Assignment 4

ECE 697 – Delta-Sigma Data Converter Design (Spring 2010)

Due on Tuesday, March 23, 2010.

Problem 1 Maximum stable amplitude (MSA) Estimation:

1. Create and add a new MATLAB function to your personal $\Delta\Sigma$ toolbox titled **estimateMSA**, similar to the toolbox function **simulateSNR**. Use the MSA estimation algorithm suggested by Lars Risbo using the slow input ramp as discussed in class. The function should return the MSA and the peak SNR. The script should also optionally produce the $log\,|y[n]|$ vs normalized input amplitude (u/FS) curve for your future design reports.
2. Write a script using these functions to characterize a given modulator resulting in a plot containing SNR and SNDR (in dB) vs the input amplitude (in dBFS) (See Assignment 2 for an example plot). The plot should also clearly mark the peak-SNR, peak-SNDR, SFDR, input amplitude for peak SNR and the Dynamic Range. Use this script for all your designs henceforth.
3. Demonstrate your function **estimateMSA** for second- and third-order NTFs of your choice.
4. Write a script to generate a plot for MSA vs OBG (i.e. out of band gain) for a third-order NTF with 3-bit, 4-bit and 5-bit quantizer. See the example plot in Figure 1. You may use the *synthesizeNTF* function for this problem. Explain your observations in the generated plot and save this script for your future use.
5. Compare the plots generated in part (4) with the loose-bound on MSA derived in the class and given by

$$u_{max} = \max_n |u(n)| \leq \frac{\triangle}{2}\left(M + 2 - \|h\|_1\right) \tag{1}$$

where Δ is the LSB size, M is the number of steps in the quantizer ($= nLev - 1$), and $\|h\|_1$ is the l_1 norm of the NTF impulse response.

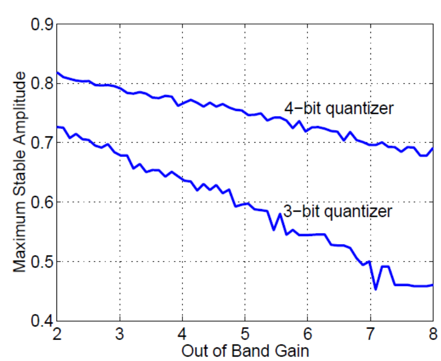

Figure 1: MSA vs OBG plot for a third-order NTF.

Problem 2 Butterworth NTF Design: Using a Butterworth highpass filter response,

1. Determine the NTF of a fourth-order $\Delta\Sigma$ modulator with an out of band gain (OBG) equal to 3. For the Butterworth response, all the NTF zeros are at DC (i.e. $z = 1$).
2. Plot the impulse response, $h(n)$ of the NTF and show that the NTF is realizable (i.e. there are no zero-delay loops). Also show the pole-zero plot for the designed NTF.
3. Assuming a 5-bit (32-level) quantizer and an OSR=16, write a MATLAB code to simulate the $\Delta\Sigma$ modulator with the NTF designed in part (1).
4. Using your **estimateMSA** function created in problem 1, estimate the maximum stable amplitude (MSA) for the design in part (1) of this problem. Determine the peak in-band SQNR in dB. Show the relevant spectrum plots with an appropriately sized Hann window.

Hint: For help on scripting the iteration on ω_N of the Butterworth high-pass NTF, to achieve a specified OBG, you may want to look at the code of the toolbox function **synthesizeNTF0**.

Problem 3 Complex NTF Zeros: This problem explores the NTF synthesis algorithm using NTF zero optimization and maximally flat poles.

1. Design a fifth-order (i.e. $L = 5$) NTF with optimized zero locations for minimizing the in-band quantization noise for a OSR=16. Let the all-zero NTF be given by

$$NTF_0 = \prod_{i=1}^{L} \left(1 - z_i z^{-1}\right) \tag{2}$$

   where z_i's are the optimized NTF zero locations while ensuring that $NTF_0(\infty) = 1$. Plot the frequency response of NTF_0. What is the OBG ? Will this NTF lead to a stable $\Delta\Sigma$ modulator with a reasonable MSA?

2. In order to reduce the OBG of the NTF to an acceptable value of OBG=3, we now introduce poles into the signal band. For a simple synthesis procedure, the poles can be added independently to the zero locations, given that the resulting response is maximally-flat in the signal band. Design a maximally-flat, all-pole, low-pass transfer function $\frac{1}{P(z)}$ (i.e. $P(z)$ is the denominator with the poles as its roots) such that the resulting NTF given by

$$NTF(z) = \frac{NTF_0(z)}{P(z)} \tag{3}$$

   has an OBG=3. Ensure that your NTF is realizable, i.e. $NTF(\infty) = 1$. Plot the frequency response of $NTF(z)$ superimposed with that of $\frac{1}{P(z)}$ and $NTF_0(z)$. Also show the pole-zero plots for these transfer functions.

   Hint: Use the following MATLAB function for generating an all-pole, maximally-flat, low-pass response :

```
[b,a] = maxflat(0,order,Wn)
```

3. Plot the impulse response, $h(n)$ of the NTF and show that the NTF is realizable (i.e. $h[0] = 1$).

4. Assuming a 4-bit (16-level) quantizer and an OSR=16, simulate the ΔΣ modulator with the NTF designed in part (2).

5. Using your **estimateMSA** function created in problem 1, estimate the maximum stable amplitude (MSA) for this design. Determine the peak in-band SQNR in dB. How much improvement is this over a design with all zeros at $z = 1$? Show the relevant spectrum plots with an appropriately sized Hann window.

6. Repeat part (2) for an OSR=4. Is the frequency response of $\frac{1}{P(z)}$ maximally-flat in the signal band ? Will this design yield minimum in-band quantization noise (IBN) with the pre-selected optimal NTF zeros ?

   Note: Simultaneous optimization of NTF zeroes and poles for low-OSR designs is still an open problem and has not been implemented in the ΔΣ toolbox.

Problem 4 Chebyshev NTF Design: This problem explores the use of inverse-Chebyshev high-pass response for NTF design.

1. Explore the MATLAB function for generating an inverse-chebyshev high-pass response.

```
[b,a] = cheby2(order,R,Wst, 'high')
```

   The inverse-chebyshev transfer function uses three parameters, viz, filter order, stop-band attenuation R and the stop-band edge-frequency ω_{st}.

2. Using an inverse-Chebyshev highpass filter response, determine the NTF of a fifth-order ΔΣ modulator with OSR=16, and OBG=3. You can achieve this by using multiple design approaches:

   (a) Use a fixed $\omega_{st} = \frac{\pi}{OSR}$, and then iterate upon R to get the desired OBG.

   (b) Use a fixed R (say equal to 60 dB), and then iterate upon ω_{st} to achieve the desired OBG.

   (c) Use optimization algorithms to find the optimal value of ω_{st} and R for a given OBG and with minimum in-band noise.

   Design approaches (a) and (b) are recommended in this problem. You may try method (c) offline as a research problem.

   It is also important to realize that the NTF zero locations for the inverse-Chebyshev high-pass response are not optimal (for a large OSR).

3. Plot the impulse response(s), $h(n)$ of the NTFs and show that the designed NTFs are realizable (i.e. there are no zero-delay loops). Also show the pole-zero plots for the NTFs.

4. Assuming a 4-bit (16-level) quantizer and an OSR=16, write a MATLAB code to simulate the ΔΣ modulator with the NTFs designed in part (2).

5. Using your **estimateMSA** function created in problem 1, estimate the maximum stable amplitude (MSA) for the design(s) in part (2) of this problem. Determine the peak in-band SQNR in dB. Show the relevant spectrum plots with an appropriately sized Hann window.

Problem 5 NTF Synthesis for low-OSR : This problem explores the limitations of the $\Delta\Sigma$ toolbox NTF synthesis algorithms.

1. Explore the following MATLAB $\Delta\Sigma$ toolbox functions for NTF synthesis and the toolbox demos (dsdemo1 to dsdemo4 and dsexample1) :

```
ntf = synthesizeNTF(order,OSR,opt,OBG,f0)
ntf = synthesizeChebyshevNTF(order,OSR,opt,OBG,f0)
```

   (a) What are the possible values for the optimization parameter 'opt' and what do they stand for ?

   (b) What happens if no value for the parameter OBG (or H_inf) is passed in the function call ?

   (c) What does the function **rmsGain()** do ? Show that the in-band quantization noise can be expressed as

$$IBN = \frac{\sigma_H^2 \cdot \sigma_e^2}{OSR} \tag{4}$$

   where σ_H is the rms noise gain in the signal-band, and $\sigma_e^2 = \frac{\Delta^2}{12}$ is the total quantization noise.

2. Design NTF of a fifth-order $\Delta\Sigma$ modulator with OSR=16, and OBG=3 using both the synthesis functions (**synthesizeNTF** and **synthesizeChebyshevNTF**) . Plot the magnitude responses and pole-zero plots for both the NTFs and compare them. Compute and compare the rms gains for both the NTFs. Which of the two synthesis functions performs better and why ?

3. Repeat part (2) for OSR=8 and OSR=4. What do you conclude from these experiments ?

4. Repeat (2) for a lower out of band gain value of OBG=1.2. What do you observe?

Bonus problem: Understand what the toolbox function **clans()** does (See reference [2] for CLANS synthesis). Design NTF of a fifth-order $\Delta\Sigma$ modulator with using the CLANS synthesis for OSR=16 and OSR=4. Use appropriate values for the function inputs Q, r_{max} and *opt*. How does the resulting performance from the CLANS synthesis method compare against the synthesizeNTF and synthesizeChebyshevNTF functions ?

Problem 6 Bode's Sensitivity Theorem : We know that for a stable and minimum-phase NTF (all poles and zeros inside the unit circle), the Bode Sensitivity Theorem states that

$$\int_0^{\pi} log\left|NTF\left(e^{j\omega}\right)\right| \cdot d\omega = 0 \tag{5}$$

which implies that the total area bounded between the log-magnitude of the NTF and the 0-dB line is zero. This can also be interpreted as that the areas above the 0-dB line and below the 0-dB line are equal.

1. Demonstrate the validity of the Bode's Sensitivity Theorem for a fifth-order NTF with OSR=16, and OBG=3 synthesized using the toolbox. Show the two areas (above and below the 0-dB line) in the NTF spectrum using the **area()** command and using different colors.

2. The amount of 'wiggling' in the time-domain output of a $\Delta\Sigma$ modulator, assuming a low-frequency input, can be defined as

$$\delta v[n] = v[n] - v[n-1] \tag{6}$$

which in z-domain is given by

$$\begin{aligned} \delta V(z) &= V(z) - V(z-1) = (1 - z^{-1})V(z) \\ &= (1 - z^{-1})STF(z)U(z) + (1 - z^{-1})NTF(z)E(z) \\ &\approx (1 - z^{-1})NTF(z)E(z) \end{aligned} \tag{7}$$

Simulate the NTF in part (1) using a 5-bit (32-level) quantizer and a sinusoidal input with MSA and OBG=2, 3 and 4. Plot the $\delta v[n]$ waveforms for the different OBG values. How does the amount of wiggling change as the OBG is increased ?

3. Using Bode's Sensitivity Theorem, show that

$$\int_0^{\pi} log\left|\left(1 - e^{-j\omega}\right) \cdot NTF\left(e^{j\omega}\right)\right| \cdot d\omega = 0 \tag{8}$$

Plot the area above and below the 0-dB line for integral in Eqn. 7. The variance of the wiggling ($\sigma^2_{\delta v}$) is exponentially related to the area above the 0-dB line (C) as [3]

$$\sigma^2_{\delta v} \geq \frac{\triangle^2}{12\pi}\left(\pi - \omega_1\right) exp\left(\frac{2C}{\pi - \omega_1}\right) \tag{9}$$

where ω_1 is the 0-dB cross-over frequency as shown in Figure 2. What happens to the area C as the OBG is increased ? Can you interpret the trend in the variance of wiggling ($\sigma^2_{\delta v}$) as the OBG is increased ?

Note: As we will see later in the course, the variance of the time-domain wiggling ($\delta v[n]$), given by $\sigma^2_{\delta V}$, contributes to in-band jitter-noise in the continuous-time implementation of the NTF. Also for multi-bit quantizers, $\sigma^2_{\delta V}$ is a good estimator of the relative time-constants in the modulator loop-filter.

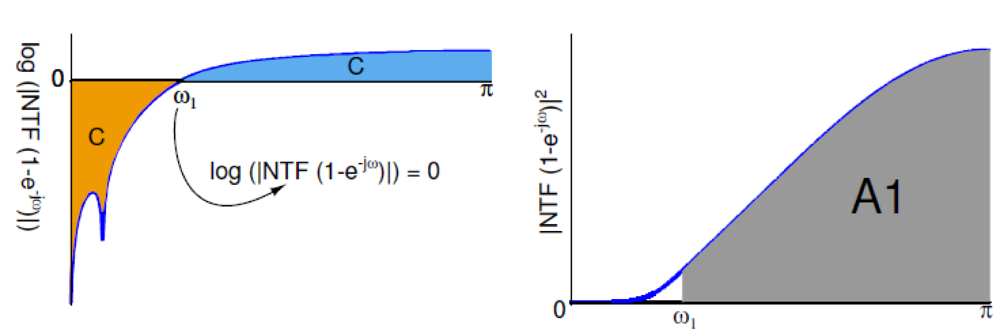

Figure 2: Log-magnitude and magnitude plots for $NTF\left(e^{j\omega}\right) \cdot \left(1 - e^{-j\omega}\right)$. Here, ω_1 is the 0-dB cross-over frequency and the area A_1 corresponds to the wiggling variance $\sigma^2_{\delta V}$ [3].

References:

[1] S. Pavan, N. Krishnapura, *EE658: Data Conversion Circuits Course*, IIT Madras. Available: [Online] http://www.ee.iitm.ac.in/vlsi/courses/ee658_2009/start.

[2] J. G. Kenney and L. R. Carley, "Design of multibit noise-shaping data converters," *Analog Integrated Circuits Signal Processing Journal*, vol. 3, pp. 259-272, 1993.

[3] K. Reddy, S. Pavan, "Fundamental Limitations of Continuous-time Delta-Sigma Modulators due to Clock Jitter," *IEEE TCAS-I*, vol. 54, no. 10, pp. 2185-2194, Oct. 2007.