Assignment 5

ECE 697 – Delta-Sigma Data Converter Design (Spring 2010)

Due on Tuesday, May 4, 2010.

Problem 1 Continuous-time $\triangle\Sigma$ Modulator with Butterworth NTF:

1. Determine the NTF of a third-order $\Delta\Sigma$ modulator with a Butterworth high-pass response and an out of band gain (OBG) equal to 3. For the Butterworth response, all the NTF zeros are at DC (i.e. $z = 1$). Assume that the design is normalized to a clock frequency (f_s) of 1 Hz.
2. The modulator is to be implemented using the **CIFB** architecture. Also, assume in this problem that the excess loop delay (ELD) is equal to zero. What is the loop-filter response $L(z) \triangleq L_1(z)$ seen by the quantization noise ?
3. Plot the impulse responses, $l(n)$ of the loop-filter seen by the quantization noise, and $h(n)$ of the NTF(z). Also show the respective pole-zero plots.
4. Now, using the forward impulse-invariant transform table, Table II in [1], analytically find the transfer function of the continuous-time loop-filter $\hat{L}(s)$ for the **NRZ** DAC pulse-shape. Sketch the ideal block-level implementation of the resulting CT $\triangle\Sigma$ modulator. Plot the frequency response of $\hat{L}(s)$ using MATLAB.
5. Drive the designed loop-filter $\hat{L}(s)$ with the DAC pulse response in time-domain and plot the continuous-time open-loop response $l(t)$. Show that the sampled loop-response (with $T_s = 1$) is same as the impulse response $l[n]$ seen in part (3).
   - (a) From this observation, what can you conclude about the NTF of the overall CT DSM?
   - (b) Is it overall a discrete-time or a continuous-time system? Can you ever write NTF and STF responses as $NTF(s)$ and $STF(s)$ respectively?
6. Determine the $STF(e^{j\omega})$ for your design and sketch the spectrum. Can you plot the STF spectrum using MATLAB?
7. Explore the MATLAB $\triangle\Sigma$ toolbox function **realizeNTF_ct** used for synthesizing CT $\triangle\Sigma$ modulators.
   - (a) Verify your answer in part (4) with the result using the toolbox. What algorithm does the toolbox function use for determining the ABCD matrix description of $\begin{bmatrix} L_0(s) & L_1(s) \end{bmatrix}^T$.
   - (b) Use **mapCtoD** toolbox function to ensure that $L(s)$ from parts (4&6) results in the desired $NTF(z)$.
8. Repeat parts (4) to (7) for a **RZ** DAC pulse-shape given by $r_{DAC}(t) = (0, 0.5)$.

Problem 2 Excess-Loop Delay in CT $\triangle\Sigma$ Modulators:

1. Determine the NTF of a fourth-order $\Delta\Sigma$ modulator with a Butterworth high-pass response and an out of band gain (OBG) equal to 2.5. Assume that the design is normalized to a clock frequency (f_s) of 1 Hz. Plot the NTF response and the pole-zero plot.
2. Assuming an **NRZ** DAC pulse-shape and a **CIFF** architecture, determine the loop-filter coefficients $\{k_1, k_2, k_3, k_4\}$ for the continuous-time loop-filter $\hat{L}(s)$. Initially assume that the excess loop delay (τ) is zero. Sketch the loop-filter block diagram. You may use MATLAB to do this computation.
3. Plot the CT and sampled loop-responses $l(t)$ and $l[n]$ respectively for the CT-DSM.
4. Determine the $STF(e^{j\omega})$ for your design and plot the spectrum. Can you explain the out of band peaking in the STF?
5. Now, due to design considerations an excess loop delay of 40% of the clock-period is introduced (i.e. $\tau = 0.4$).
   (a) Find the equivalent $NTF(z)$ and plots its response and the pole-zero plot. What do you observe?
   (b) Plot $l(t)$ and $l[n]$ for the CT-DSM with the excess loop-delay.
6. Find a method to compensate for the ELD so that the NTF is restored to the response seen in part (1).
   (a) Use analytical calculations to determine the resulting loop-filter coefficients. You will realize that the usual method of computing the modified loop-filter coefficients using the Table III in [1] is unwieldy for this design. You may use the algorithm introduced in [2].
   (b) Show that the direct path in the loop-filter arising due to the ELD compensation, can be realized as a direct feedback path around the quantizer.
   (c) Verify your hand-calculations with the toolbox function **realizeNTF_ct.**
   (d) Plot $l(t)$ and $l[n]$ for the CT-DSM, and demonstrate that the ELD is fixed.
7. Repeat parts (3) to (6) for a **RZ** DAC pulse-shape given by $r_{DAC}(t) = (0, 0.5)$ with $\tau = 0.4$. Do you need the direct path around the quantizer ?

References

[1] J. Cherry and W. Snelgrove, "Excess loop delay in continuous-time delta-sigma modulators," *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing*, vol. 46, no. 4, pp. 376–389, 1999.

[2] S. Pavan, "Excess Loop Delay Compensation in Continuous-Time Delta-Sigma Modulators," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 55, no. 11, pp. 1119–1123, 2008.