

Stability Considerations and Multi-Bit Modulators

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Equation 1 depicts the linearized model of the delta-sigma modulator where the stability of the modulator depends only upon the loop-gain $L_1(z)$ and thus the NTF. However, we need to accommodate the non-linear behavior of the quantizer in order to gain further insight into the modulator stability. Also the stability depends upon the amplitude of the input signal, $u(n)$, as a large enough input can lead to integrator saturation and quantizer overload, which in turn destabilizes the modulator loop.

$$Y(z) = STF(z)U(z) + NTF(z)E_Q(z) \quad (1)$$

For the delta-sigma modulators with single-bit quantizer, a heuristic result called *Lee's criterion* states that modulator is likely to be stable if the out-of-band gain (*OBG*) follows the condition that $OBG = \max |NTF(e^{j\omega})| < 1.5$ [2]. However, this is neither necessary nor sufficient condition but it serves as a simple design rule and the modulator stability must be confirmed through extensive simulations [4].

So far we assumed that the gain (k_q) of the quantizer is constant and equal to unity. However the gain of the quantizer strongly depends upon its input. As illustrated in Figure 1, the gain of a single-bit quantizer varies widely with its input due to the hard non-linearity. However for the multi-bit quantizer case, the quantizer gain is restricted to a narrower range, which gets tighter as the quantizer levels are increased. The statistics of the quantizer gain depends upon the PDF of its input, $f_Y(y)$, in the closed-loop operation of the modulator, and the average gain is estimated from the simulations as

$$k_q = \frac{E[|y|]}{E[y^2]} \quad (2)$$

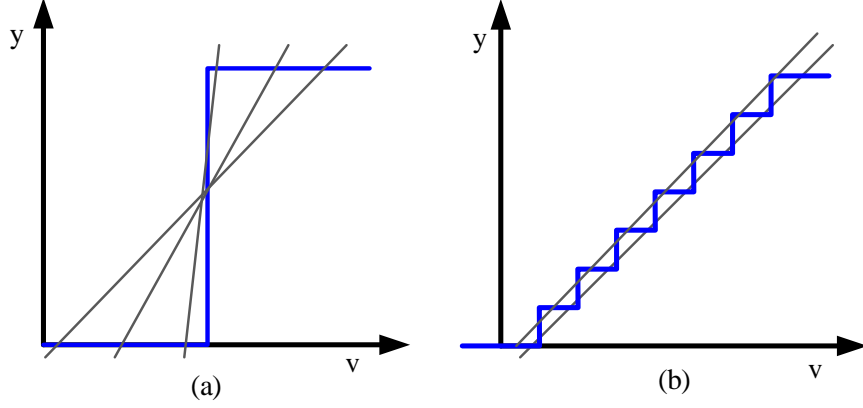


Figure 1: Illustration of the variation of quantizer gain with respect to the input signal for (a) a single-bit quantizer and (b) a multi-bit quantizer.

The modified NTF response after accommodating the quantizer gain variation is given as

$$NTF_{k_q}(z) = \frac{1}{1 - k_q L_1(z)} = \frac{NTF_1(z)}{k_q + (1 - k_q) NTF_1(z)} \quad (3)$$

where $NTF_1(z)$ is the noise-transfer function with a constant quantizer gain of 1. Figure 2 shows the root locus of a third-order modulator with the quantizer gain k_q varying from 0 to 1. We can observe that for values of $k_q < 0.34$, a pair of poles of NTF are out of the unit circle and thus rendering the modulator unstable. As we have seen earlier in the stability analysis lecture, the low quantizer gain results due to quantizer overload, when the amplitude of the quantizer input exceeds its output range. Also we can observe that a large variation in quantizer gain undermines the stability of a higher-order modulator.

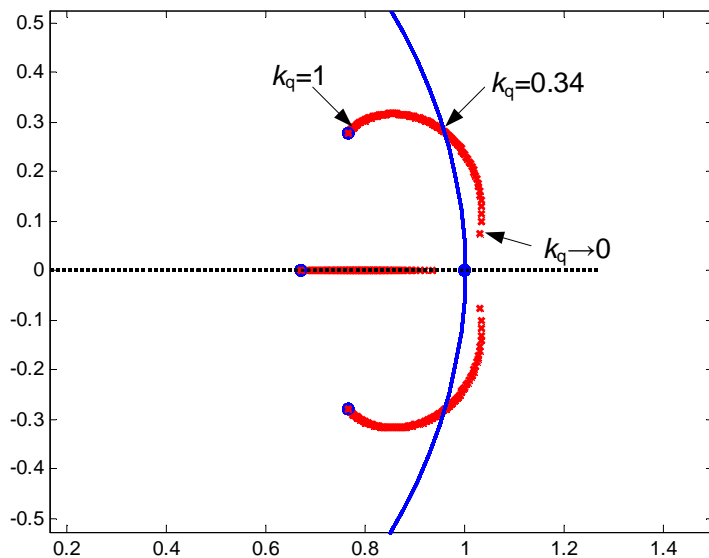


Figure 2: Root locus of the NTF for a third-order modulator with varying quantizer gain.

A single-bit modulator results in a simple quantizer design where only a single comparator is employed for quantization. A single bit quantizer does away with the circuitry required for offset cancellation and bubble correction in a Flash ADC. Also, the corresponding single-bit DAC is inherently linear due to only two possible output levels which further does away with logic and element redundancy dedicated to dynamic element mismatch shaping (DEM). However, multi-bit quantizers in the delta-sigma loop offer significant advantages over their single-bit counterparts which are listed as follows [4]:

1. *Lower In-Band Quantization Noise:* The multi-bit quantizer increases the modulator SQNR by 6 dB for every bit increase in its resolution. This correspondingly lowers the quantization noise floor and directly increases the SNDR. Also the roll-off requirements on the decimation filter, used to suppress the out of band noise, are relaxed.
2. *Linear Loop Filter Behavior:* As illustrated earlier, the variations in the gain are reduced in a multi-bit quantizer. This makes the modulator loop more linear and stable. Also, now the real modulator response closely follows the predicted performance of the linearized model and the design is more robust. This is also useful in the case of the cascaded (or MASH) modulator, where the design of the noise cancellation filter (NCF) can be done using the simple linear analysis and the noise leakage due to the variations in the effective NTFs is minimized. Thus, from a system design perspective multi-bit quantizers are preferable.

3. *Aggressive Noise Shaping*: Since the loop stability is more robust with a multi-bit quantizer, an aggressive NTF can now be employed in the design with an out-of-band gain (*OBG*) greater than 1.5. This fact is extremely useful when designing 5th or higher order modulators where significantly large resolution is desired with a relatively lower *OSR* (i.e. wideband high-resolution conversion). Here a 11 to 16 level quantizer can help achieve over 60 dB SNDR with an *OSR* of 8 [1].
4. *Lower Slew Rate Requirements in the Loop-Filter*: Since the DAC feedback to the loop filter changes in smaller steps ($LSB = V_{range}/2^N$) when using a multi-bit design, the slew rate requirements on the input op-amp (the *golden* op-amp which sets the overall linearity of the modulator) are reduced. This also relaxes the linearity requirements on the input loop filter $L_0(z)$.
5. *Higher Maximum Stable Amplitude*: When using multi-bit quantizer, the maximum stable amplitude (MSA) of the input tolerated by the modulator can be large. This is due to the fact that the smaller LSB values result in higher tolerance to loop-filter saturation and quantizer overload.
6. *Reduced Jitter-Noise Sensitivity*: In the case of continuous-time modulators, the in-band noise due to clock jitter with NRZ DAC pulse shape is given by [3]

$$IBN_{\sigma_{jt}|NRZ,MB} \approx \frac{V_{range}^2}{(2^N - 1)^2} \left(\frac{\sigma_{jt}}{T_s} \right)^2 \frac{2}{OSR} \quad (4)$$

which is clearly reduced when higher quantizer resolution is employed. This is due to the fact that the amount of DAC pulse-width difference ($\delta y[n] = y[n] - y[n-1]$) modulated by the clock jitter is reduced when more quantization levels are used.

References

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