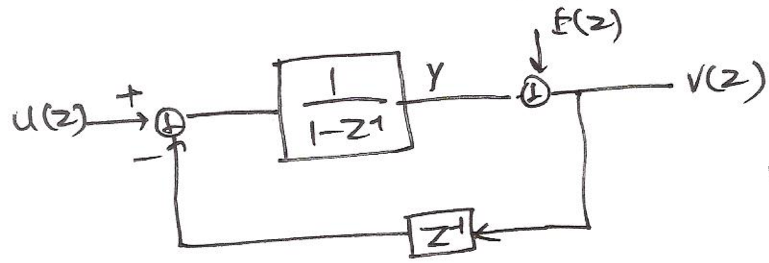
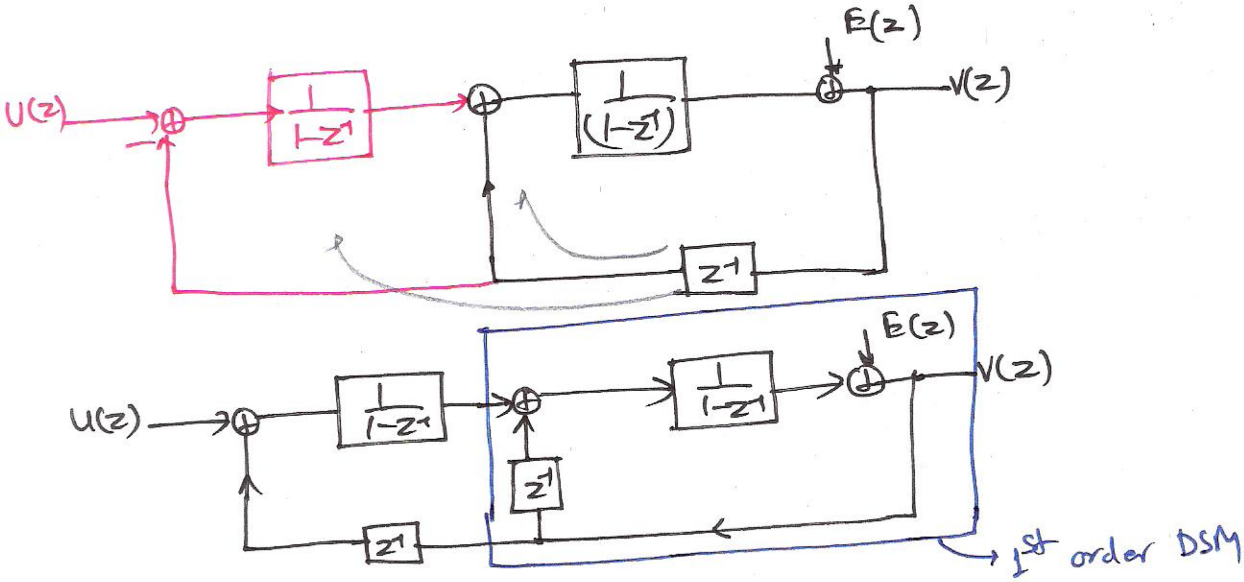
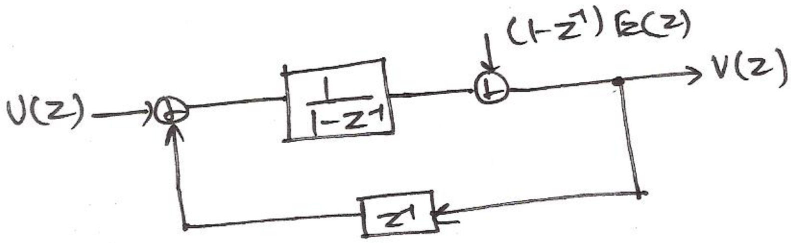
2nd order DSM



$$V(z) = U(z) + (1-z^{-1})E(z)$$





"Double-differentiation of quantization noise"

$$V(z) = X(z) + (1-z^{-1})(1-z^{-1})E(z)$$
$$= X(z) + \underbrace{(1-z^{-1})^2}_{NTF(z)} E(z)$$

$$\Rightarrow \quad NTF(z) = (1-z^{-1})^2$$

⇒ In band quantization noise

$$IBN = \frac{\Delta^2}{12\pi}\int_0^{\pi/OSR} \left|(1-e^{-j\omega})^2\right|^2 d\omega = \frac{\Delta^2}{12\pi}\cdot\int_0^{\pi/OSR} \omega^4 d\omega$$

$$= \frac{\Delta^2}{12\pi}\cdot\frac{\omega^5}{5}\Big|_0^{\pi/OSR} = \frac{\Delta^2}{12\pi}\left(\frac{\pi}{OSR}\right)^5\cdot\frac{1}{5} = \boxed{\frac{\Delta^2}{60}\pi^4\, OSR^{-5}}$$

2x OSR ⇒ 15 dB ↑ SQNR ⇒ 2.5 bits ↑ in ~~resolution~~ resolution.

Example:

quantizer $N_0 = 4$ bits resolution

$OSR = 64$.

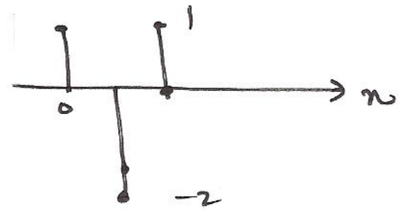
$N_{inc} = 2.5 \log_2(64) = 6 \cdot \frac{5}{2} = 15$ bits

$\Rightarrow N_{eff} = N_0 + N_{inc} = 4 + 15 = 19$ bits !

$\Rightarrow$ 16 levels quantizer $\xrightarrow{\Delta\Sigma^2}$ $2^{19} = 512 \times 10^3$ levels

Can't get this much resolution with Nyquist rate ADC's.

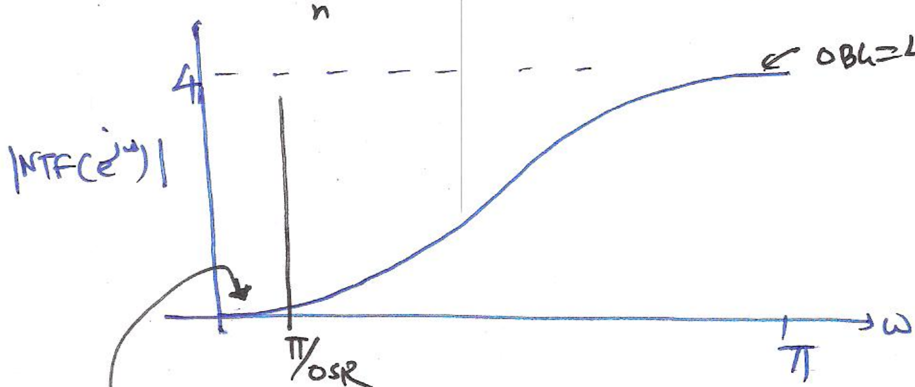
$NTF(z) = (1-z^{-1})^2$ $h[n] = [1, -2, 1]$.



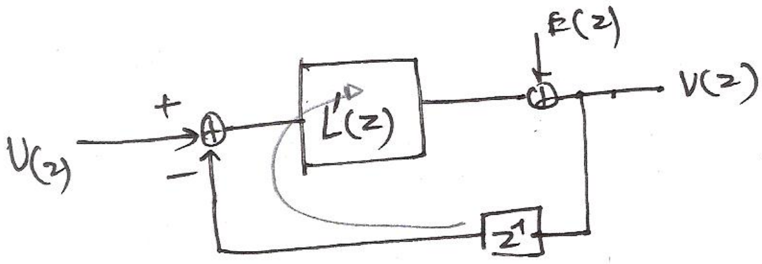
NTF gain at $\omega = \pi$

$= |NTF(e^{j\omega})|_{\omega=\pi}$

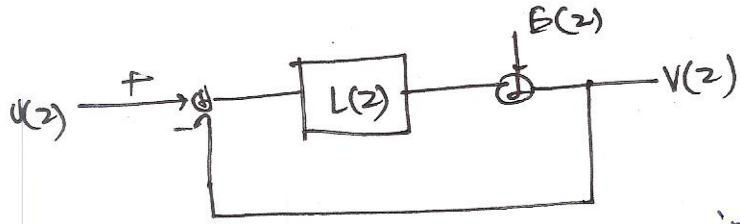
$= \sum_n (-1)^n h(n) = 4. = \sum_{n=0}^{\infty} |h[n]|$?? → we'll see later



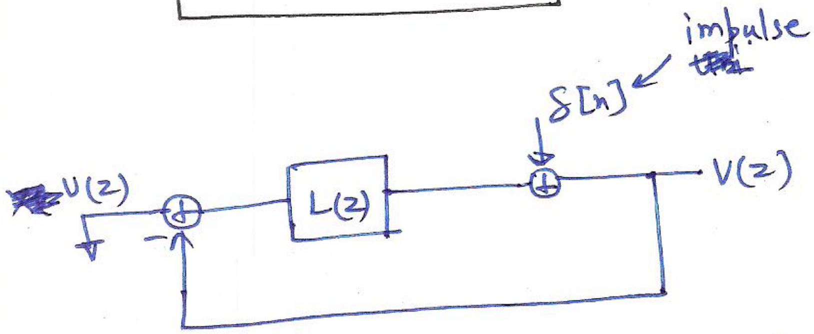
- Lots of gain at the frequencies where we wish the quantization noise to be very low.



move the delay (z^{-1}) into the loop filter



⇒ NO DELAY FREE LOOPS!



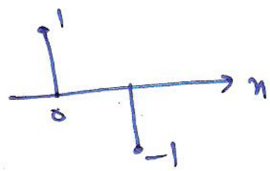
⇒ first sample of the impulse response = 1

↳ no zero-delay loops.

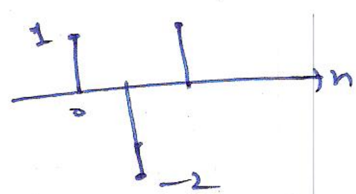
⇒ $\boxed{h[0]=1}$

Any output through L(z) comes out with at least a unit delay
⇒ first sample is always δ[n].

Ex. $NTF(z) = 1 - z^{-1}$



Ex. $NTF(z) = (1-z^{-1})^2 = 1 - 2z^{-1} + z^{-2}$



<u>Notice that</u>.

$$\sum_n h[n] = 0$$

⇒ dc gain = 0 ⇒ HIGH-PASS RESPONSE

⇒ NTF response is always a HP response

$h[0]=1$

and $\sum_n h[n] = 0$.

$h[0]=1 \Rightarrow$ ~~NTF~~ $\boxed{NTF(z\to\infty) = 1}$.

Fundamental result.

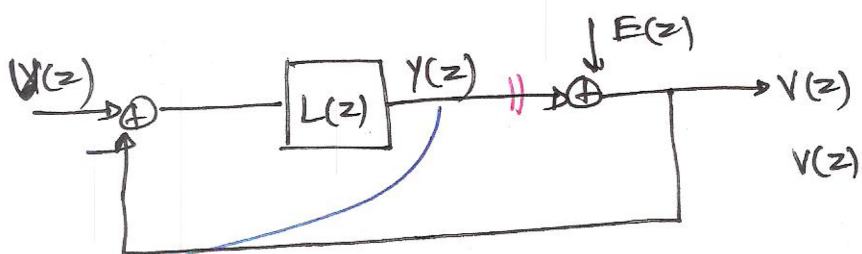
∵

$$NTF(z) = h[0] + h[1]z^{-1} + h[2]z^{-2} + \dots$$

$$\Rightarrow NTF(z\to\infty) = h[0] = 1.$$

If $h[0] \neq 1 \Rightarrow$ NOT a physically realizable NTF.

~~More~~ Further Understanding of the quantization noise in the loop.



$$V(z) = STF(z)\cdot U(z) + NTF(z)\cdot E(z)$$

$$Y(z) = V(z) - E(z) = STF(z)\cdot U(z) + NTF(z).\ E(z) - E(z)$$

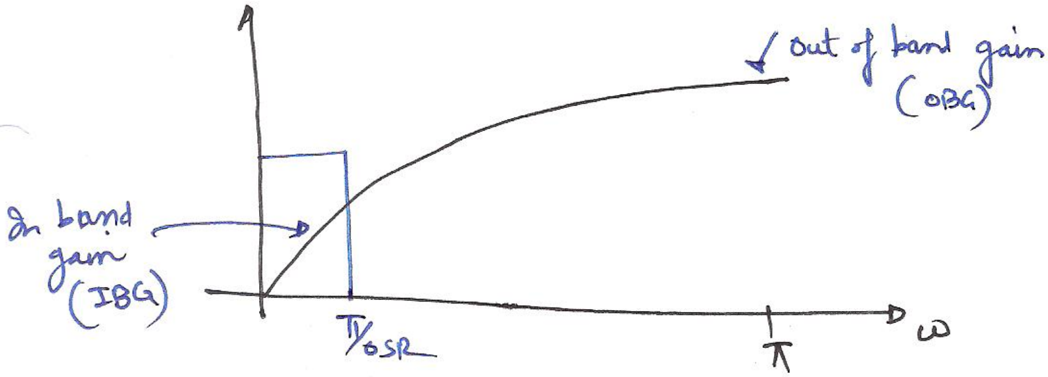
$$= \underbrace{STF(z).U(z)}_{\text{input}} + \underbrace{(NTF(z)-1)\ E(z)}_{\text{quantization noise circulating in the loop}}$$

In the time-domain:

- $STF \cong 1$ at low frequencies
  $\Rightarrow$ input will appear without any change at the output of the loop filter.
- How about the quantization noise?

$$NTF(z) - 1 \xleftrightarrow{\mathcal{Z}^{-1}} h[n] - \delta[n].$$

The quantization noise is injected as $e[n]$ into the loop and appears back at the loop-filter output $y[n]$.



Ex.

$NTF(z) = 1 - z^{-1}$

$IBG \simeq \omega$

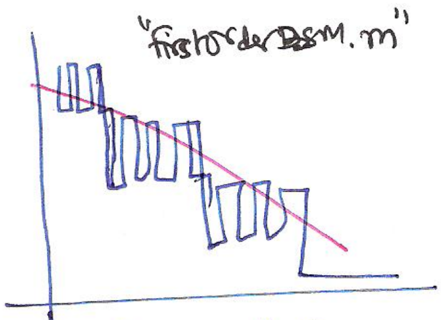
$OBG = 2$

$NTF(z) = (1 - z^{-1})^2$

$IBG \simeq \omega^2$

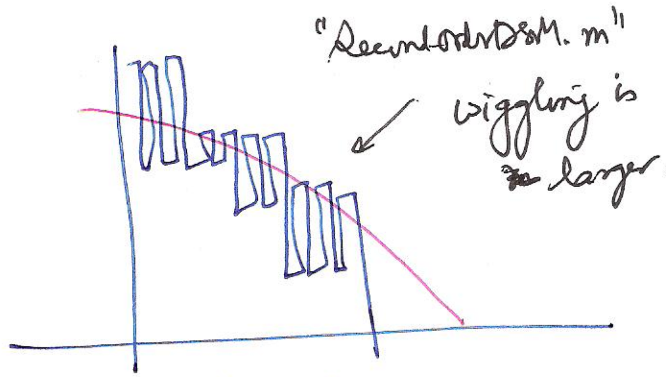
$OBG = 4.$

SEE MATLAB



first order
$(1 - z^{-1})$

1 LSB jumps



2nd order
$(1 - z^{-1})^2$

larger LSB jumps

* More OBG ⇒ larger jumps in terms of the LSB's.

- How to find the jump magnitude from the NTF(z)?
  ↳ given by the maximum accumulation of the quantization error at the output of the loop filter.

$$y[n] = u[n] + e[n] \circledast (h[n] - 1).$$

Consider only noise

$$\Rightarrow \text{noise} = e[n] \circledast (h[n] - 1) = e[n] \circledast g[n]$$

where $g[n] = h[n] - 1$.

Accumulated Noise $= e[n] \circledast (h[n] - 1) = e[n] \circledast g[n]$

$= \sum_{i=0}^{\infty} g[i]\, e[n-i]$, $g[n]$ is causal

$\leq \sum_{i=0}^{\infty} |g[i]|\,|e[n-i]|$.

$\leq \frac{\Delta}{2} \sum_{i=0}^{\infty} |g[i]|$, → $\|g[n]\|_1$ 1-norm of $g[n]$. $g[0] = h[0] - 1 = 0$

$= \frac{\Delta}{2} \cdot \sum_{i=1}^{\infty} |g[i]|$ or $\boxed{\frac{\Delta}{2} \cdot \|h[n] - 1\|_1}$ → Key quantity.

After some "hand-waiving" intuition

Max. LSB jump = 2 × Accumulated noise = $\Delta \cdot \|h[n] - 1\|_1$

Example: ① $NTF(z) = (1 - z^{-1})$

Max LSB jump = $\Delta \cdot 1 = \Delta$. = 1 LSB

② $NTF(z) = (1 - z^{-1})^2 = 1 - 2z^{-1} + z^{-2}$

Max LSB jump = $\Delta \cdot (|2| + |1|) = 3\Delta$ = 3 LSB's

~~What~~ How about third- or higher order?

$NTF(z) = (1 - z^{-1})^N$

$$\Rightarrow IBN \approx \frac{\Delta^2}{12\pi} \int_0^{\pi/OSR} \omega^{2N}\, d\omega$$

$$= \frac{\Delta^2}{12\pi} \cdot \frac{\omega^{2N+1}}{(2N+1)} \Big|_0^{\pi/OSR}$$

$$= \frac{\Delta^2 \, \pi^{2N+1}}{12\pi\,(2N+1)} \cdot OSR^{-(2N+1)}$$

$\Rightarrow$ $3 \times (2N+1)$ dB ↑ per 2x OSR

$\Rightarrow$ $(N + \frac{1}{2})$ bit increase in resolution for 2x OSR.

Stability issues! will come back to it later.

ECE 697 Delta-Sigma Converters Design

Lecture#8 Slides

Vishal Saxena
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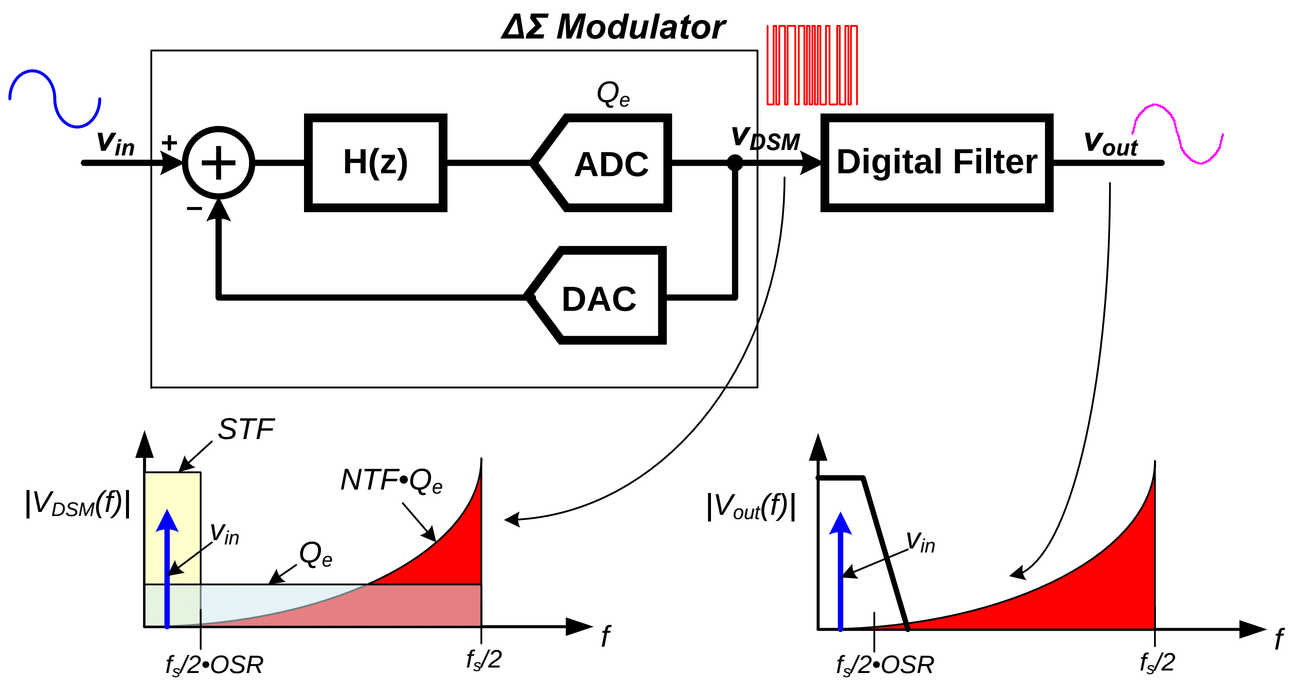
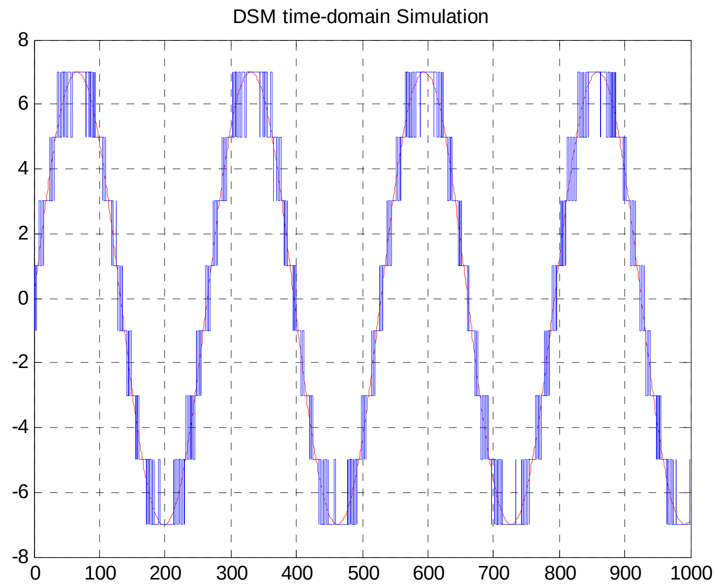
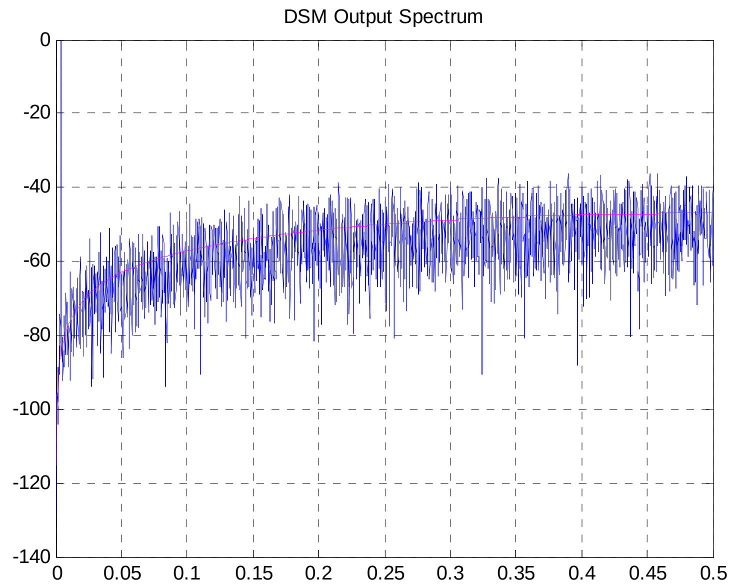
Delta-Sigma (ΔΣ or DS) Modulation



- Use oversampling ($f_s=2{\cdot}OSR{\cdot}BW$) to shape the quantization noise out of the signal band.
- Use low-resolution ADC and DAC to higher much higher resolution
  - ✓ In MATLAB, Quantizer = ADC + DAC
- Digitally filter away the out-of band shaped (modulated) noise.
- Trades-off SNR with oversampling ratio.

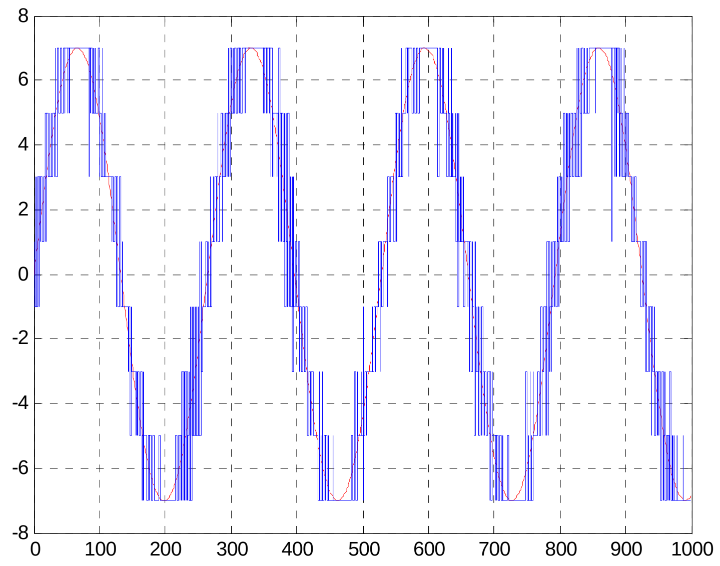
First-order Noise Shaping

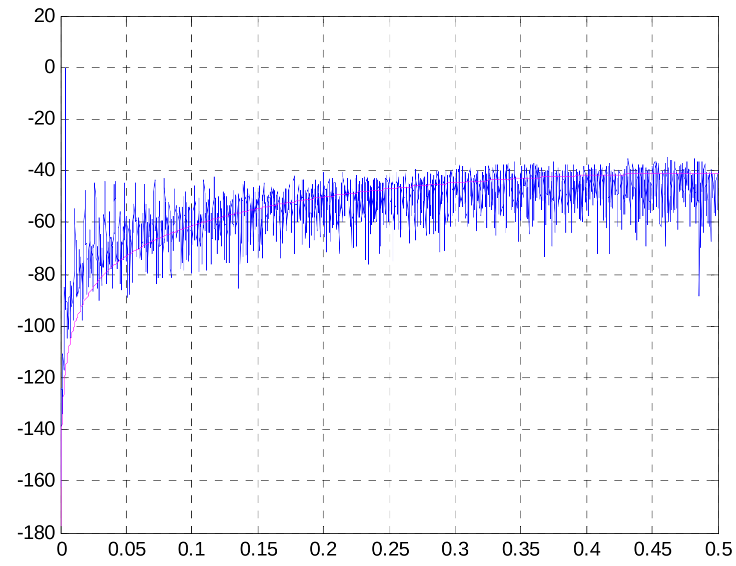




File: First_Order_DSM.m

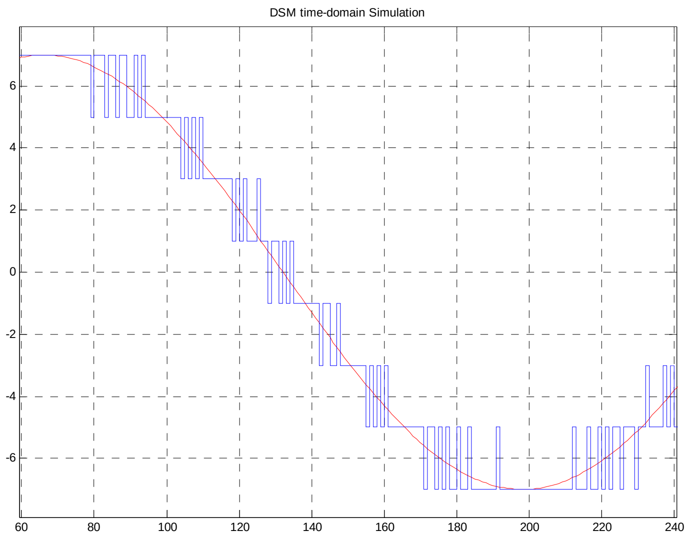
Second-order Noise Shaping

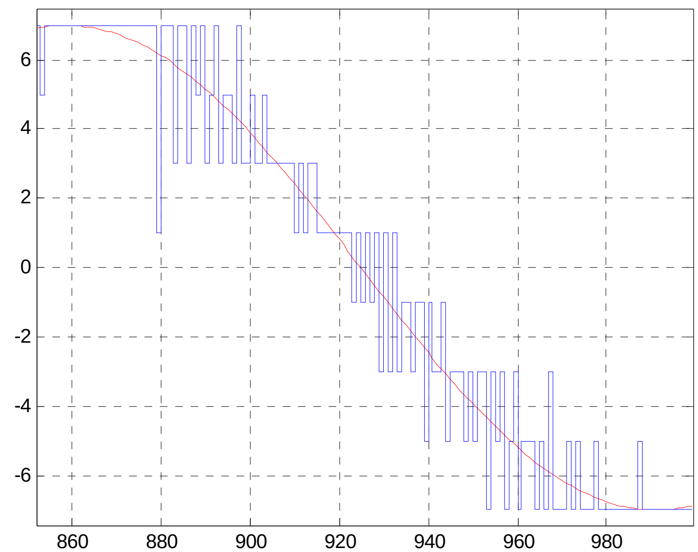




File: Second_Order_DSM.m

Comparison: 1^{st} and 2^{nd} order modulator waveforms

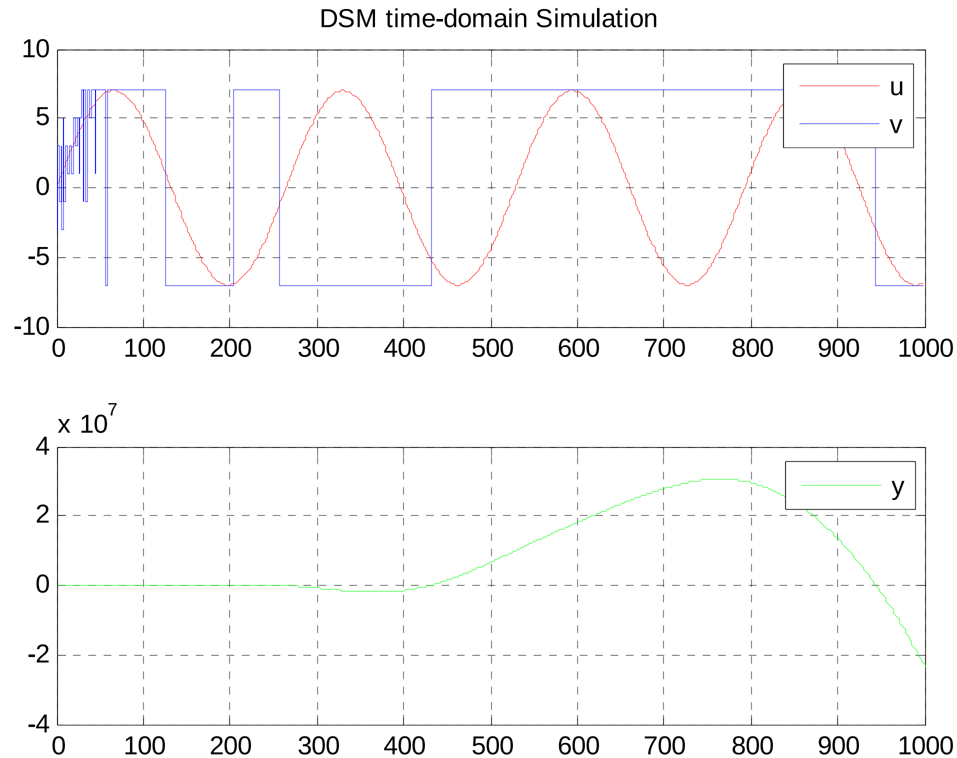




- NTF(z) = $(1-z^{-1})$
- OBG = 2
- Max LSB jump = 1

- NTF(z) = $(1-z^{-1})^2$
- OBG = 4
- Max LSB jump = 3

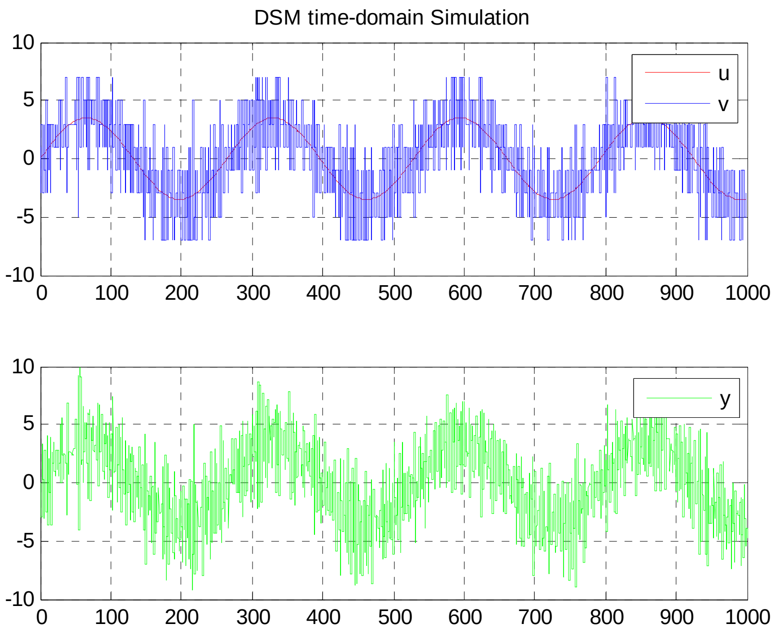
Third-order Noise Shaping (trivial design)

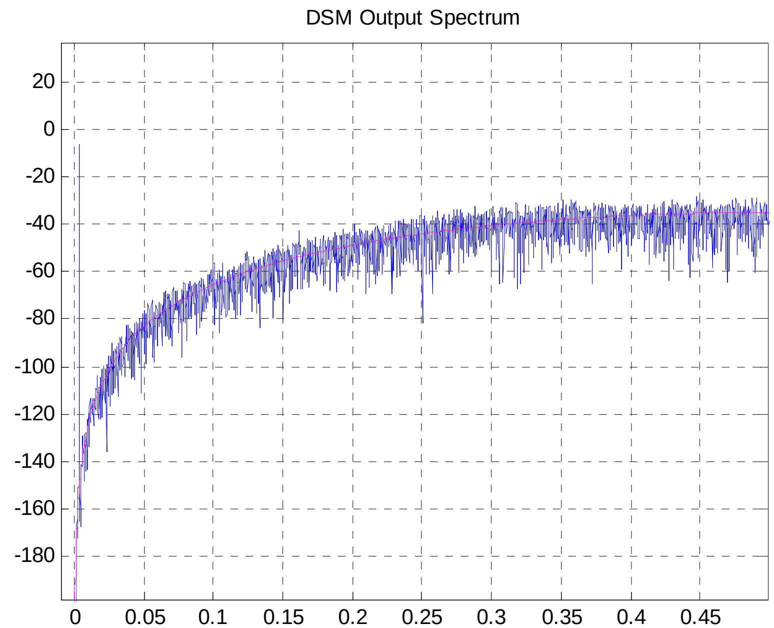


- NTF(z) = $(1-z^{-1})^3$
- OBG = 8, Full-scale input.
- Unstable after few samples (look at y[n] blowing up).
  - ✓ Worst for a single-bit quantizer.

File: Third_Order_DSM.m

Third-order Noise Shaping contd.





- Input amplitude = 0.5·FS
- Signal dependent stability.
  - Need to develop intuition for modulator stability.
  - Reference: Stability theory from the Yellow book of delta-sigma.

File: Third_Order_DSM.m