Assignment 2

ECE 697 – Delta-Sigma Data Converter Design (Spring 2010)

Due on Thursday, February 18, 2010.

Problem 1 DTFT and DFT:

1. Consider the sequence

$$r[n] = \begin{cases} 1, & 0 \leq n \leq N \\ 0, & otherwise \end{cases} \tag{1}$$

   (a) Sketch $r[n]$, determine the DTFT $R\left(e^{j\omega}\right)$ and sketch its magnitude and phase.

   (b) Find DFT $R[k]$ of the sequence $r[n]$ and sketch its magnitude and phase response. Show that $R[k]$ can be obtained by sampling $R\left(e^{j\omega}\right)$ in frequency domain as $\omega = \frac{2\pi}{N}$.

   (c) Confirm your DTFT and DFT plots with MATLAB for $N = 8$.

   (d) Notice that $r[n]$ is a ractangular window. What is the mainlobe width and the first sidelobe depth ?

   Hint:

```
r = [1 1 1 1 1 1 1 1];  % 8-point rect signal
fvtool(r);              % plot DTFT
R=fft(r)                % find DFT
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Problem 2 Spectral Windows: Some commonly used FFT spectral windows available in MATLAB are Hann (or Hanning), Bartlett, Hamming, Blackman and Blackman-Harris.

1. Look up the MATLAB documentation on spectral windows (*command: doc window*). List the equations of the discrete sequences $w[n]$ for each of these windows.

2. Using the command *wvtool(hann(N), blackman(N),...., rectwin(N))* overlay the time-domain and frequecy domain plots for all of the windows, along with the rectangular window (rectwin). Here, the variable $N = 64$ is the window length.

3. Compare the first side-lobe depth of these windows and rank them in the increase order of side-lobe suppression.

4. Find the number of non-zero FFT bins for these windows, including the rectangular window.

5. Which of these windows will you prefer for plotting FFT of the results obtained from a simulation (where the ratio of the input and sampling frequency can be precisely controlled)? Which window is the best for processing experimental results where the relation between input and sampling frequency can not be precisely controlled? Explain.

6. In the text book, there is another window called $Hann^2$ window which is given by

$$w[n] = \begin{cases} \left(\frac{1-cos\left(\frac{2\pi n}{N}\right)}{2} \right)^2, & 0 \le n \le N \\ 0, & otherwise \end{cases} \tag{2}$$

Using MATLAB, compare the $Hann^2$ window with the Hann window. How much is the first side-lobe suppression? What are the number of non-zero FFT bins? Is this window better than Blackman-Harris window for side-lobe suppression?

Problem 3 Quantization error distribution: We discussed in the class that the quantization error ($e = v - y$) can be modeled as a random process (i.e. noise) with a uniform distribution given as $e \sim U\left[-\frac{\Delta}{2}, \frac{\Delta}{2}\right]$ where Δ is the LSB size. In this problem, we will study the range of validity of this assumption.

1. Drive a 4-bit quantizer with a full-scale sine wave ($A = FS$), while ensuring that the input and the sampling frequency are rationally related (i.e. $f_{in} = \frac{m}{N_{FFT}} f_s$, $m, N \in I$ and m and N_{FFT} are mutually prime). Choose a large value of N_{FFT}, say $N_{FFT} = 2^{13}$. The input to the quantizer is y and the quantized output is v. The quantization error is $e = v - y$. Divide the time-domain quantizer error $e[n]$ into 100 bins and plot a histogram. Is the histogram uniform ?
2. Repeat (1) for a 6-bit, 8-bit, 10-bit and 12-bit quantizer. How does the histogram (which approximates the PDF of e) behave as the quantizer resolution if increased ? Intuitively explain your observations.
3. Repeat (1) for overloading input amplitudes when $\frac{A}{FS}$= 1, 1.2, 1.5 and 2.0. What do you observe ?

Problem 4 Quantization noise spectrum: In our simple model of the quantization noise, we assumed that the quantization noise is uncorrelated with the input and is *white* with a flat PSD in the frequency band $f\epsilon[-\frac{f_s}{2}, \frac{f_s}{2}]$ where f_s is the sampling frequency. Here, we will study this assumption.

1. Drive a 4-bit quantizer with a full-scale sine wave, while ensuring that the input and the sampling frequency are rationally related. To observe the spectral properties of the quantizer choose a large value of N_{FFT}, say $N_{FFT} = 2^{10}$ and set the input tone bin to $m = 1$. The input to the quantizer is y and the quantized output is v. The quantization error is $e = v - y$. Plot the time-domain plots for y,v and the quantization error v for one cycle of the input and observe the periodicity of $e[n]$ (see Fig. 1). Plot magnitude of the FFT of $e[n]$ given by $E[k]$. Is the quantization noise spectrum uniformly distributed in the frequency band $[-\frac{f_s}{2}, \frac{f_s}{2}]$? Can you explain the 'tonal' behavior of the quantization noise spectrum ?

2. From the time-domain plot (1) show that the quasi-periodic quantization error waveform has a number of cycles given roughly by

$$n_{cyc} = \frac{4A}{\triangle} \tag{3}$$

where A is the amplitude of the input sine wave and $\triangle$ is the LSB size of the quantizer. Observe that these number of cycles are related to the bin locations of the dominant tones in the FFT $E[k]$. Argue that these tones will be located near the frequency given by $f_{tone} = \frac{n_{cyc}}{N_{FFT}} f_s$.

3. Repeat (1) for a 6-bit, 8-bit, 10-bit and 12-bit quantizer. How does the quantization noise spectrum behave as the quantizer resolution if increased ? Comment on the bin location of the dominant tones n_{cyc} in each case.

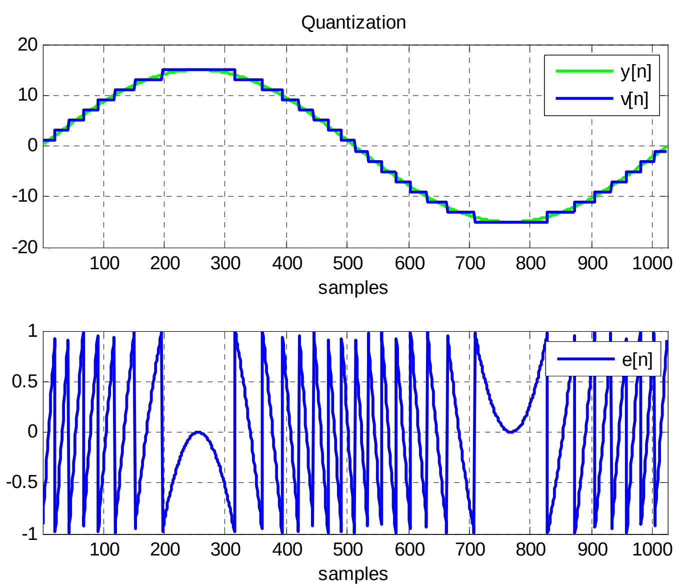

Figure 1: Quantization error waveform

Problem 5 SQNR calculation: You are given an ideal quantizer with N-bits of resolution. In your test setup, the quantizer is driven by a full-scale sine wave input with a frequency within the Nyquist bandwidth. You have captured 1024 output data points of the quantizer, and expressed the output as a Discrete Fourier Series (DFS) computed using the FFT algorithm. The magnitudes of the DFS coefficients are plotted after normalizing with the input tone magnitude (peak at 0 dBFS), on a log scale. The resulting plot is shown in Figure 2. In the plot, assume that the quantization noise floor is uniform at -63 dBFS.

1. If the sampling rate $f_s$was 100 MHz, find the frequency of the sine-wave input f_{in}.

2. Estimate the resolution of the quantizer N. Show your calculation steps clearly.

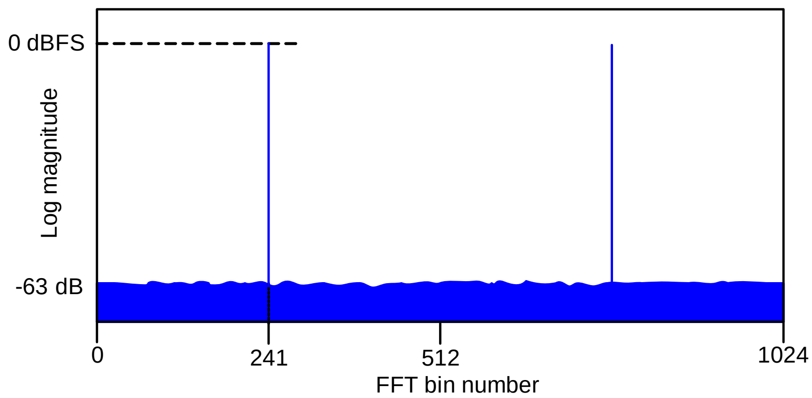

Figure 2: DFS log-magnitude plot.

Problem 6 SQNR estimation using MATLAB: Using MATLAB compute the peak Signal to Quantization Noise ratio for quantizers with resolutions of 2, 4, 6, 8, 10, 12 bits. Ensure that that the input and the sampling frequency are rationally related. Choose the FFT size of the form $N_{FFT} = 2^p$ for faster DFT computation. Make sure you add the FFT coefficient power in the signal and noise bins separately to compute the SQNR. How do the computed SQNR values compare with the relation

$$SQNR = 6.02N + 1.76 \; dB \tag{4}$$

derived in class ?

Problem 7 Quantizer SFDR: Consider an ideal N-bit quantizer with N ranging from 4 to 14, and driven by a full-scale sine wave. Find the ratio of the powers of the fundamental and the largest spur in the quantizer spectrum. This is called the Spurious Free Dynamic Range (SFDR). For consistency, use $N_{FFT} = 2^{15}$ with input at approximately $f_s/4$ (ensuring that the input and sampling frequency are rationally related).

1. Plot SFDR (in dB) versus N. Can you explain the slope of the curve ?
2. In your MATLAB code, introduce a compressive non-linearity just before a 10-bit quantizer with its normalized input-output relationship given by

$$y = A\left(\frac{x}{A} - 0.001\left(\frac{x}{A}\right)^3\right) \tag{5}$$

where A is the full-scale amplitude. Use $N_{FFT} = 2^{15}$ with input at approximately $f_s/8$ (ensuring that the input and sampling frequency are rationally related). What is the SFDR in the presence of the non-linearity? Identify the location of the third-harmonic tone. Can you hand-calculate this distortion limited SFDR using Eqn. 5 ?

Hint: $sin^3\theta = \frac{1}{4}sin3\theta - \frac{3}{4}sin\theta$

Problem 8 Dynamic Measurements: Figures 3 and 4 show the test results for a wideband continuous-time delta-sigma modulator recently published in literature (reference to be provided later). From these two plots, find the approximate values of the peak-SNR, peak-SNDR, SFDR, dynamic-range (DR) and the input signal level for the peak-SNR.

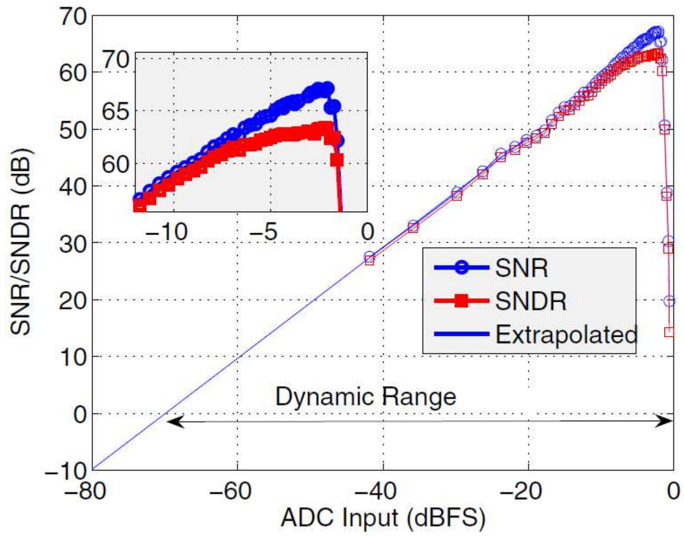

Figure 3: Measured SNR, SNDR and dynamic range for a wideband CT DSM.

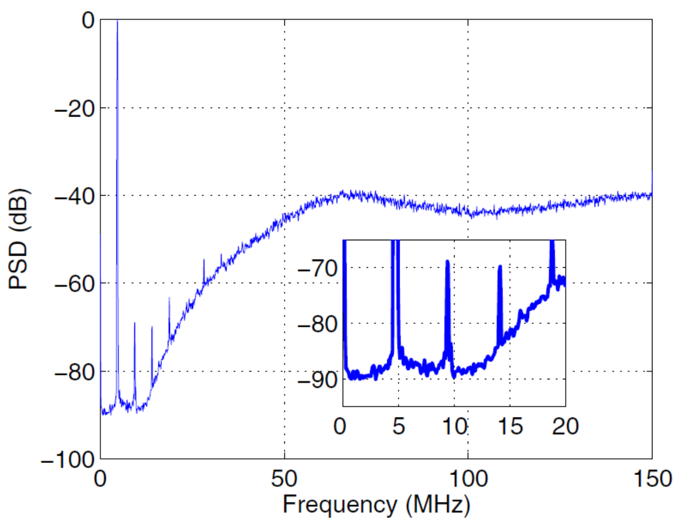

Figure 4: PSD of the tested CT DSM for a 4.7 MHz input tone with the maximum stable amplitude (MSA).

Bonus Problem: Hann Window: Continued from problem 1.

1. Consider the sequence

$$w[n] = \begin{cases} \frac{1}{2}\left[1 - cos\left(\frac{2\pi n}{N}\right)\right], & 0 \leq n \leq N \\ 0, & otherwise \end{cases} \tag{6}$$

   (a) Sketch $w[n]$ and express $W\left(e^{j\omega}\right)$, the DTFT of $w[n]$, in terms of $R\left(e^{j\omega}\right)$, the DTFT of $r[n]$. Sketch the magnitude of $W\left(e^{j\omega}\right)$.

   (b) Confirm your DTFT plot with MATLAB for $N = 8$.

   (c) Notice that $w[n]$ is the Hann window. What is the main-lobe width and the first sidelobe depth ? What is the number of non-zero FFT bins when using this window?

   Hint: First express $w[n]$, in terms of $r[n]$ and the complex exponentials $e^{j\left(\frac{2\pi n}{N}\right)}$ and $e^{-j\left(\frac{2\pi n}{N}\right)}$. Then use the result

$$r[n]e^{j\omega_0 n} \longleftrightarrow 2\pi R\left(e^{j(\omega-\omega_0)}\right)$$

References:

[1] S. Pavan, N. Krishnapura, *EE658: Data Conversion Circuits Course*, IIT Madras. Available: [Online] http://www.ee.iitm.ac.in/vlsi/courses/ee658_2009/start