VOFDM/OTFS and 6G Modulation

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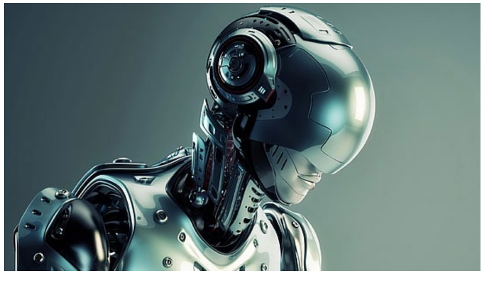
I think …

- The most important changes in the last 40 years in USA are electronics, in particular smart phones.

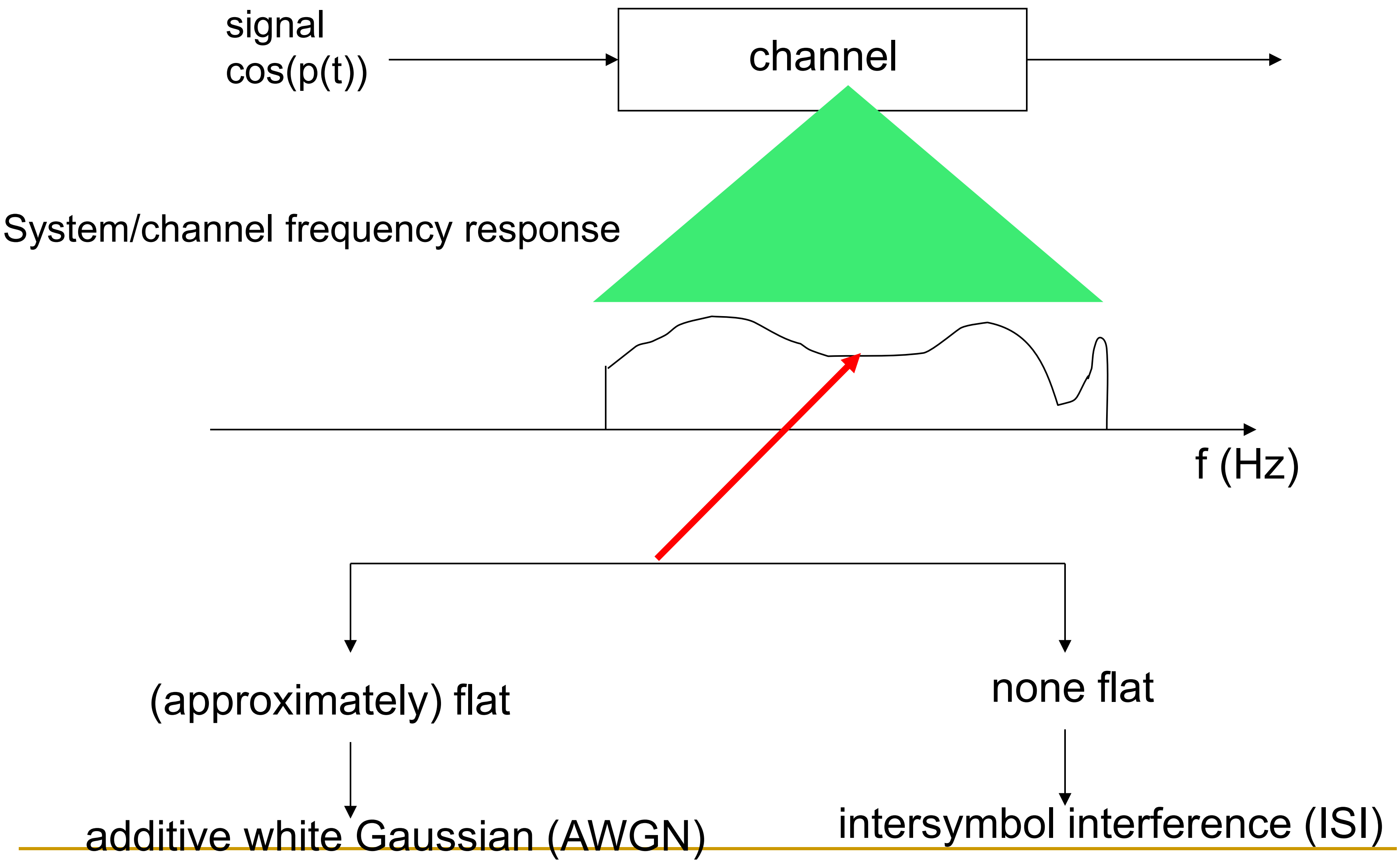
- The most important changes in the next 40 years will be humanoid robots that can take care of people.



Outline

- Brief Review the Past Standards
  - Wireline and Wireless Standards
- OFDM Overview
- Vector OFDM (VOFDM)
- Linear Receivers
- Conclusion

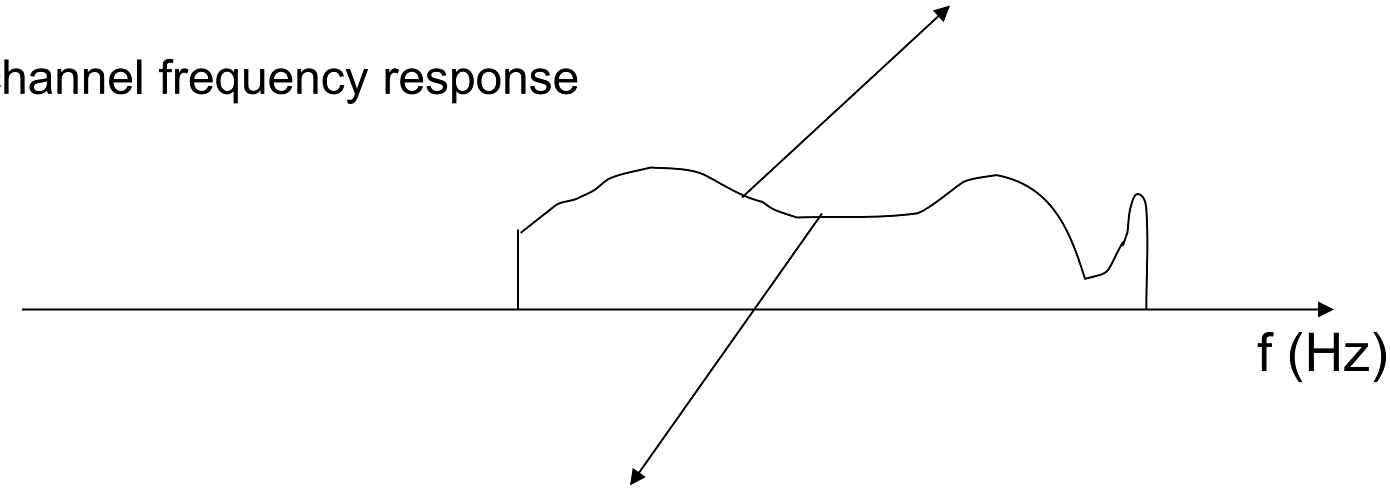
A communication channel is approximately a linear system



In broadband systems

this is wide

channel frequency response



It is not possible to be flat ⟶ ISI occurs

Wireless: Multipath

Wired: None flat ISI channel
None ideal

The Wave Propagation Problem

- For any wave propagation, the most serious problem is the multiple reflections/paths
  - Optical images: ghost images
  - Acoustic communications: severe multipaths
  - Radar: multipaths, range ambiguity, inter-range cell interference
  - Telecommunications: intersymbol interference (ISI)
- The most important problem in physical layer communications in the past has always been in dealing with the ISI, or equalization.
- OFDM is the only modulation scheme to convert an ISI channel to multiple ISI free subchannels

Wired (modem): Channel is fixed and has high SNR

Telnet

< 9.6 kbs/s — equalization (Lucky 60s)

9.6 kbs/s — TCM +equalization (DFE)

14.4 kbs/s

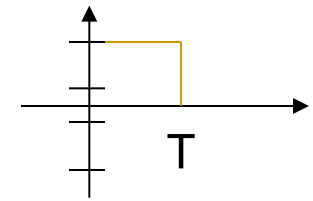
28.8 kbs/s — TCM + equalization

33.6 kbs/s

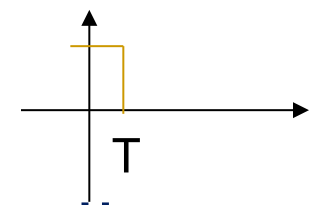
56 kbs/s — TCM/shaping+equalization

Mod/Code Demod/decode

Squeeze more bits to a symbol



time to send one symbol



Use more bandwidth

Asymmetric Digital Subscriber Line (ADSL)

Ethernet

6 Mbs/s

orthogonal frequency division multiplexing (OFDM)
or called discrete multi-tone (DMT)

Data Rate	Wire Size	Distance
1.5 or 2 Mbps	0.5 mm	5.5 Km
1.5 or 2 Mbps	0.4 mm	4.6 Km
6.1 Mbps	0.5 mm	3.7 Km
6.1 Mbps	0.4 mm	2.7 Km

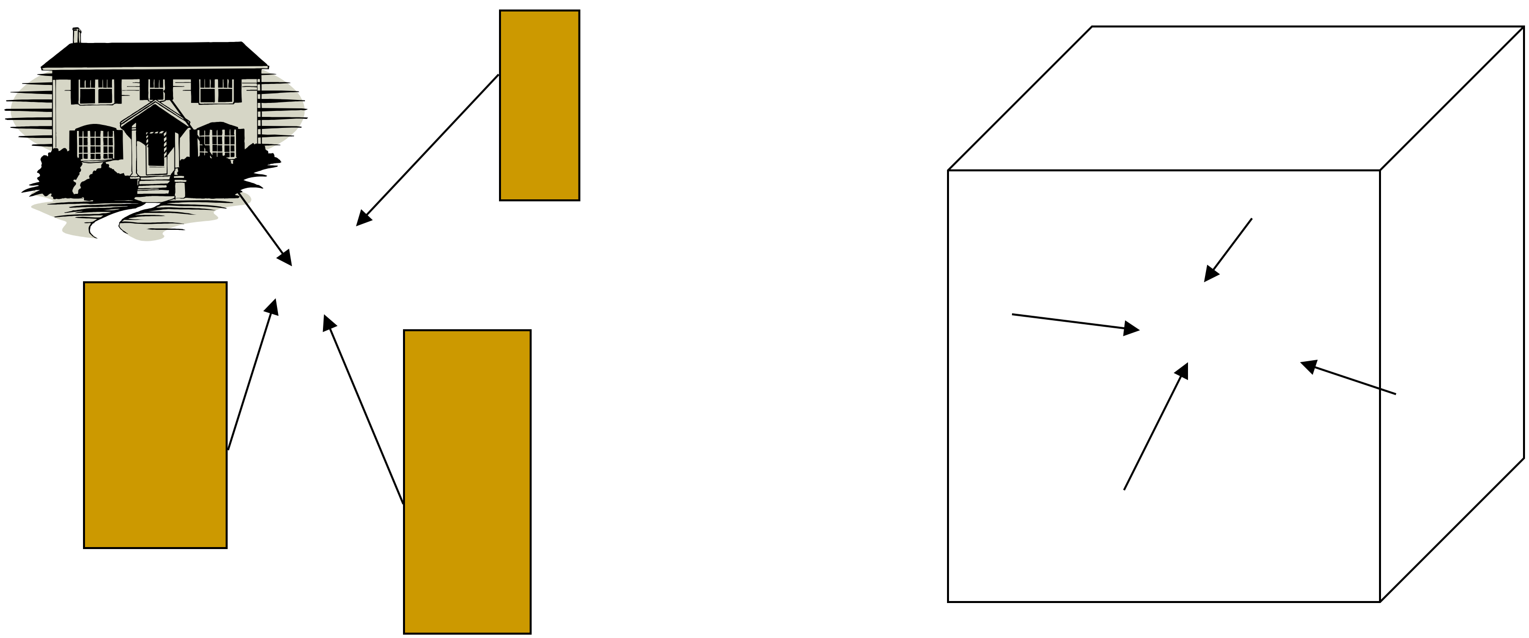
Computer modem was the most important business in communications in the 1990's

Wireless Systems: Channel varies/fades and not high SNR

outdoor

indoor

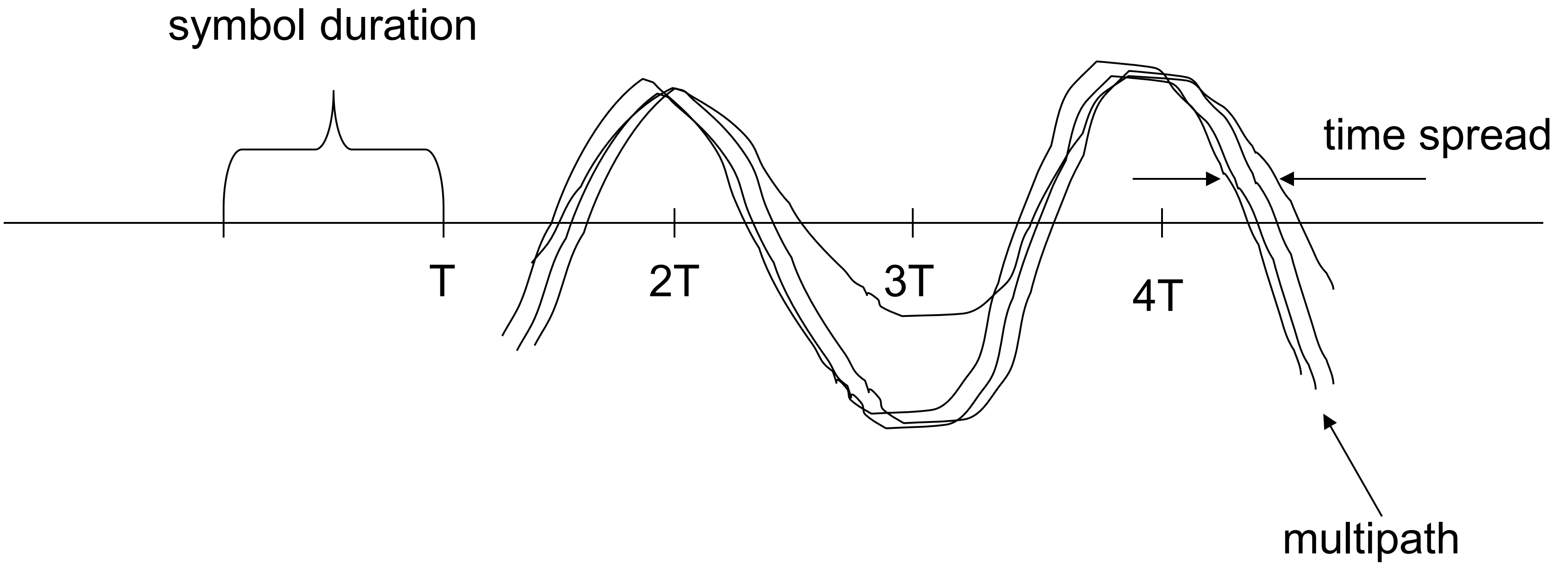
multipaths



multiple reflections

Multipath

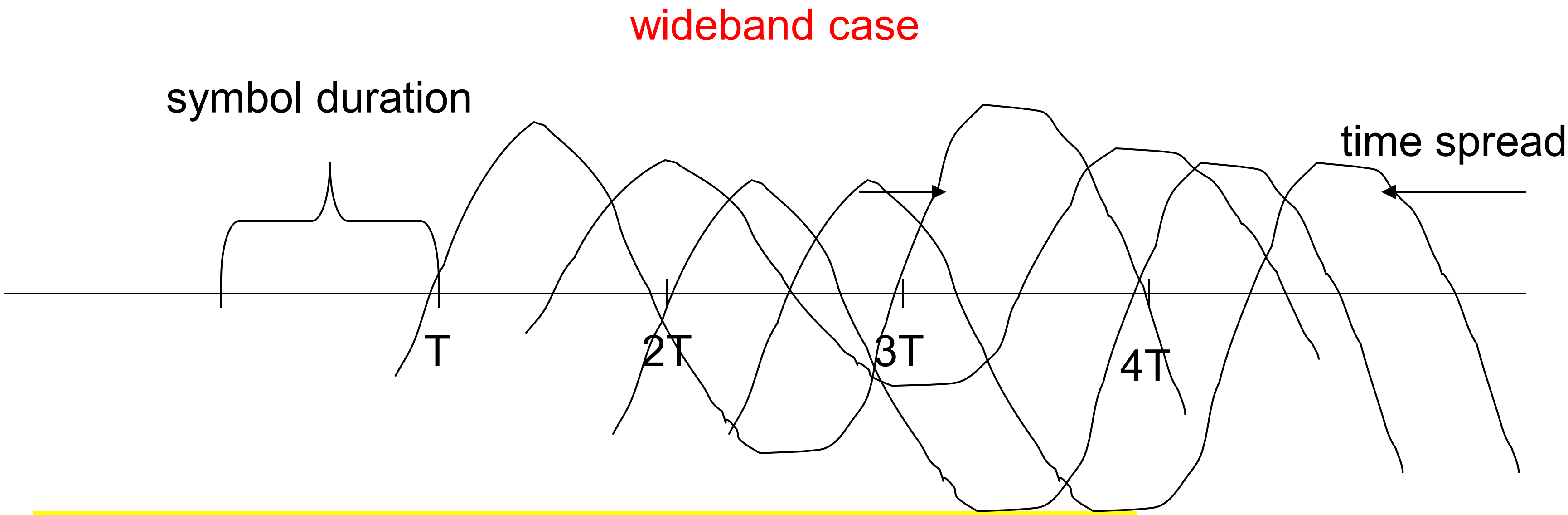
narrowband case



1/T bandwidth

No intersymbol interferences

When the bandwidth is too wide (T is too small), the time spread may be across over multiple symbols. In this case, intersymbol interference (ISI) occurs.



x(t): transmitted signal; *r(t)*: received signal

$$r(t)=\sum_{n=0}^{L} h(n)x(t-nT)+W(t)$$

ISI

AWGN

In practice, *h(n)* may be time-varying

Number of Multipath vs. Modulation Methods in Wireless Applications

2G (IS-95)	1.23 MHz	Almost optimal for single path (or equivalent)
3G (WCDMA CDMA2000)	< 11 MHz	6--8 multipaths (or equivalent) almost the break point to use CDMA
IEEE 802.11b (LAN)	similar to 3G	
IEEE 802.11a (LAN)	20 MHz	16 multipaths (or equivalent) OFDM
IEEE 802.11n (LAN)	20 & 40MHz	40MHz doubles everything in 20MHz OFDM
4G LTE	20 MHz	16 multipaths (or equivalent) OFDM and SC-FDE
5G	100 MHz	OFDM
6G	????	????

Complexities (for block length N)

- 3G or earlier: N
- 4G and 5G: $N \log N$
- 6G: ??

Digital Wireless Standards vs. Bandwidth (#of Multipaths)

- A standard is determined by a bandwidth (so far)
- 2G: 1.23MHz, almost the highest for non-ISI (or highest for TDMA in cellular systems)
  - Both TDMA and CDMA (DS spread spectrum) work well
- 3G: ~10 MHz, a few multipaths, highest for CDMA
  - Due to the ISI and wireless varying channels, time domain equalization may not work well, TDMA is not used, but CDMA (DS spread spectrum) is used in all standards since it is good to resist a few chip level time delays (RAKE receiver)
- 4G: 20 MHz, more multipaths
  - **Even CDMA RAKE receiver may not work well**
  - **OFDM is adopted (down link)**
    - Due to wireless channel varying, the number of subcarriers, N=64, is used, 25% data overhead for the cyclic prefix (CP) to deal with the multipaths
- 5G: 100 MHz, OFDM

Some Comments on These Standards

- The modulation schemes for all these standards are determined by the way to deal with ISI.
  - In the past several decades, the most important task in physical layer communications has been always on dealing with ISI.
  - In my opinion, multi-access or multi-cells is NOT the problem to determine which basic modulation is used.
  - Adding more antennas or not is the hardware choice and may not determine a basic modulation.
  - A basic modulation has to be simple.
- Dealing with ISI is the key !

6G: Bandwidth **>>100** MHz (?)

- Extremely low latency communication with fast data rate is urgently needed
  - Important Applications
    - Humanoid Robots that can be remotely controlled
      - Taking care of the elderly
      - Completely independent humanoid robots may not be possible in any short time range
    - VR/AR

6G: Bandwidth **>>100** MHz (?)

- To have extremely low latency and fast data rate, large bandwidth is necessary
- Can OFDM Still Work Well?
  - Much more multipaths exist

    → much large CP length to deal with multipaths

    → much large number *N* of subcarriers/IFFT_size

    → may lead to break down OFDM??
      - High PAPR (?)
      - Time varying channels (?)
- What bandwidth will be the breakpoint for OFDM in **cellular systems**? How large will a bandwidth go?
  - Can we make it work with a **fixed *N*** while it still can deal with the increased # of multipaths? Can we make it ***scalable to a bandwidth***?
    - **Single antenna VOFDM** [Xia, TCOM, Aug. 2001, also ICC 2000]

This is also the same as the current OTFS

OFDM vs. Vector OFDM System

x_0 ⋮ x_{N-1} $\xrightarrow[\text{IFFT}]{\text{N-point}}$ X_0 ⋮ X_{N-1} $\xrightarrow[\text{adding}]{\text{cyclic prefix}}$ X_0 ⋮ X_{N-1} X_0 ⋮ $X_{\Gamma-1}$ $\xrightarrow{\text{P/S}}$

scalar

OFDM, when

data rate overhead $\Gamma \geq L$

Receiver:

$$Y_k = H_k x_k + \eta_k$$

N many scalar channels/equations

$\underline{x}_0$ ⋮ $\underline{x}_{N-1}$ $\xrightarrow[\text{IFFT Component wise}]{\text{N-point}}$ $\underline{X}_0$ ⋮ $\underline{X}_{N-1}$ $\xrightarrow[\text{adding}]{\text{cyclic prefix}}$ $\underline{X}_0$ ⋮ $\underline{X}_{N-1}$ $\underline{X}_0$ ⋮ $\underline{X}_{\tilde{\Gamma}-1}$ $\xrightarrow{\text{P/S}}$

M by 1 vector

VOFDM, when

data rate overhead $\tilde{\Gamma} \geq \tilde{L} \approx \frac{L}{M}$

Receiver:

$$\underline{Y}_k = \underline{H}_k \underline{x}_k + \underline{\eta}_k$$

N many M by 1 vector channels/equations

This is a simple way to see the transmission of OTFS.

Comments on VOFDM and OTFS

- An analog pulse $g(t)$ is skipped here but can be always added to VOFDM signals in real transmission similar to OFDM.

- The CP length does not have to be exactly a multiple of the vector size M: $\tilde{\Gamma} M$.
  - The CP part can be truncated to any length that is not less than the channel length L to avoid the inter-block-interference.

- The transmission of VOFDM is exactly the same as that of OTFS that is a recent active topic in 6G.

OTFS (Orthogonal Time Frequency Space): R. Hadani, S. Rakib, M. Tsatsanis, A. Monk, A. J. Goldsmith, A. F. Molisch, and R. Calderbank, "Orthogonal time frequency space modulation," in Proc. IEEE Wireless Commun. Netw. Conf. (WCNC), San Francisco, CA, USA, pp. 1-6, Mar. 2017.

VOFDM: Vectorized Channel

- The ISI channel *H(z)* is converted into *N* vector channels

M symbols in each vector are in ISI

$$\underline{Y}_k = \underline{H}_k \underline{x}_k + \underline{\eta}_k \ , \quad k=0,1,\ldots,N-1, \quad (1)$$

where $\underline{H}_k$ is the *M* by *M* blocked version of the original frequency responses of the ISI *H(z)*:

$$\underline{H}_k = \underline{H}(e^{j2\pi k/N}),$$

$$\underline{H}(z) = \begin{bmatrix} H_0(z) & z^{-1}H_{M-1}(z) & \cdots & z^{-1}H_1(z) \\ H_1(z) & H_0(z) & \cdots & z^{-1}H_2(z) \\ \vdots & \vdots & \vdots & \vdots \\ H_{M-1}(z) & H_{M-2}(z) & \cdots & H_0(z) \end{bmatrix}$$

$$H_m(z) = \sum_{l=0}^{\tilde{L}'} h(Ml+m)z^{-l}, \quad 0 \le m \le M-1.$$

$$\tilde{L}' = \left\lfloor \frac{L}{M} \right\rfloor$$

*m*th polyphase component of *H(z)*

Vectorized Channel Example

If $H(z) = 1 + 0.9z^{-1} - 0.8z^{-2} + 0.6z^{-3} + 0.5z^{-4} - 0.4z^{-5}$, vector size M=2,

then, its polyphase components are

$$H_0(z) = 1 - 0.8z^{-1} + 0.5z^{-2}, \quad H_1(z) = 0.9 + 0.6z^{-1} - 0.4z^{-2}$$

and the vector channel coefficient matrices are

$$L = 5$$

$$\tilde{L} = \left\lceil \frac{L}{M} \right\rceil = \left\lceil \frac{5}{2} \right\rceil = 3$$

$$\underline{H}(z) = \begin{bmatrix} H_0(z) & z^{-1}H_1(z) \\ H_1(z) & H_0(z) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0.9 & 1 \end{bmatrix} + \begin{bmatrix} -0.8 & 0.9 \\ 0.6 & -0.8 \end{bmatrix} z^{-1} + \begin{bmatrix} 0.5 & 0.6 \\ -0.4 & 0.5 \end{bmatrix} z^{-2} + \begin{bmatrix} 0 & -0.4 \\ 0 & 0 \end{bmatrix} z^{-3}$$

Why VOFDM Is Good for Time Varying Channels

- The vectorized channel matrix $\underline{H}(z)$ is pseudo-circulant and can be diagonalized by *M*-point DFT matrix $\mathbf{W}_M$ with a diagonal phase shift matrix $\Lambda(z) = \mathrm{diag}(1, z^{-1}, \cdots, z^{-M+1})$ as follows:

$$\underline{H}(z^M) = \left(\mathbf{W}_M^* \Lambda(z)\right)^{-1} \mathrm{diag}(H(z), H(zW_M), \cdots, H(zW_M^{M-1}))\mathbf{W}_M^* \Lambda(z)$$

where $W_M = e^{-j\frac{2\pi}{M}}$. Thus, matrices $\underline{H}_k = \underline{H}(W_N^{-k})$ can be diagonalized by $\mathbf{W}_M^* \Lambda(W_N^{-k})$, *k=0,1,…,N-1*.

- The receiver equation (1) becomes

$$\underline{\tilde{Y}}_k = \underline{\mathrm{diag}(H(W_{MN}^{-k}), H(W_{MN}^{-k} W_M), \ldots, H(W_{MN}^{-k} W_M^{M-1}))} \; \underline{\mathbf{W}_M^* \mathrm{diag}(1, W_{MN}^{k}, \ldots, W_{MN}^{k(M-1)})} \underline{x}_k + \underline{\tilde{\eta}}_k$$

This frequency domain part is similar to the channel in time domain for single antenna systems
Or
diagonal space-time coded MIMO systems

This part is similar to the precoding to achieve signal space diversity for time-varying channel **Or**
diagonal space-time block coding to achieve spatial diversity

When channel varies with Doppler spread, it can collect multipath diversity and/or signal space diversity. This can be seen later even with the MMSE receiver.

Delay Doppler Channels

- For a path of delay τ, the path gain at time *t* is

$$h(\tau, t) = g(\tau)e^{-j\Omega(\tau)t},$$

where $\Omega(\tau)$ is the Doppler shift and a function of delay τ.

- In this case, the received signal is

$$y(t) = \int g(\tau)s(t-\tau)e^{-j\Omega(\tau)t}\mathrm{d}\tau + n,$$

where $s(t)$ is the transmitted signal.

Delay Doppler Channels

- When all the Doppler shifts $\Omega(\tau)$ for all the paths are the same (or approximately the same) for all delays τ, i.e., $\Omega(\tau)=\Omega$, is a constant, then the received signal model becomes

$$
\begin{aligned}
y(t) &= \int g(\tau)s(t-\tau)e^{-j\Omega t}\mathrm{d}\tau + n \\
&= e^{-j\Omega t}\int g(\tau)s(t-\tau)\mathrm{d}\tau + n
\end{aligned}
$$

  - In this case, the Doppler effect $e^{-j\Omega t}$ can be compensated at either transmitter or receiver and then the channel becomes the conventional time spread only channel. It is called *trivial Doppler spread,* otherwise, *non-trivial Doppler spread.*

Delay Doppler Channels

- For a non-trivial Doppler spread channel, its Doppler shifts $\Omega(\tau)$ for different path delays τ are different. For example, let us consider the case of linear in terms of τ:

$$\Omega(\tau) = \Omega\tau$$

- Then, the received signal model without considering the noise is

$$y(t) = \int g(\tau)s(t-\tau)e^{-j\Omega\tau t}\mathrm{d}\tau$$

- Comparing with the short-time Fourier transform

$$\int g(\tau)s(\tau - t)\mathrm{e}^{-j\Omega\tau}\,\mathrm{d}\tau$$

Received Signal over Delay Doppler Channel with Non-Trivial Doppler Spread

No matter what a transmitted signal *s*(*t*) is,
it cannot do much about the Doppler spread !!

$$\int g(t-\tau)s(\tau)\mathrm{e}^{-j\Omega\tau t}\,\mathrm{d}\tau\ \ e^{-j\Omega t^2}$$

Transmitted signal

Doppler

Delay Doppler Channels

- For a non-trivial Doppler spread channel
  - the received signal form is much different with STFT that is a typical joint time-frequency analysis tool in signal processing
  - It is not possible to compensate the Doppler effect at either transmitter or receiver
  - In fact, STFT corresponds to GFDM

- Neither GFDM nor VOFDM/OTFS can deal with a non-trivial Doppler spread channel well

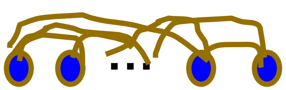
X.-G. Xia, "Delay Doppler transform," *IEEE Wireless Commun. Letters*, June 2024.
X.-G. Xia, "Rethink delay Doppler channels and time-frequency coding," *Journal of Information and Intelligence*, vol. 3, no. 3, pp. 189-193, May 2025.

VOFDM, OFDM, SC-FDE

- When M=1, VOFDM=OFDM
- When M>L and the FFT size N is 1, VOFDM=SC-FDE:
  - at the transmitter, no IFFT is implemented (so the PAPR is not changed) but just CP of the information symbols is inserted; low PAPR.
  - at the receiver, both FFT and IFFT, and frequency domain equalizer are implemented.
- VOFDM is a bridge between OFDM and SC-FDE.
  - Its ML receiver complexity is also in the middle.

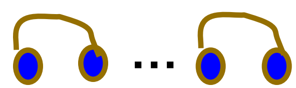
Time domain single carrier vs. equalization

Maximum # symbols in ISI



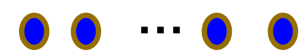
VOFDM

No, or 2, or 3, …, or Maximum # (**you choose**) symbols in ISI



Frequency domain OFDM

No ISI



Single antenna VOFDM is in the middle between single carrier and OFDM in terms of dealing with ISI

VOFDM: Some Other Advantages

- Cyclic prefix data rate overhead reduction when the FFT/IFFT size is fixed
  - For OFDM, it is $\frac{L}{N}$
  - For VOFDM, it is $\frac{L}{MN}$

- For fixed cyclic data rate overhead, the FFT/IFFT size can be reduced by M times
  - The IFFT size reduction reduces the peak-to-average power ratio (PAPR), which is important in cellular communications.
- Scalable to a bandwidth: IFFT/FFT size can be fixed.

Diversity Order of MMSE and ZF Receivers

- Definition of the diversity order

D=L

$$d\left(R,M,D,N\right) = -\lim_{\rho\to\infty}\frac{\log P_{ser}(R,M,D,N)}{\log\rho}.$$

  - *R* is the spectrum efficiency defined as bits/sec/Hz

- **Theorem** 5: *For MMSE-V-OFDM, the diversity* $d^{MMSE}(R,M,D,N)$ *equals*

the diversity order

$$d^{MMSE}(R,M,D,N) = \min\left\{\lfloor M2^{-R}\rfloor, D\right\} + 1.$$

- **Theorem** 6: *For ZF-V-OFDM, the diversity order* $d^{ZF}(R,M,D,N) = 1.$

Yabo Li, Ngebani, Xia and Host-Madsen, *IEEE Trans. on Signal Processing*, Oct. 2012

Summary on VOFDM

- VOFDM can be used either to reduce the PAPR by reducing the IFFT size while at a fixed the CP data rate overhead; or reduce the CP data rate overhead while at a fixed the IFFT size.
- VOFDM provides a tradeoff between the receiver complexity, performance, PAPR, CP overhead for an ISI channel.
- VOFDM is in the middle between single carrier and OFDM systems in terms of dealing with ISI, and scalable to bandwidth.
- Good for channels with both time and Doppler spreads
- The transmitted signals of OTFS and VOFDM are identical.
  - **VOFDM was also studied over time-varying channels in Section 7.4:**

  X.-G. Xia, *Modulated Coding for Intersymbol Interference Channels*, New York, Marcel Dekker, 2000.

  When the receiver criterion is fixed, such as ML, the receivers of VOFDM and OTFS are the same as well despite a channel.
- CDD VOFDM for multi-antennas can collect both spatial and multipath diversities, where CDD OFDM is not be able to do so in a large bandwidth system.

Another Thought for 6G Modulation

- For 6G, its bandwidth may be very wide.
- The ISI channel will be very long.
- Will it be possible to use a small size infinite impulse response (IIR) channel to approximate such a long channel?
- I recently proposed an OFDM system for IIR channels
  - X.-G. Xia, "A new OFDM system for IIR channels," *IEEE Wireless Comm Letters*, May 2023.

R. Khanzadeh, S. Angerbauer, A. Springer, and W. Haselmayr, "End-to-end learning of communication systems with novel data-efficient IIR channel identification," *Proc. Asilomar Conf.*, 2023.

Modulation and Bandwidth

- Wireless Communications Can Be Categorized as
  - Narrowband: both TDMA and CDMA work well
    - 2G Complexity: *N*
  - Low wideband: CDMA
    - 3G Complexity: *N*
  - Wideband: OFDM
    - 4G, 5G Complexity: *N* log *N*
  - High wideband: VOFDM (it is scalable with bandwidth) ?
    - 6G? Complexity ? Flexible (vector size 1, it is OFDM vector size >L, it is SC-FDE)

The larger vector size, the better performance, the higher complexity

Correspondence between DSP and Modulations

Digital Signal Processing Concepts	Digital Communications Modulation Concepts
DFT filterbanks	OFDM
Cosine modulated filterbanks	OQAM-OFDM
Multirate filterbanks	FBMC/Transmultiplexers
Discrete Gabor transforms/expansions	GFDM
Vector DFT filterbanks	VOFDM/OTFS
Discrete Fractional Fourier transform (FrFT) Discrete Chirp-Fourier Transform (DCFT) Linear canonical transform	AFDM/OCDM

Thank you!