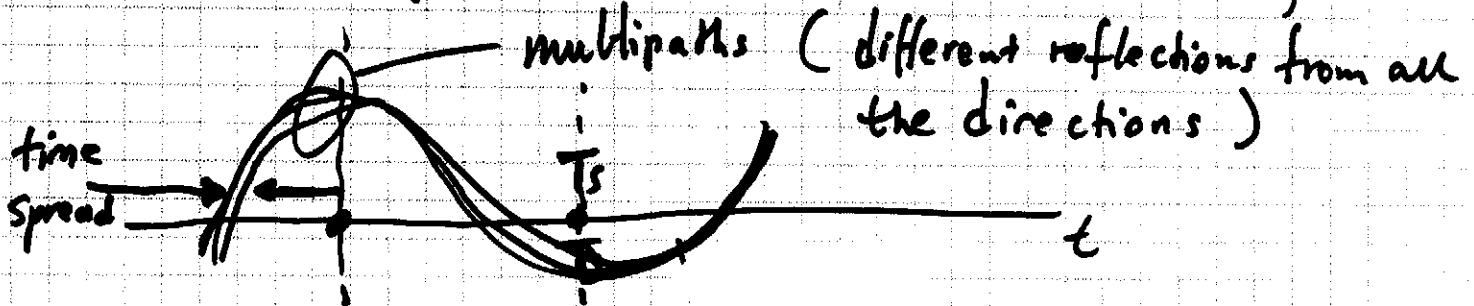




Section 3

Frequency-Selective Multipath-Fading



Flat fading: narrow-band transmission

$\Leftrightarrow$ $\frac{1}{\text{signal bandwidth}}$ $\gg$ time spread of the propagation path delays

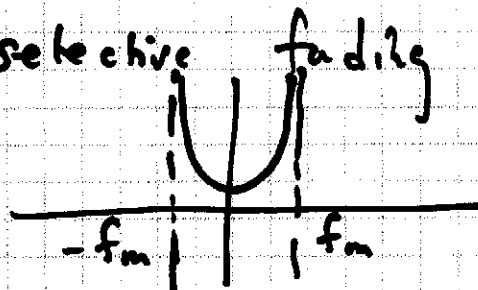
$\Leftrightarrow$ The duration of a modulated symbol T_s is much greater than the time spread of the propagation path delays

In this case, all frequencies in the transmitted signal will experience the same random attenuation and phase shift.

$$y(t) = h(t) x(t - \tau) + n(t)$$

frequency - non-selective fading

$$H(f) =$$





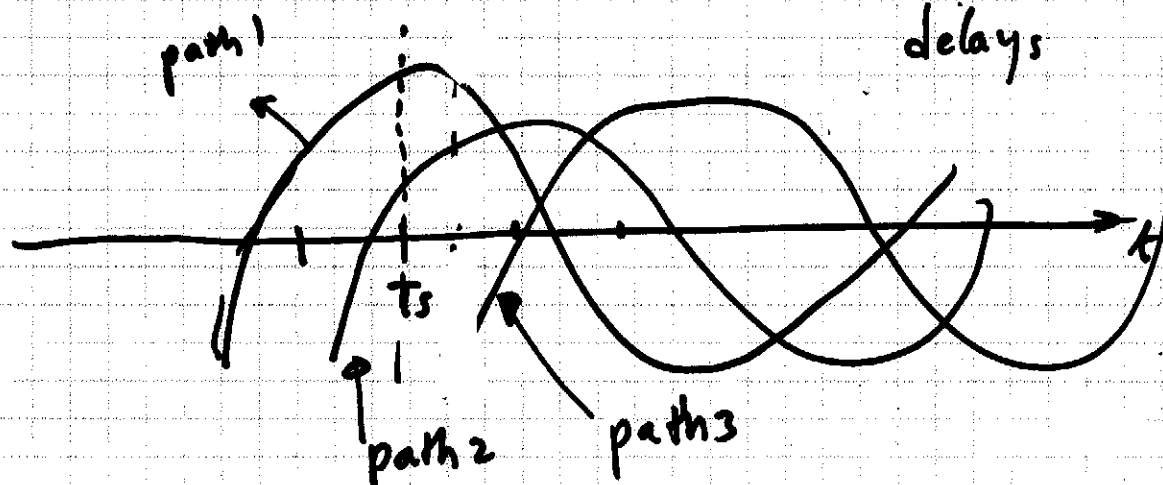
f_m is the maximal Doppler frequency caused by the motion of MS (mobile station)

$$f_m = \frac{v}{\lambda_c}$$

v ← speed of MS
 λ_c ← Carrier wave length

Frequency - selective fading

If $T_s = \frac{1}{\text{Signal bandwidth}}$ < time spread of the propagation path delays



Then,
$$y(t) = \sum_{k=0}^N h_k(t) x(t - \tau_k) + w(t)$$

and each $h_k(t)$ is the same as the frequency-non-selective fading case for each k .

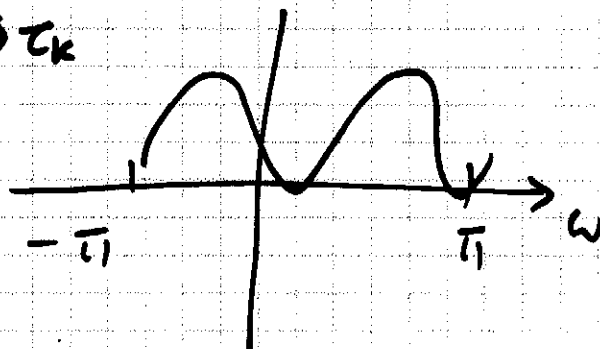
In this case, the channel is (in the time domain)

$$h(t) = \sum_{k=0}^N h_k(t) \delta(t - \tau_k)$$



and in the frequency domain

$$H(\omega) = \sum_{k=-\infty}^{\infty} h_k e^{-j\omega \tau_k}$$



Whether a fading channel is frequency-selective or not depends on the signal bandwidth.

Roughly

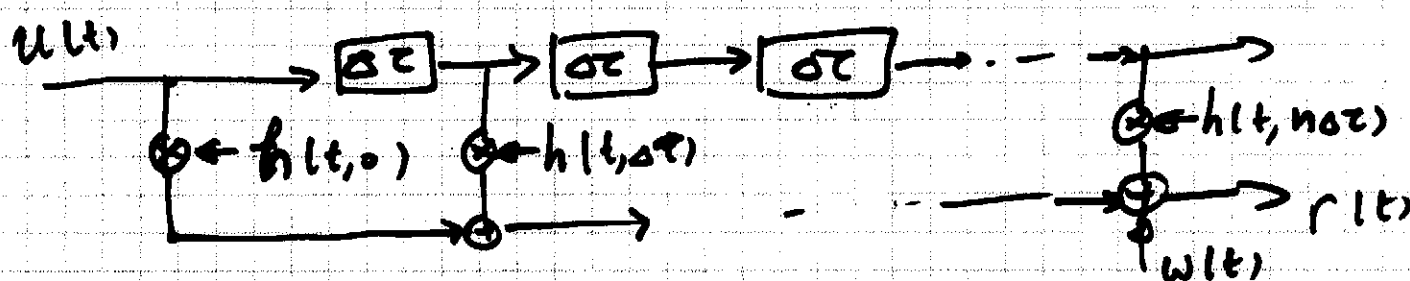
$$B = \frac{1}{T_s} \ll 1.3 \text{ MHz, frequency-non-selective}$$

$$B = \frac{1}{T_s} > 1.3 \text{ MHz, frequency selective}$$

In general, the input delay-spread function $h(t, \tau)$. Then the received signal

$$r(t) = \int_{-\infty}^{\infty} u(t-\tau) h(t, \tau) d\tau + w(t)$$

$$= \sum_{k=0}^N u(t - k\Delta\tau) h(t, k\Delta\tau) \Delta\tau + w(t)$$





Let $\phi_h(t, s; \tau, \eta)$ denote the correlation function of the input-delay spread function as

$$\phi_c(t, s; \tau, \eta) = \frac{1}{2} E[h(t, \tau) h^*(s, \eta)].$$

The average delay can be calculated as

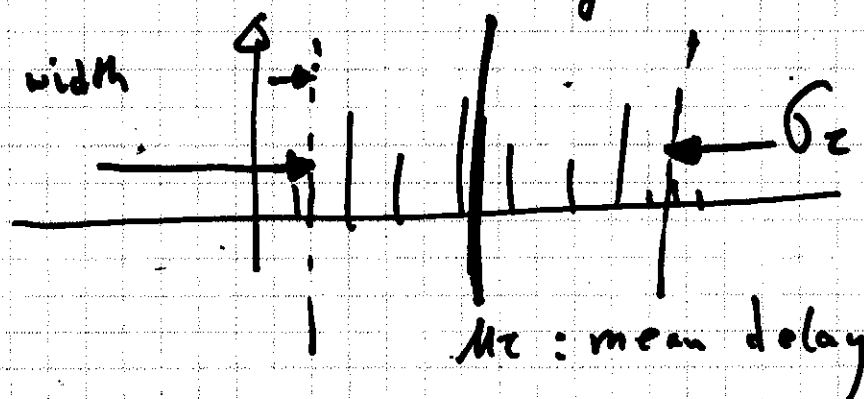
$$\mu_\tau = \frac{\int_0^\infty \tau \phi_c(\tau) d\tau}{\int_0^\infty \phi_c(\tau) d\tau}$$

The rms (root mean square) delay spread is

$$\sigma_\tau = \sqrt{\frac{\int_0^\infty (\tau - \mu_\tau)^2 \phi_c(\tau) d\tau}{\int_0^\infty \phi_c(\tau) d\tau}}$$

where $\phi_c(\tau) = \phi_c(0, \tau)$ is the multipath intensity profile or power delay.

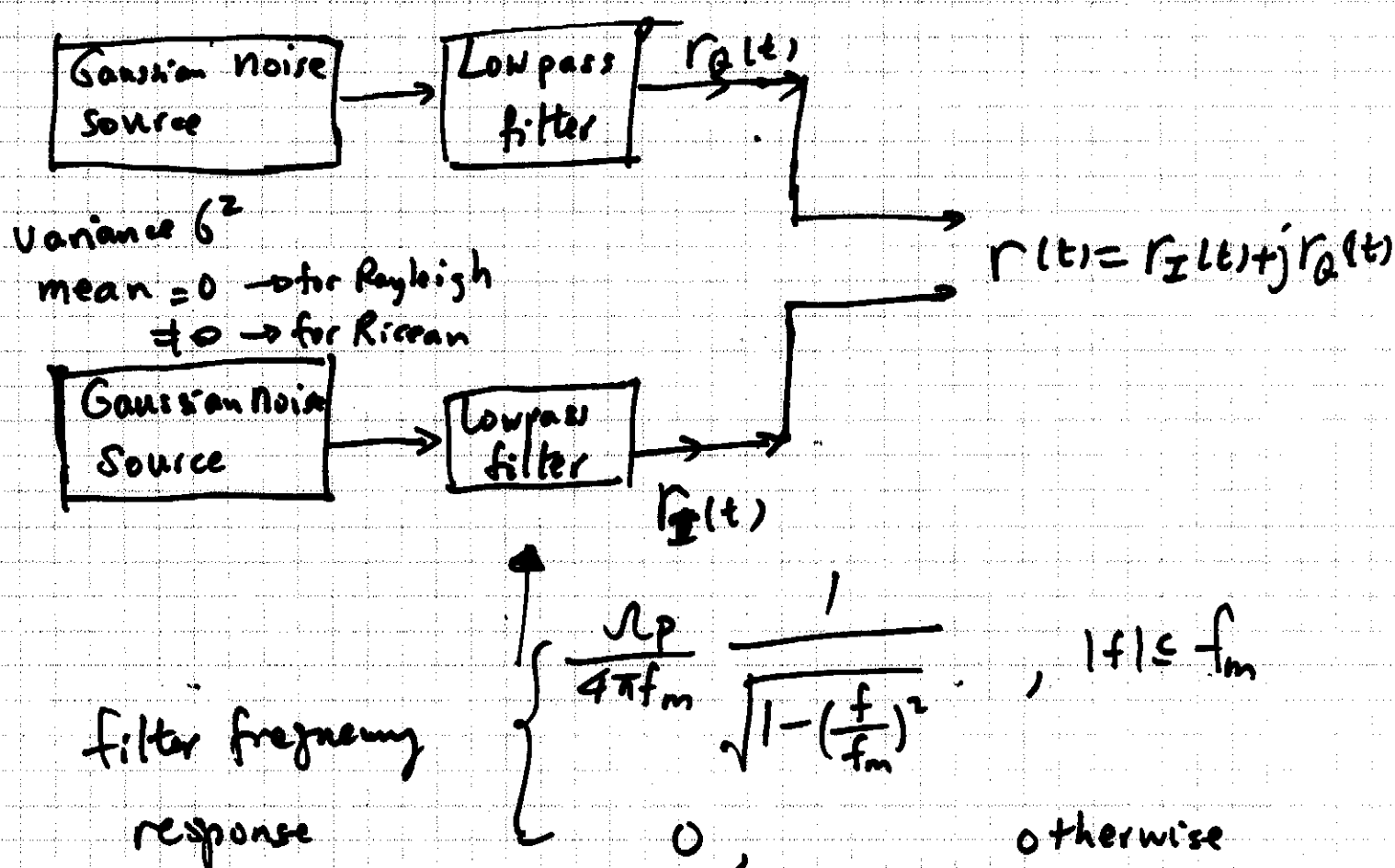
σ_τ measures roughly the time-delay spread width





Section 4. Laboratory Simulation of Multipath Fading Channels

§1. Filtered Gaussian



One way to implement this lowpass filter is to use first order recursive filter:

Let $r_I[k] = r_I(kT)$
 $r_Q[k] = r_Q(kT)$, $k \in \mathbb{Z}$, T is the simulation step size



The first order lowpass filter is

$$r_I[k+1] = \zeta r_I[k] + (1-\zeta) w_I[k]$$

$$r_Q[k+1] = \zeta r_Q[k] + (1-\zeta) w_Q[k]$$

The filtered noise $r_I(k)$ should have correlation function $\frac{\sigma^2}{2} J_0(2\pi f_m nT)$

The received signal spectrum is

$$S_{pp}(f) = \frac{\sigma^2}{1 + \zeta^2 - 2\zeta \cos 2\pi fT}$$

$$\text{Set } \zeta = 2 - \cos(2\pi f_m T) - \sqrt{(2 - \cos 2\pi f_m T)^2 - 1}$$

$$\sigma^2 = \frac{1+\zeta}{2(1-\zeta)}$$

§2. Jake's Method (Model)

(1). Single fading signal

* Assume the stationarity ($f_{D,n}(t) = f_{D,n}, \alpha_n = 1$)

In this case, the received signal (base band)

$$r(t) = \sum_{n=1}^N e^{-j(\hat{\phi}_n + 2\pi f_m t \cos \theta_n)}$$

where $\hat{\phi}_n = 2\pi(f_c + f_m)\tau_n$

↑
arriving
angle



* Assume the isotropic scattering model.
Then, the N components are uniformly distributed

$$\Theta_n = \frac{2\pi n}{N}, \quad n=1, 2, \dots, N.$$

* Let $\frac{N}{2}$ be an odd number and $M = \frac{1}{2}(\frac{N}{2} - 1)$

By rearranging the summation

$$r(t) = r_I(t) + j r_Q(t)$$

where $r_I(t) = 2 \sum_{n=1}^M \cos(\beta_n) \cos 2\pi f_n t + \sqrt{2} \cos \alpha \cos 2\pi f_m t$

$$r_Q(t) = 2 \sum_{n=1}^M \sin(\beta_n) \cos 2\pi f_n t + \sqrt{2} \sin \alpha \cos 2\pi f_m t$$

where β_n is chosen as $\frac{\pi n}{M}$

$$\alpha = 0$$

$$f_n = f_m \cos(2\pi n/N)$$

(2) Multiple fading signals

$$r_i(t) = r_{I,i}(t) + j r_{Q,i}(t), \quad i=1, \dots, M.$$

where $r_{I,i}(t) = 2 \sum_{n=1}^M \cos(r_{n,i} + \beta_n) \cos 2\pi f_n t + \sqrt{2} \cos \alpha \cos 2\pi f_m t$

$$r_{Q,i}(t) = 2 \sum_{n=1}^M \sin(r_{n,i} + \beta_n) \cos 2\pi f_n t + \sqrt{2} \sin \alpha \cos 2\pi f_m t$$

where $r_{n,i} = \frac{2\pi(i-1)n}{M}$, $n=1, \dots, M$, other parameters are the same as before.



HomeWork Assignment #1

Use $v = 90 \text{ km/h}$, $f_c = 900 \text{ MHz}$ to generate the baseband received signal $r(t)$ with Rayleigh fading

- ① FIR lowpass with 64 coefficients
- ② IIR lowpass filter of the first order
- ③ Jake's method by using $N=34$ or $M=8$ for both single path and multipaths.

Plot $r(t)$ and calculate/plot its power spectrum density function and compare it to the derived theoretical formula $S_{xx}(f)$.



Section 5. Shadowing

$x(t)$: received signal (complex)

$$z(t) = |x(t)|$$

$$\Omega_p = E(z^2(t)) \quad \text{average power}$$

$$\Omega_v = E(z(t)) \quad \text{average mean}$$

maybe variables

With shadowing, Ω_v itself is a random variable.

Empirical studies have shown that Ω_v follows a log-normal distribution, such that the pdf of Ω_v is

$$p(\Omega_v) = \frac{2\zeta}{\Omega_v 6\sigma \sqrt{2\pi}} \exp\left\{-\frac{(\log_{10} \Omega_v^2 - \mu_{\Omega_v})^2}{2\sigma^2}\right\}$$

where $\mu_{\Omega_v} = E(\Omega_v)$ denotes the mean value of Ω_v (dB) and $\zeta = \frac{10}{\ln 10}$

After a transformation

$$p(\Omega_v(\text{dB})) = \frac{1}{\sqrt{2\pi} 6\sigma} \exp\left\{-\frac{(\Omega_v(\text{dB}) - \mu_{\Omega_v})^2}{2\sigma^2}\right\}$$

Some people consider $\Omega_v = E(z(t))$
 some other people consider $\Omega_v = E(z^2(t))$



* One may composite shadowing-fading distributions

$$P_{zc}(x) = \int_0^{\infty} P_{z/\Omega_v}(x|\omega) P_{\Omega_v}(\omega) d\omega$$

For the case of Rayleigh fading

$$\Omega_v = E(z|h) = \sqrt{\frac{\pi}{2}} \sigma$$

$$\text{and } P_{z/\Omega_v}(x|\omega) = \frac{\pi x}{2\omega^2} \exp\left\{-\frac{\pi x^2}{4\omega^2}\right\}$$

$$P_{zc}(x) = \int_0^{\infty} \frac{\pi x}{2\omega^2} e^{-\frac{\pi x^2}{4\omega^2}} \frac{2s}{\omega \sigma_n \sqrt{2\pi}} e^{-\frac{(10 \log_{10} \omega^2 - \mu_{sv})^2}{2\sigma_n^2}} d\omega$$

