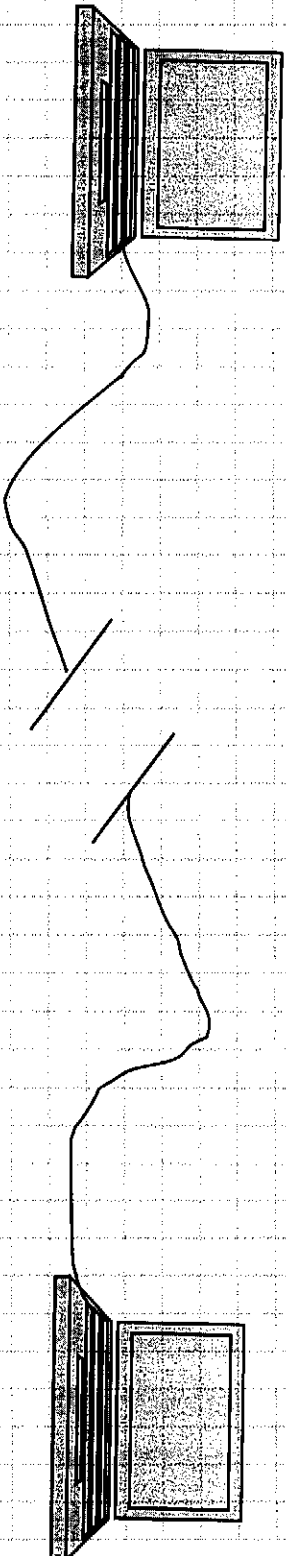
Communications

wireless



transmission through EM wave propagation

wired



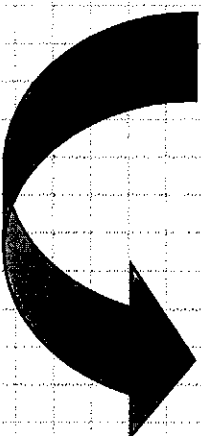
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Signal:

$\cos(p(t))$

carry information

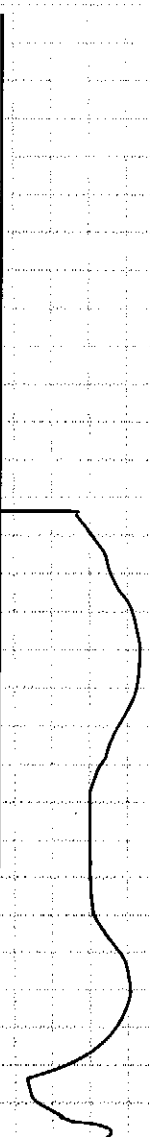


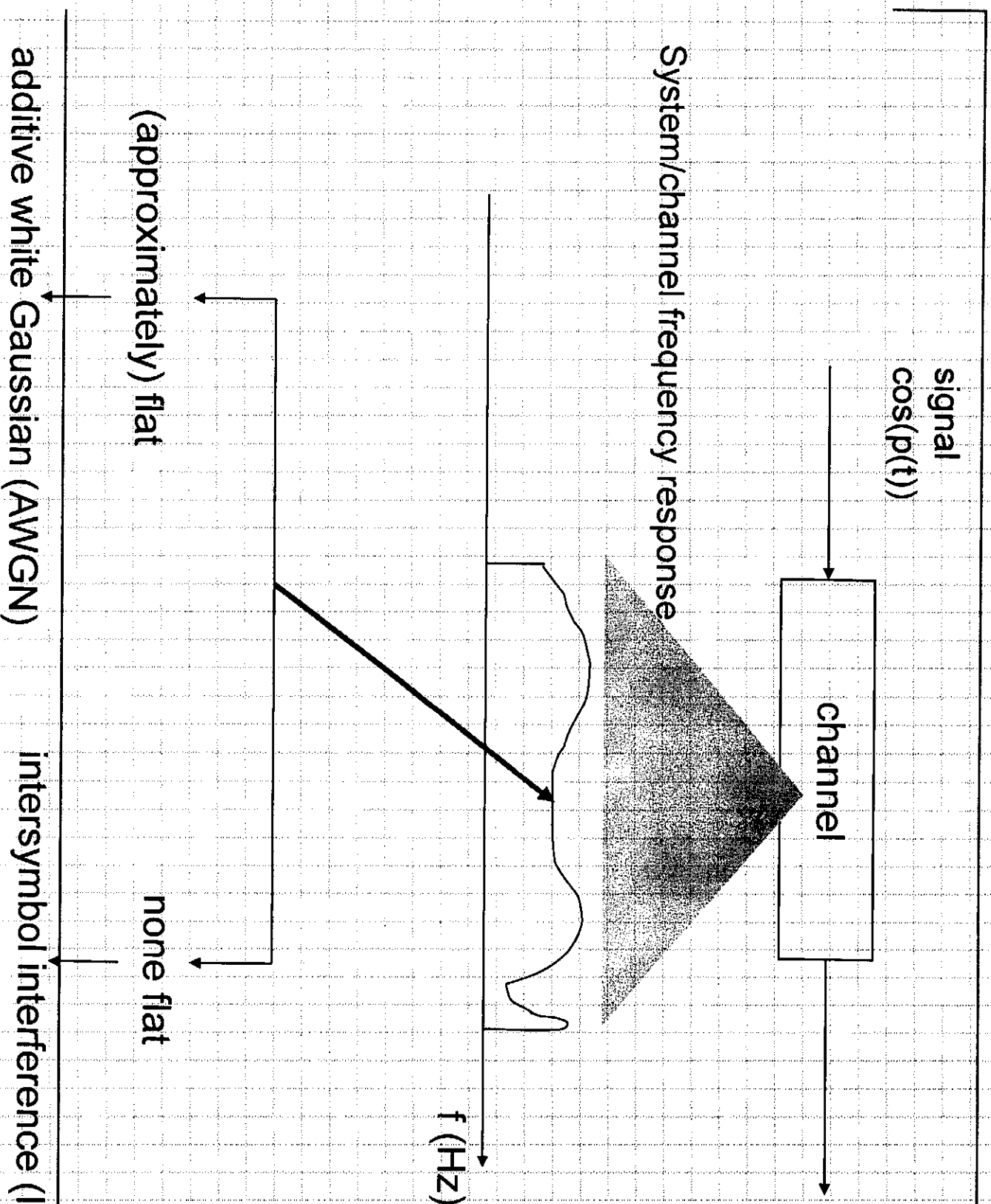
Channel:

a media that wave carrying information propagates through

→ Approximately a system

system frequency response







In broadband systems

this is wide

channel frequency response

f (Hz)

It is not possible to be flat $\rightarrow$ ISI occurs

Wireless:

Multipath

Wired:

None flat ISI channel

None ideal

Wired (modem)

< 9.6 kbs/s

equalization (Lucky 60s)

9.6 kbs/s

TCM + equalization (DFE)

14.4 kbs/s

28.8 kbs/s

56 kbs/s

TCM + equalization

Asymmetric Digital Subscriber Line (ADSL)

6 Mbps/s

orthogonal frequency division
multiplexing (OFDM)
or called discrete multi-tone (DMT)

Data Rate	Wire Size	Distance
1.5 or 2 Mbps	0.5 mm	5.5 Km
1.5 or 2 Mbps	0.4 mm	4.6 Km
6.1 Mbps	0.5 mm	3.7 Km
6.1 Mbps	0.4 mm	2.7 Km





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Wireless

HDTV Broadcasting

Europe

OFDM

Good and no ghost

Japan

USA

Equalization but Not OFDM

Ghost/unstable
(multipath problem)

LAN IEEE 802.11a OFDM

54 Mbs/s

4G ?

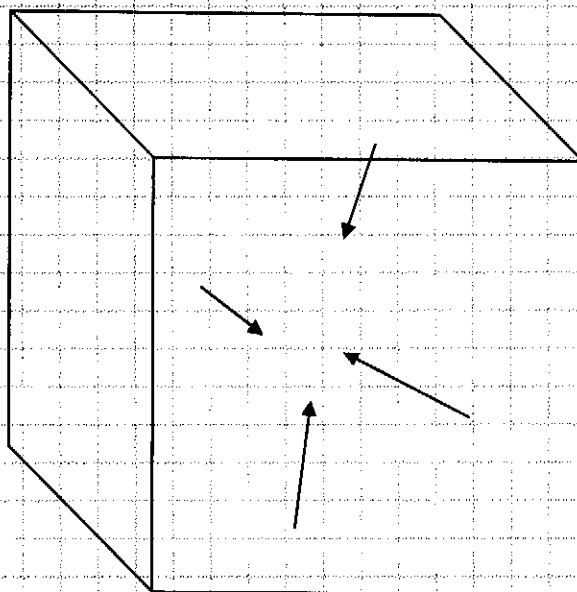
OFDM ?

VOFDM ?

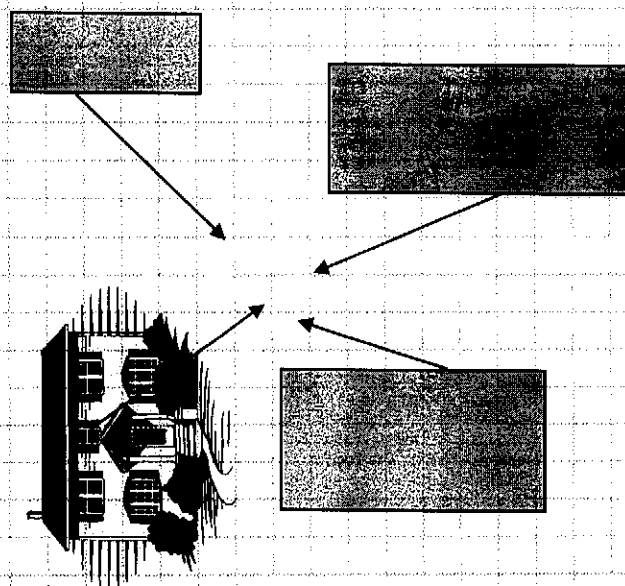


Why Multipath in Wireless Systems

indoor

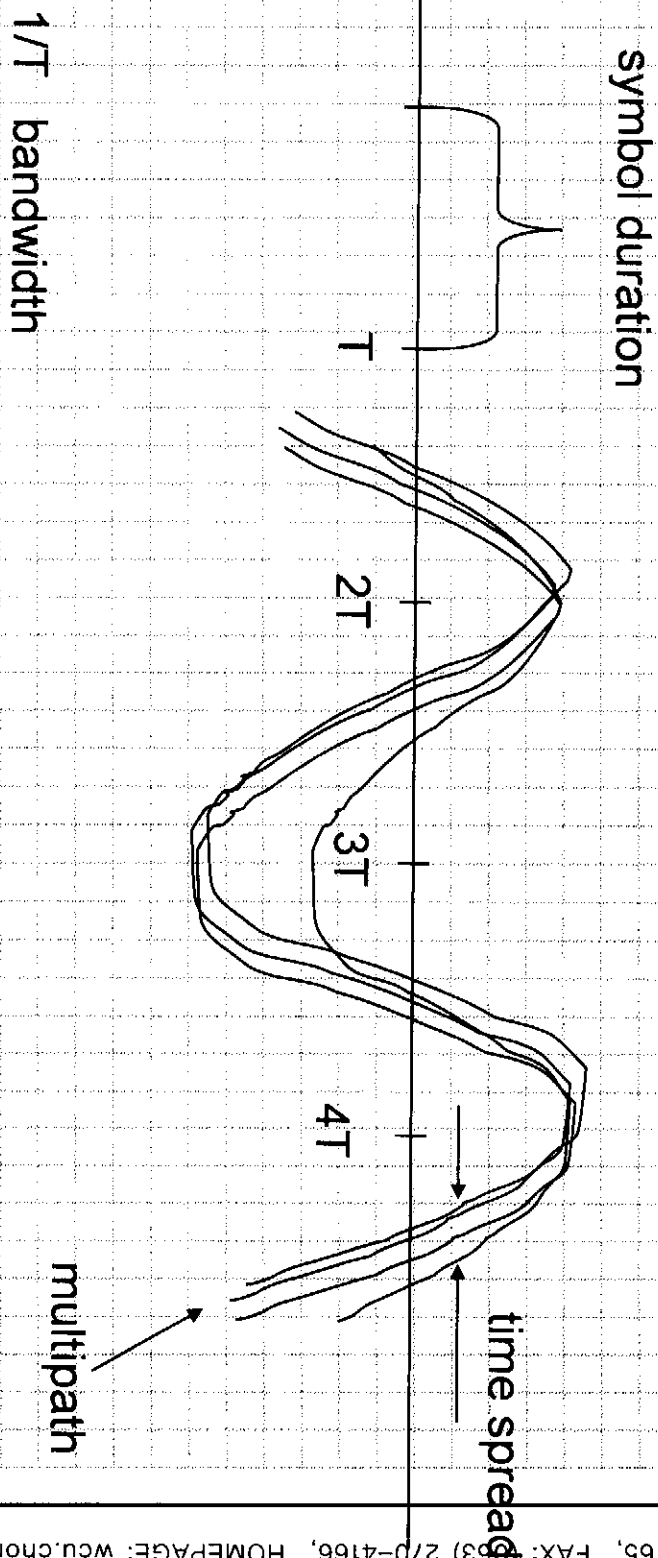


outdoor

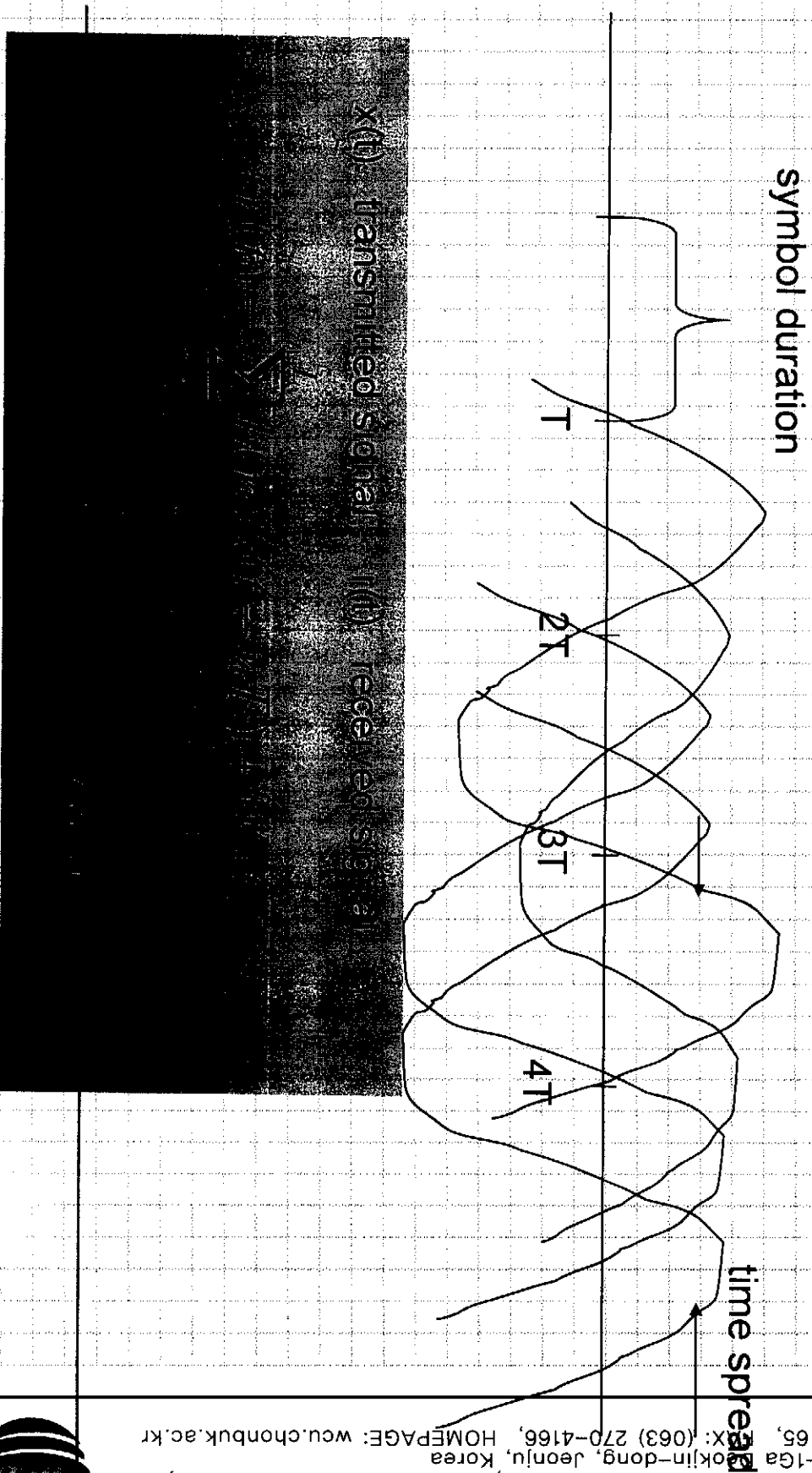


multiple reflections

Multipath



When the bandwidth is too wide or T is too small, the time spread may be across over multiple symbols. In this case, intersymbol interference (ISI) occurs.

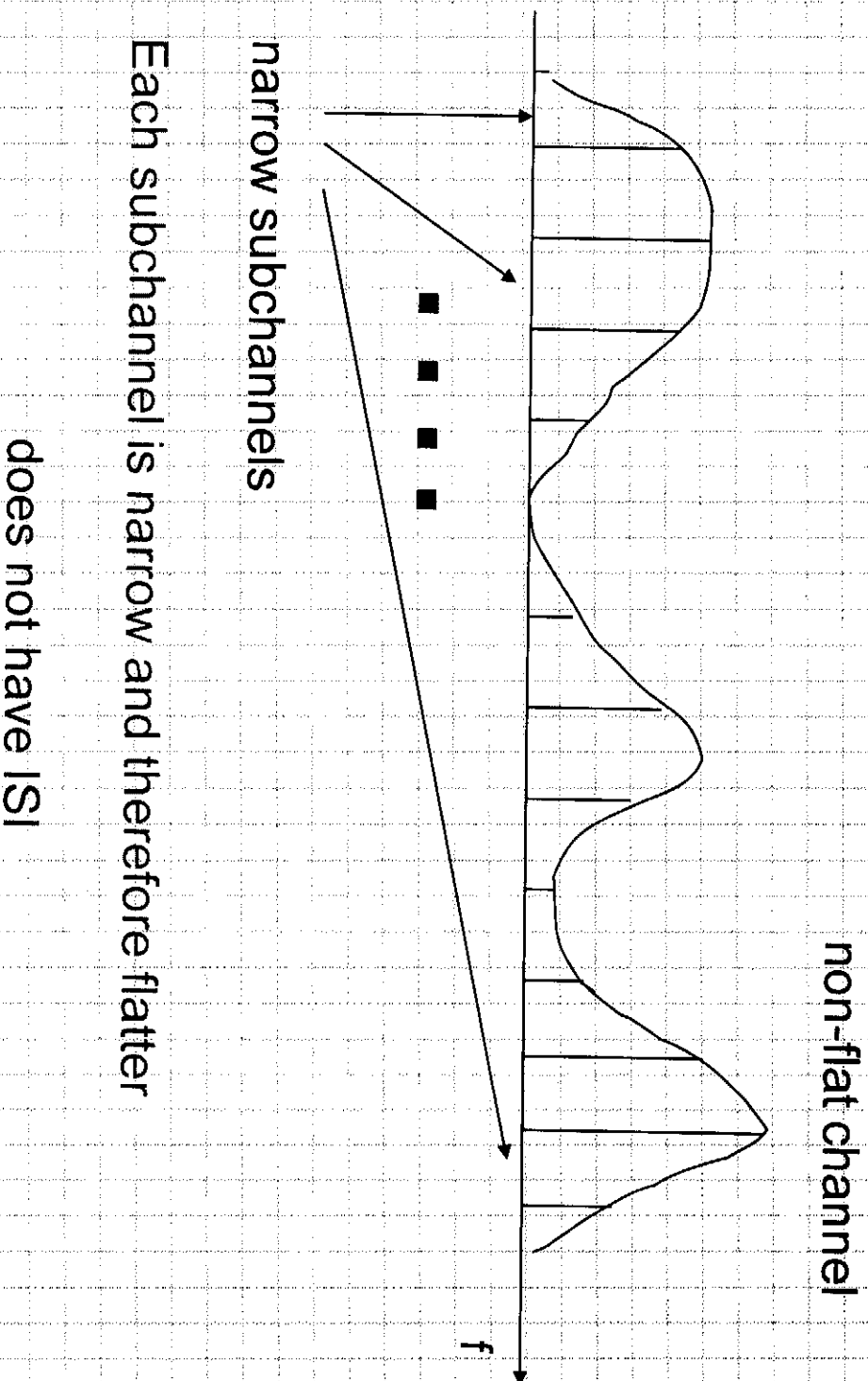


Number of Multipath vs. Modulation Methods in Wireless Applications

2G (IS-95)	1.3 MHz	Almost optimal for single path
3G (WCDMA)	< 11 MHz	6--8 multipath
IEEE 802.11b (LAN)	similar to 3G	almost the crashing point to use CDMA
IEEE 802.11a (LAN)	20 MHz	16 multipath OFDM
4G	> 20 MHz	VOFDM ???

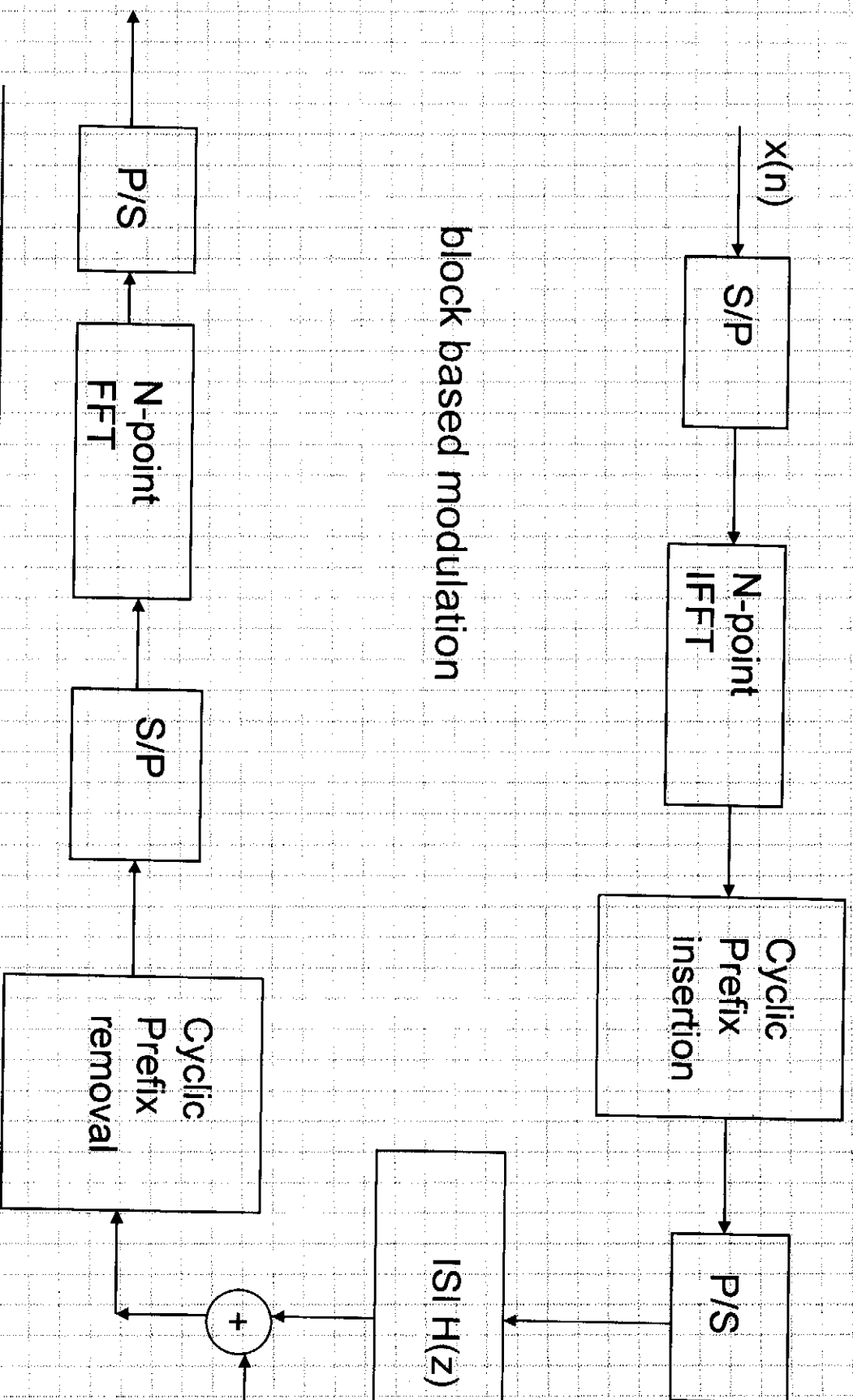


Why OFDM ? ----- Rough Idea



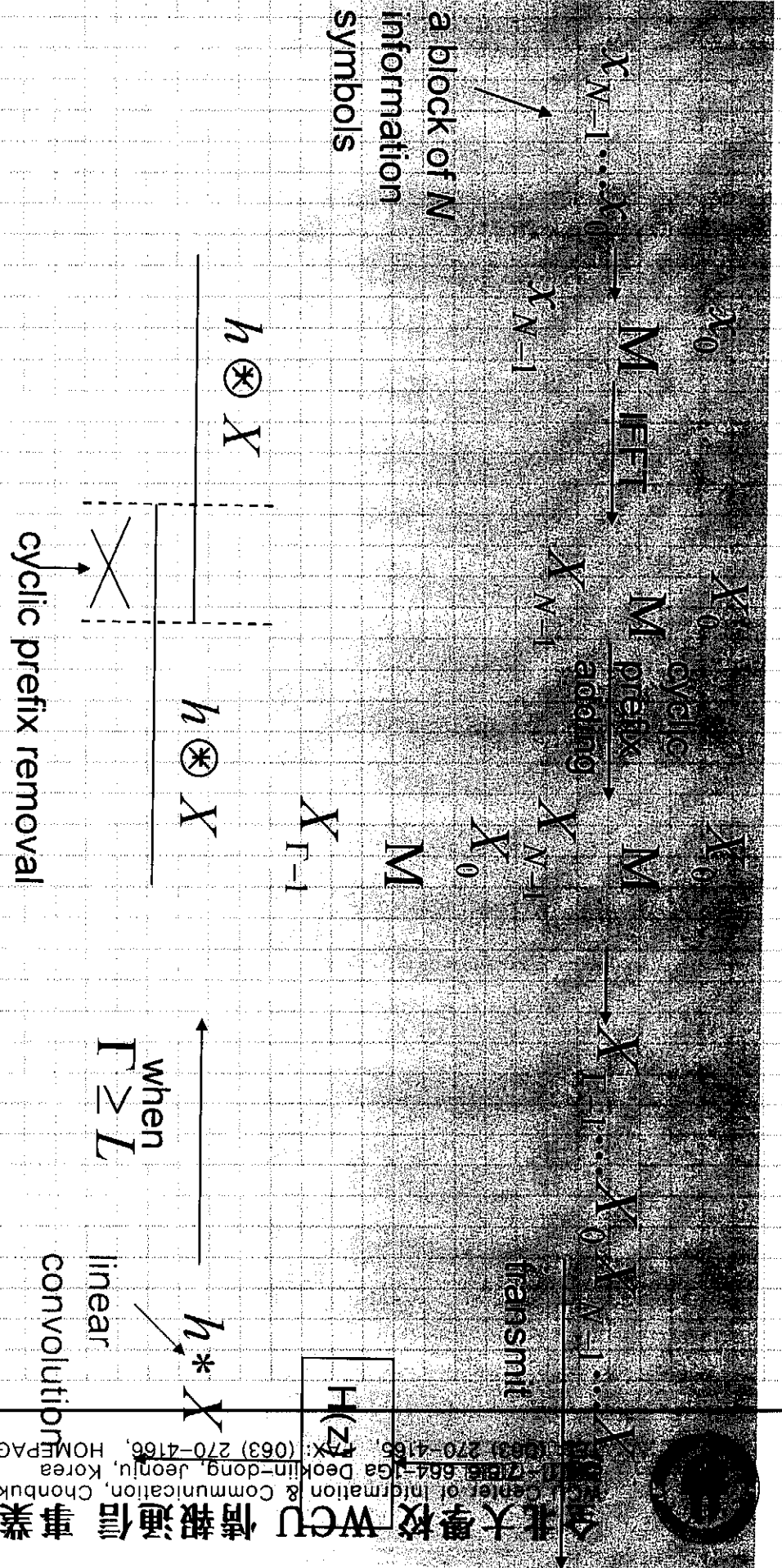
OFDM: One of the simplest communication systems

block based modulation





a block of N information symbols



$$Y_k = H_k \cdot x_k, k = 0, 1, \dots, N-1$$

FFT

$$h \otimes X$$

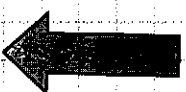
$$H_k = H(e^{j2\pi \frac{k}{N}}) = \sum_{n=0}^L h(n)e^{-j2\pi \frac{nk}{N}}$$

cyclic convolution



ISI channel

$$y(k) = \sum_{n=0}^L h(n)x(k-n) + w(k)$$



adding cyclic prefix as an additional data rate overhead

ISI-free subchannels

$$Y_k = H_k \cdot x_k + W_k, \quad k = 0, 1, \dots, N-1$$

Each subchannel corresponds to a DFT component H_k of the ISI channel.
If the frequency component H_k is small (bad), then this subchannel is bad.

In IEEE 802.11a,

$$L \leq 16$$

$$N=64$$

$$\Gamma=L=16, \quad 25\% \text{ data rate overhead}$$

Among 64 subchannels, only 52 (fixed indices) of them are used

12 subchannels H_k , $k=0, 27, \dots, 37$, are not used, i.e.,

no information is sent over these subchannels





OFDM, when $\Gamma \geq L$
data rate overhead

Receiver:

$$Y_k = H_k x_k + W_k$$

N scalar channels/equations

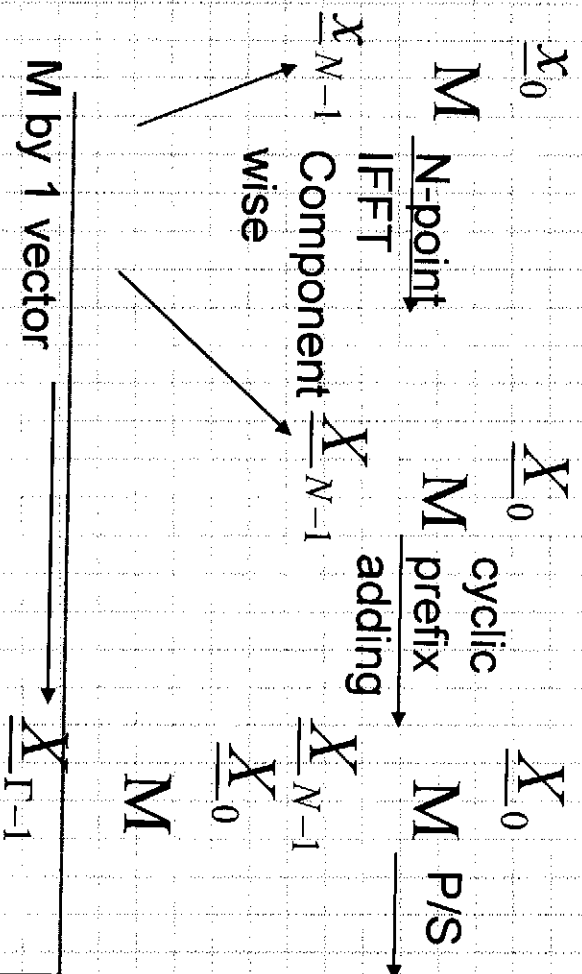
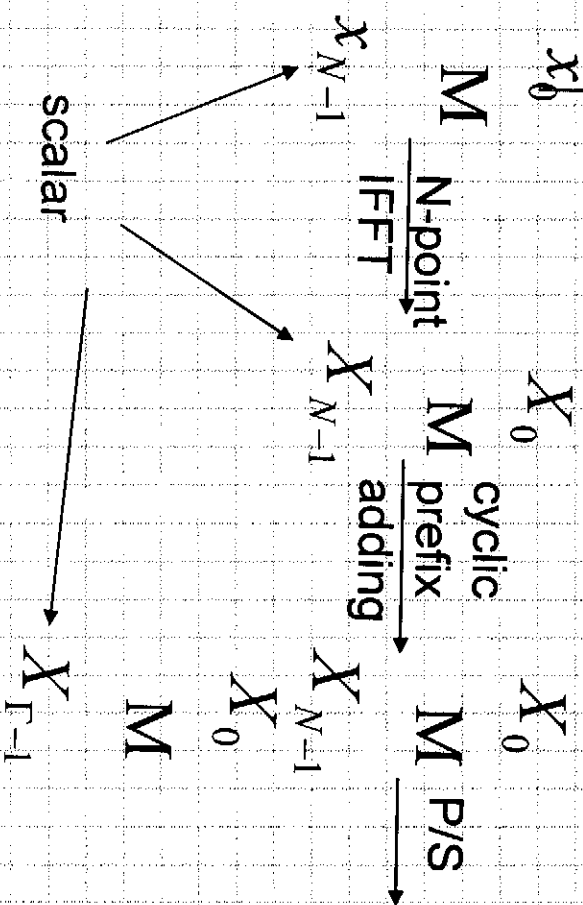
VOFDM, when

data rate overhead $\tilde{\Gamma} \geq \tilde{L} \approx \frac{L}{M}$

Receiver:

$$Y_k = H_k x_k + W_k$$

N/M by 1 vector channels/equations



Why Single Transmit Antenna (Not Multiple)

- Simplicity in implementation
- Simpler sync.
- Simpler channel estimation
- Achieve similar multipath diversity as in multiple antennas.

