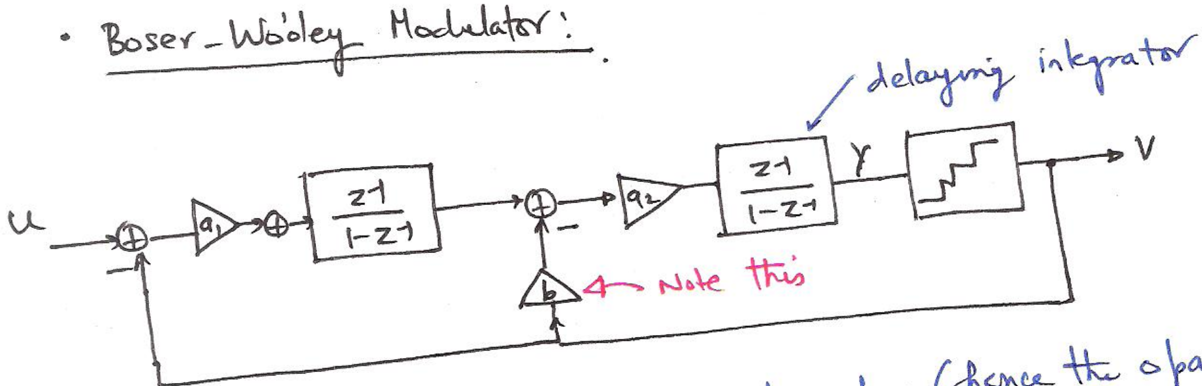
Alternative 2^{nd} order ~~ΣΔ~~ Modulators.

- Large number of structures for 2^{nd} order noise-shaping with $STF(z) = 1$ or z^{-1} or z^{-2}.
- Be careful to avoid delay-free loops.
- Should have reasonable robustness against practical circuit requirements/limitations like finite opamp f_{un}, gain, SR, comparator delay (or quantizer delay), etc.

- Boser-Wooley Modulator:



- Settling requirements on the integrators (hence the opamps) is reduced.

$$NTF(z) = \frac{(1-z^{-1})^2}{D(z)} \qquad STF(z) = \frac{a_1 a_2 z^{-2}}{D(z)}$$

where, $D(z) = (1-z^{-1})^2 + a_2 b z^{-1}(1-z^{-1}) + a_1 a_2 z^{-2}$

for $STF(z) = z^{-2}$ and $NTF(z) = (1-z^{-1})^2$ we have the conditions $\boxed{a_1 a_2 = 1}$ and $\boxed{a_2 b = 2}$.

$\Rightarrow D(z) = 1$

$\Rightarrow$ infinite solutions possible

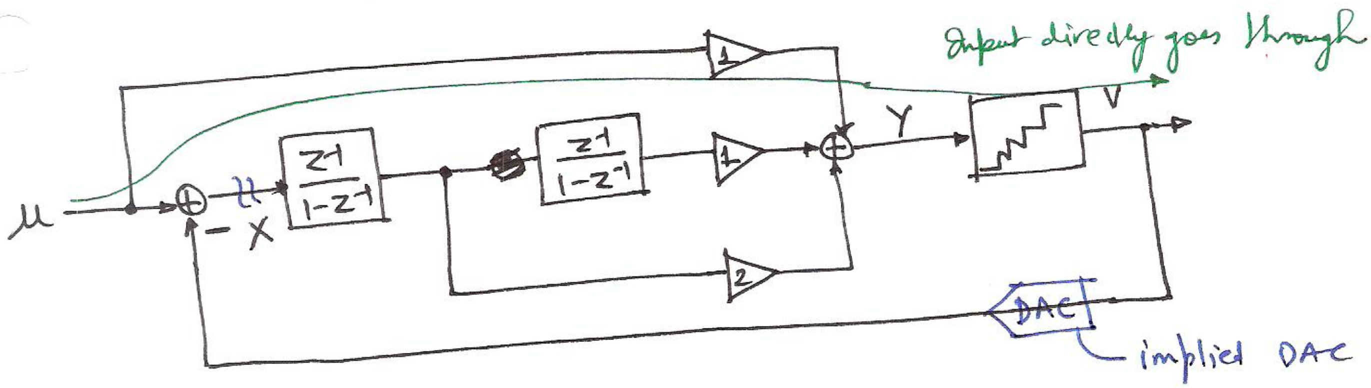
Sol^n ①: $a_1 = a_2 = 1$ & $b = 2$

Sol^n ②: $a_1 = 1/2$, $a_2 = 2$, $b = 1$

* In actual design "Range-Scaling" eliminates these ambiguities in designs

Here, $U(z) - V(z) = \underbrace{(1-z^{-2})U(z)}_{\text{check this spectrum}} - (1-z^{-1})^2 E(z)$

The Silva-Steensgaard Structure:



- Note the direct feed-forward path.
- $V(z) = U(z) + (1-z^{-1})^2 E(z)$ ← Note $U(z)$ is not delayed even when using delaying integrators
- Input signal to the loop filter?

  $= U(z) - V(z)$

  $= -(1-z^{-1})^2 E(z)$ ← contains only noise, no signal content

- Advantages:

  ① requirements on loop-filter linearity are reduced. ⇒ Lower power

  ② ↳ signal swing at X is reduced → lower SR requirements from the opamps.

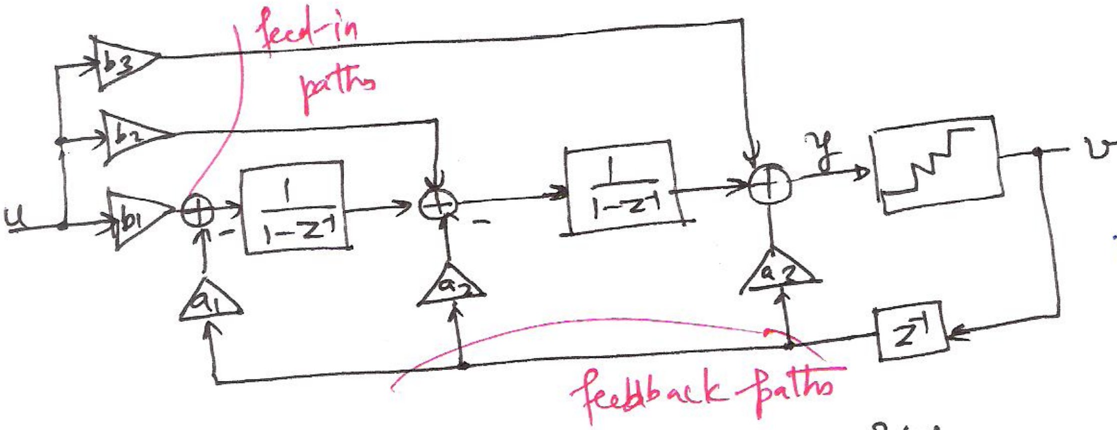
- output of the second-integrator:

  $-z^{-2} E(z)$ → can be directly used in a MASH. without any differencing.

- Disadvantages:

  ① Extra ADDER before the quantizer

  ↳ passive adders using capacitors (for S/C design)



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$$NTF(z) = \frac{(1-z^{-1})^2}{A(z)}, \qquad STF(z) = \frac{B(z)}{A(z)}.$$

$$B(z) = b_1 + b_2(1-z^{-1}) + b_3(1-z^{-1})^2$$

$$A(z) = 1 + (a_1+a_2+a_3-2)z^{-1} + (1-a_2-2a_3)z^{-2} + a_3 z^{-3}$$

→ Used only in CT-DSM for Excess-Loop Delay Compensation (later).

~~By using~~ ~~feed-forward FBs~~

By using multiple feed-in and feed-back paths, more flexibility is obtained for enhancing stability and Dynamic Range.

Which topology/architecture is the best for 2^{nd} order modulator?

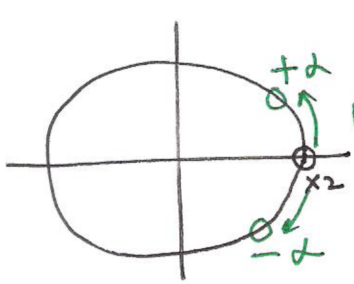
↳ what is the optimal NTF? (STF is secondary)

⇒ Find an NTF which minimizes the IN BAND Quantization NOISE (IBN)

$$|NTF(z)| = \frac{|(1-z^{-1})^2|}{|A(z)|} \simeq \frac{\omega^2}{|A(1)|} \quad \text{for} \quad \omega \ll \pi$$

$$= k\omega^2, \quad k = \frac{1}{|A(1)|}$$

- $|A(1)|$ is the DC gain of $A(z)$



Move the NTF zeros. for $z=1$ to $z=e^{\pm j\alpha}$

⇒ $|NTF(z)|$ in the signal band $\simeq k(\omega+\alpha)(\omega-\alpha)$

$$= k(\omega^2 - \alpha^2)$$

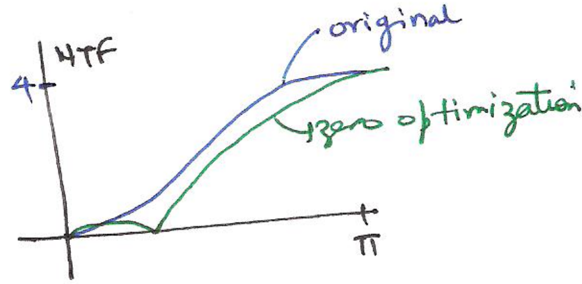
$\omega_B = \frac{\pi}{OSR}$

$$\Rightarrow IBN = \frac{\Delta^2}{12\pi} \int_0^{\omega_B} |k(\omega^2 - \alpha^2)|^2 d\omega$$

$$= \frac{\Delta^2 k^2}{12\pi} \int_0^{\omega_B} (\omega^2 - \alpha^2)^2 d\omega = \frac{\Delta^2}{12\pi} k^2 \cdot I(\alpha)$$

where the integral $I(\alpha) = \int_0^{\omega_B} (\omega^2 - \alpha^2)^2 d\omega$

$\Rightarrow$ for the least IBN, $I(\alpha)$ must be minimized



$$\Rightarrow \frac{dI(\alpha)}{d\alpha} = 0$$

$$\Rightarrow \frac{d}{d\alpha} \int_0^{\omega_B} (\omega^2 - \alpha^2)^2 d\omega = 0$$

$$\Rightarrow \int_0^{\omega_B} \frac{d}{d\alpha} (\omega^2 - \alpha^2)^2 d\omega = 0$$

$$\Rightarrow 4\alpha \int_0^{\omega_B} (\omega^2 - \alpha^2) d\omega = 0$$

$$\Rightarrow \frac{\omega_B^3}{3} - \alpha^2 \omega_B = 0$$

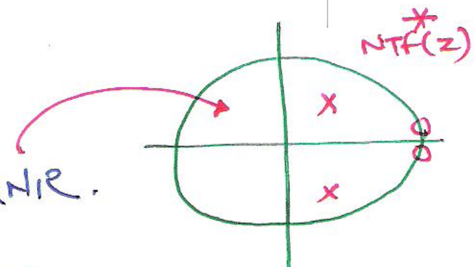
$$\Rightarrow \boxed{\alpha_{opt} = \frac{\omega_B}{\sqrt{3}}}$$

Now find, $\frac{I(0)}{I(\alpha_{opt})} = \frac{9}{4} \Rightarrow$ SQNR improvement $= \log_{10}(9/4)$
$= 3.5$ dB

Now, what if we also optimize the pole locations?
↳ MATLAB based design using exhaustive search.

optimal denominator

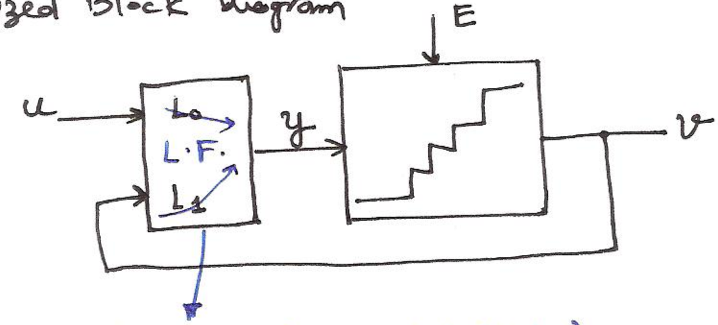
$A_{opt}(z) = 1 - 0.5z^{-1} + 0.16z^{-2}$
↳ 6 dB higher SQNR.



* We will study this algorithm in detail later.

Describing function Analysis

Generalized Block Diagram



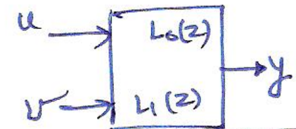
- So far we have assumed Linear Model of noise.

$$Y(z) = L_0(z)\,U(z) + L_1(z)\,V(z)$$

where,

$$L_0(z) = \frac{STF(z)}{NTF(z)}, \qquad L_1(z) = \frac{NTF(z)-1}{NTF(z)}$$

Loop-filter
↳ 2 inputs & 1 output



* for Second-order DSM

$$L_0(z) = \frac{1}{(1-z^{-1})^2}$$

$$L_1(z) = \frac{-z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{(1-z^{-1})^2}$$

* for special cases!

$L_0(z) = L_1(z) = L(z)$

Ex. first-order DSM

for the whole modulator:

$$\Rightarrow \quad V(z) = Y(z) + E(z)$$

$$= L_0(z)\,U(z) + L_1(z)\,V(z) + E(z)$$

$$\Rightarrow \quad V(z) = \underbrace{\frac{L_0(z)}{1-L_1(z)}}_{STF} \cdot U(z) + \underbrace{\frac{L_1(z)}{1-L_1(z)}}_{NTF} \cdot E(z)$$

$$= STF(z)\cdot U(z) + NTF(z)\cdot E(z)$$

So far a linear model. But how to model the quantizer so as to understand the effects of its non-linearity.

- Overload (or saturation) of the quantizer causes instability!
  ↳ when input exceeds the range of the quantizer, the output of the quantizer doesn't change at all.
  ↳ feedback breaks down!

- Definition of stability:
  for LTI systems, Bounded input → Bounded output (BIBO)

$$\Rightarrow \sum_n |h(n)| < \infty$$

ECE 697 Delta-Sigma Converters Design

Lecture#9 Slides

Vishal Saxena
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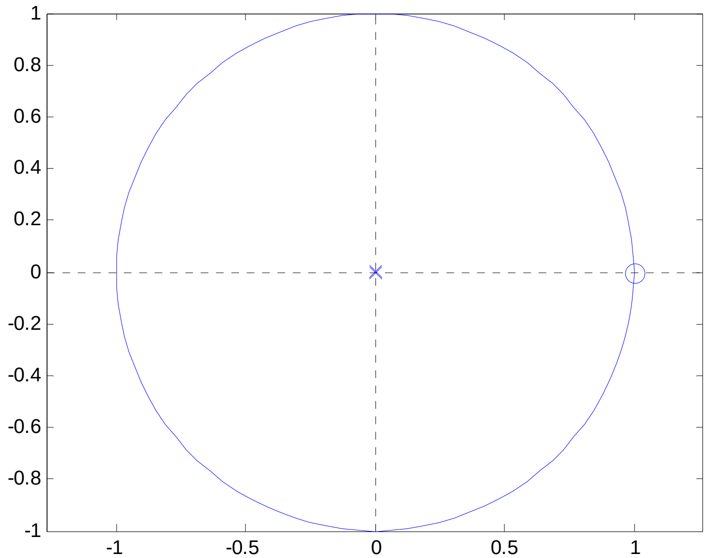
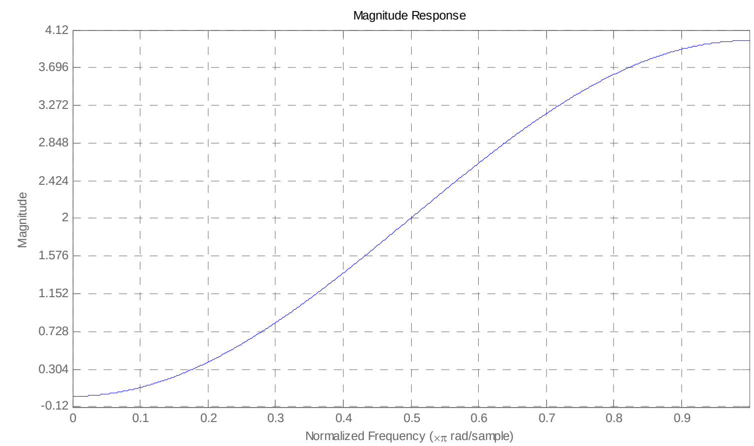
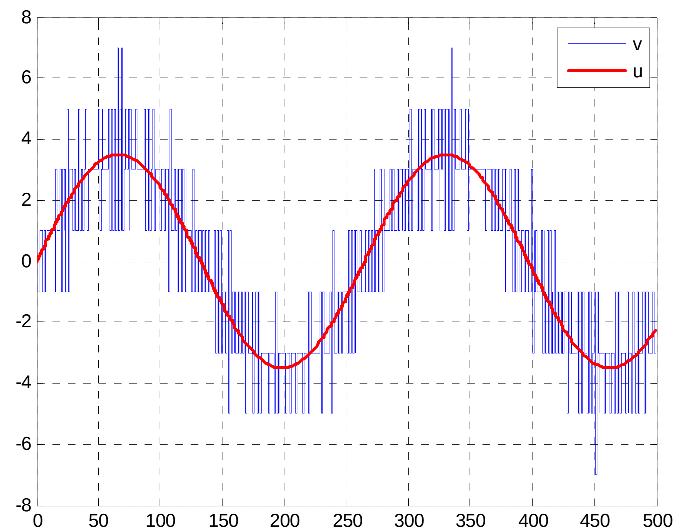
2^{nd} order DSM



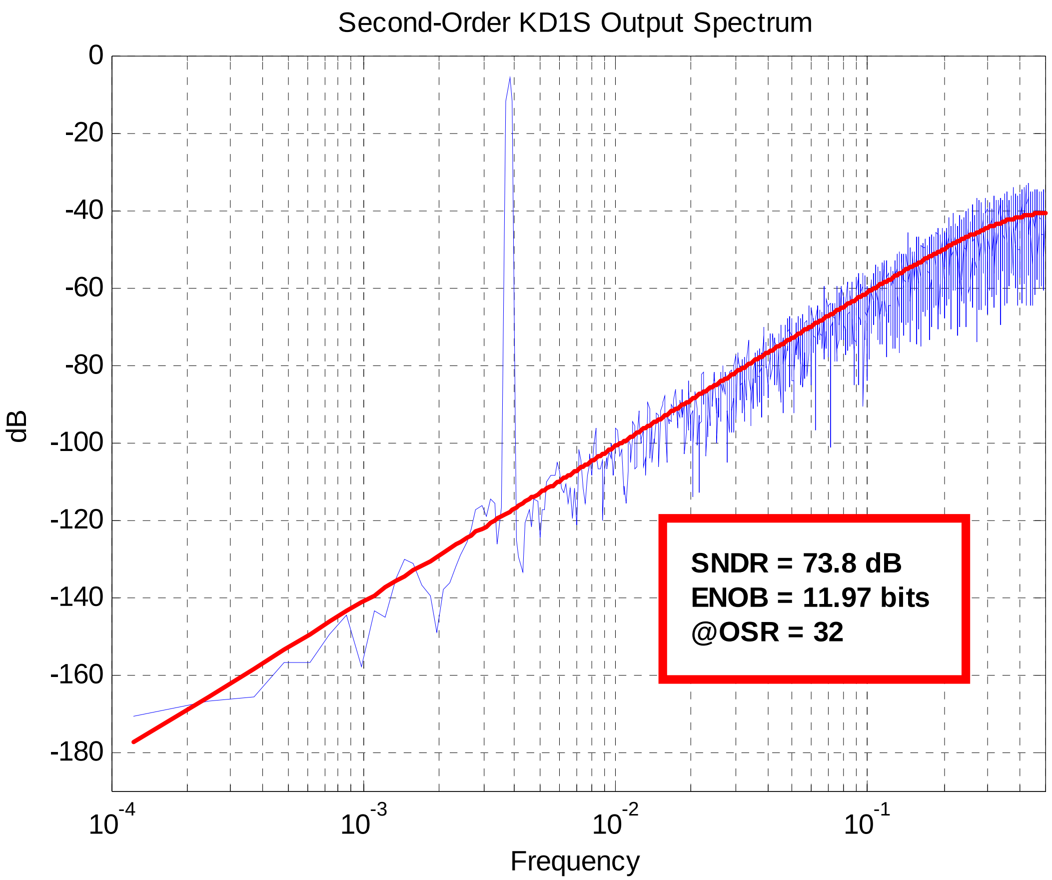




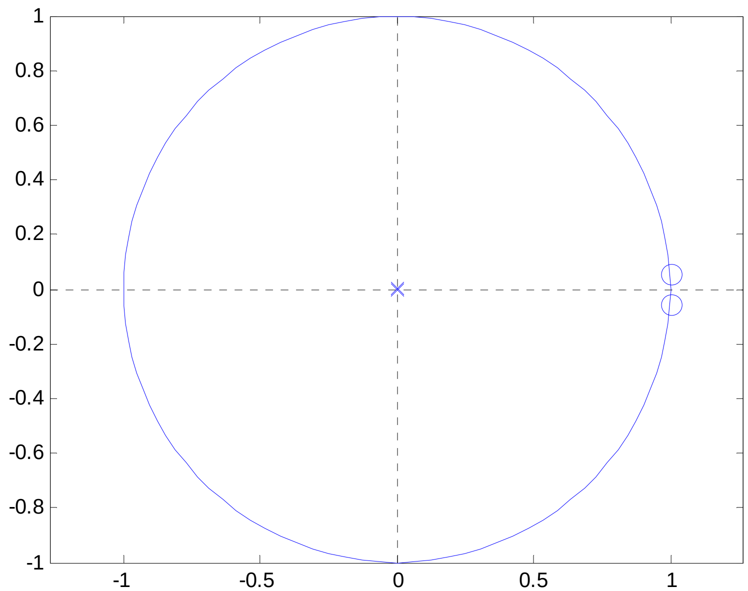
$$NTF(z) = (1 - z^{-1})^2$$

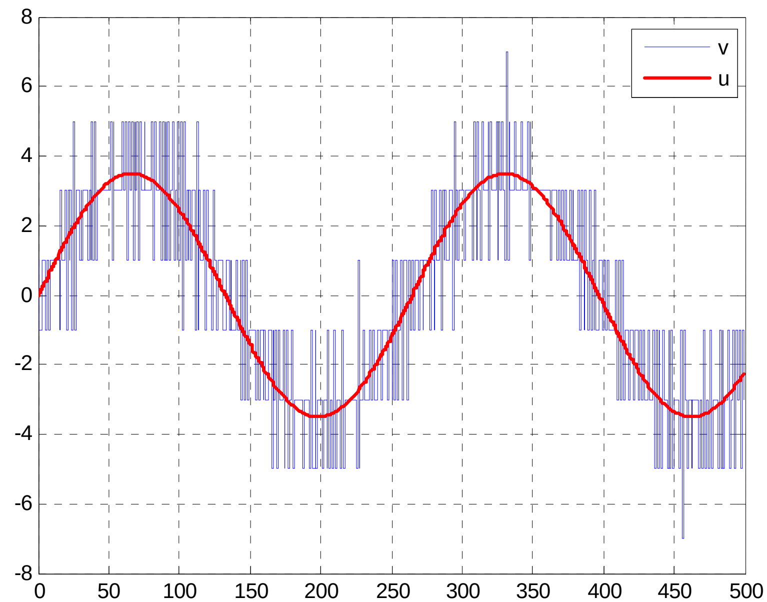
File: Second_Order_DSM_Zero_Opt.m
Set variable opt=0.

2^{nd} order DSM: contd.



2nd order DSM: NTF Zero Optimization

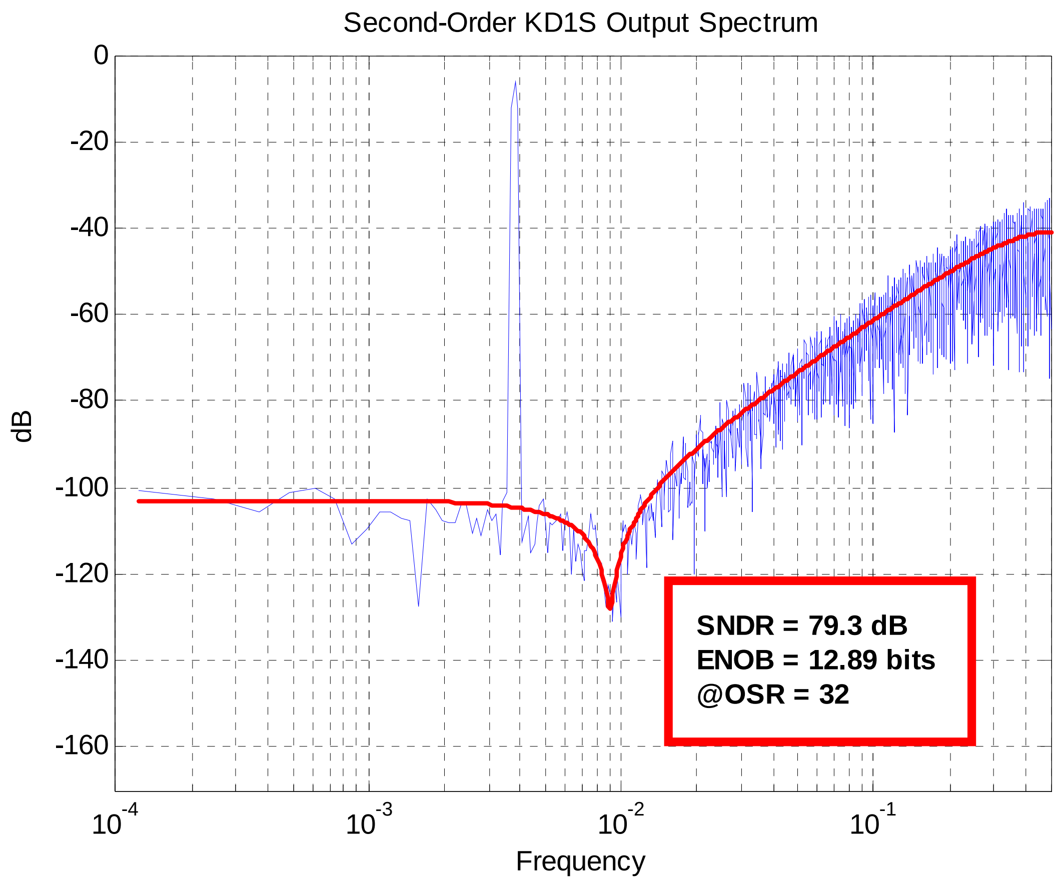




$$NTF(z) = (1 - e^{j0.06}z^{-1})(1 - e^{-j0.06}z^{-1})$$

File: Second_Order_DSM_Zero_Opt.m
Set variable opt=1.

2nd order DSM: NTF Zero Optimization contd.



•5.5 dB increase in SQNR.

NTF pole (if any) optimization to be discussed later.