Discrete Chirp-Fourier Transform

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For a finite length signal $x(n), n = 0, 1, \ldots, N-1,$ of length N, its discrete chirp-Fourier transform (DCFT) is defined in [1] as

$$X_c(k,l) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(n) \exp(-2\pi j(kn + ln^2)/N), \;\; 0 \le k, l \le N-1.$$

DCFT is used to estimate the chirp rate for a quadratic chirp signal. If signal $x(n)$ is a single discrete chirp of the form

$$x(n) = \exp(2\pi j(k_0 n + l_0 n^2)/N), \;\; 0 \le n \le N-1,$$

and N is a prime, then [1]

$$|X_c(k,l)| = \begin{cases} \sqrt{N}, & \text{when } k = k_0, l = l_0 \\ 1, & \text{when } l \ne l_0 \\ 0, & \text{when } l = l_0, k \ne k_0 \end{cases}$$

where $0 \le k, l \le N-1.$ This means that the main lobe is $\sqrt{N}$, while the side lobe is not above 1, which holds only when the signal length N is prime. If the signal length N is not a prime, then the maximum side lobe in the range $0 \le k, l \le N-1$ is at least as high as $\sqrt{2}$ [1]:

$$\max_{(k,l) \ne (k_0, l_0)} |X_c(k,l)| \ge \sqrt{2}.$$

From the inverse discrete Fourier transform (IDFT), the inverse DCFT is obvious:

$$x(n) = \exp(2\pi j l n^2/N) \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_c(k,l) \exp(2\pi j k n/N), \;\; 0 \le n \le N-1,$$

where $l, 0 \le l \le N-1,$ is an arbitrarily fixed integer. More details and properties about DCFT can be found in [1], [2].

References

[1] Xiang-Gen Xia, "Discrete Chirp-Fourier Transform and Its Application to Chirp Rate Estimation," *IEEE Transactions on Signal Processing*, vol. 48, no. 11, pp. 3122-3133, Nov. 2000.

[2] Xiang-Gen Xia, "Discrete Chirp-Fourier Transform," *Transforms and Applications Handbook*, Third Ed. (edited by D. Poularikas), CRC Publisher, 2010.