Assignment 1

ECE 697 − Delta-Sigma Data Converter Design (Spring 2010)

Due on Tuesday, February 9, 2010.

Problem 1 Ideal sampling review : A band-limited signal $x(t)$ is sampled using an impulse train

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \tag{1}$$

such that the sampled signal is given as

$$y(t) = x(t) \cdot p(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s) \tag{2}$$

1. Show that the frequency domain representation of the impulse train, $p(t)$, is given by

$$P(f) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} \delta(f - kf_s) \tag{3}$$

   where the sampling frequency $f_s = \frac{1}{T_s}$. Sketch both $p(t)$ and $P(f)$.
2. Derive the expression for the sampled spectrum $Y(f)$. Sketch $|Y(f)|$ (assume any shape for the band limited spectrum $X(f)$, like we used a triangle in the class).
3. Nyquist sampling theorem : If the input signal is bandlimited to bandwith f_B, derive the relation between f_B and f_s so that there is no aliasing and the signal $x(t)$ can be recovered from $y(t)$.
4. What are the reasons for exmploying an anti-aliasing filter (AAF) before the sampler in practical scenarios ?
5. Can the AAF be implemented using a discrete-time (switched-capacitor) filter topology ? Explain.

Problem 2 Ideal Sample-and-Hold : Consider a generalized S/H with an RZ sampling pulse-shape $h(t)$ as shown in Figure 1. The S/H samples an arbitrary, band-limited signal $x(t)$ at $t = nT_s$ and then holds the sampled value for a time T, $0 < T \leq T_s$.

1. Show that the sampled-and-held signal $y(t)$ can be expressed as

$$y(t) = [x(t) \cdot p(t)] \otimes h(t) \tag{4}$$

2. Find the frequency response $H(f)$ of the sampling pulse $h(t)$ and sketch it.
3. Derive an expression for the sampled spectrum $Y(f)$. Sketch $|Y(f)|$ (assume any shape for the bandlimited spectrum $X(f)$) showing the sinc distortion due to $|H(f)|$.
4. Sketch $|Y(f)|$ for the case of a zero-order hold (ZOH) (where $T = T_s$), showing the sinc distortion due to $|H(f)|$.
5. Sketch $|Y(f)|$ for the case when $T = T_s/10$ and comment on the sinc distortion in this case. What happens to the sinc distortion and the output signal power as $\frac{T}{T_s} \to 0$.
6. Now, this S/H is employed as the front-end of an ADC where it is immediately followed by a quantizer. The quantizer picks the sampled values only during the hold phase. Will the sinc distortion due to the S/H affect the performance of the ADC ? Explain.

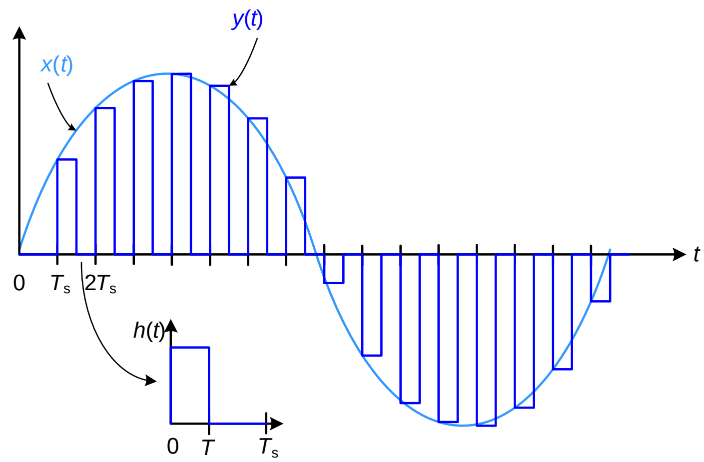

Figure 1: The sample-and-hold output waveform using a RZ sampling pulse-shape.

Problem 3 Ideal Track-and-Hold : Consider a generalized T/H as shown in Figure 2. The T/H processes an arbitrary, band-limited signal $x(t)$ with a sampling rate of $f_s = 1/T_s$. During the track phase, the output tracks the input and the last tracked value is held during the hold phase. Note that the hold phase output is analogous to the output of the RZ-S/H seen in problem 3. The T/H output can be written as a sum given by

$$y(t) = y_H(t) + y_T(t) \tag{5}$$

where $y_H(t)$ and $y_T(t)$ are the outputs of the hold and track phases respectively. In order to directly use the result for $y_H(t)$ from problem 3, without any loss of generality, assume that the hold occurs for the first T time-interval of the clock period. During the remaining time $(T_s - T)$ of the clock-period, the output tracks the input.

1. Show that the track phase output $y_T(t)$ can be expressed as

$$y_T(t) = x(t) \cdot [h_T(t) \otimes p(t)] \tag{6}$$

where

$$h_T(t) = rect\left(\frac{t - \left(\frac{T+T_s}{2}\right)}{T_s - T}\right) \tag{7}$$

2. Evaluate the spectrum $H_T(f)$ and then the spectrum of the track-phase output $Y_T(f)$. Sketch $|H_T(f)|$.

3. Find an expression for the spectrum of the T/H output $Y(f)$ using the sum of the track and hold phases components

$$Y(f) = Y_H(f) + Y_T(f)$$

4. Let the input be a sinusoid given as $x(t) = A \cdot sin(2\pi f_{in} t)$, with $f_{in} < f_s/2$. Using the fact that the spectrum of $x(t)$ is

$$X(f) = \frac{A}{2j}\left[\delta(f - f_{in}) - \delta(f + f_{in})\right] \tag{8}$$

simplify the T/H output spectrum $Y(f)$ and sketch it. Comment on the the additional contribution to the output spectrum from the tracked input.

5. Now, this T/H is employed as the front-end of an ADC where it is immediately followed by a quantizer. The quantizer picks the held values in every clock phase. Will the sinc distortion due to the T/H degrade the performance of the ADC ? Explain.

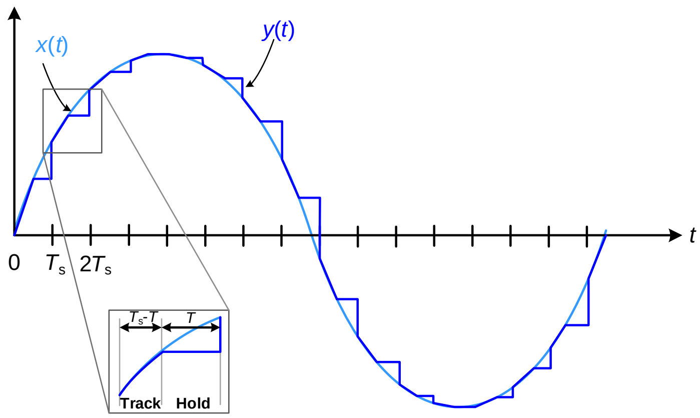

Figure 2: The track-and-hold output waveform. Inset shows the track and hold phases.

Problem 4 Anti-alias filter design with oversampling : Assume the AAF is an N^{th} order Butterworth filter with a transfer function given by

$$H(s) = \frac{1}{1 + \left(\frac{s}{\omega_{3dB}}\right)^N} \tag{9}$$

and thus its magnitude response is

$$|H(f)| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3dB}}\right)^{2N}}} \tag{10}$$

where $f_{3dB}(\omega_{3dB})$ is the 3-dB bandwidth (angular frequency). The input signal bandwidth is $f_B = 1$ MHz. The specification for the AAF states that the attenuation in the first "alias-band" should be at least 60 dB. Also, the acceptable attenuation in the signal band is less than 0.5 dB. In class, we discussed that it is practically impossible to design a brick-wall AAF so that the sampling frequency (f_s) could be equal to the Nyquist rate (i.e. $f_s = 2f_B$= 2 MHz).

1. If the sampling rate (f_s) is 4 MHz, what is the oversampling ratio (OSR) in this case? What is the minimum order (N_{min}) required of the AAF? What is the bandwidth (f_{3dB}) of the AAF ? For the minimum order, N_{min}, how much the filter bandwidth can vary while still meeting the attenuation specification?
2. The sampling rate is now increased to 64 MHz. What is the OSR ? What is the minimum order (N_{min}) required of the AAF? What is the bandwidth (f_{3dB}) of the AAF ? For the minimum order, N_{min}, how much the filter bandwidth can vary while still meeting the attenuation specification?

Hint: For analog filter design, explore the following commands in MATLAB :

```
[n,Wn] = buttord(Wp,Ws,Rp,Rs, 's');
[z,p,k] = butter(n, Wn,'s');
[b,a] = zp2tf(z,p,k);
```

Bonus Problem : Understand the MATLAB code for the sample-and-hold simulation (*SampleHoldDemo.m*).

1. Modify the code to simulate the track-and-hold response.
2. If the S/H is followed by an ideal quantizer of infinite resolution, the quantizer picks the held value to obtain $x[n] \triangleq x(nT_s)$ from the sampled version of $x(t)$. Simulate this operation (ADC = S/H + Quantizer) and observe if the sinc distortion due the the S/H response still persists.
3. Repeat (2) for the T/H simulation.