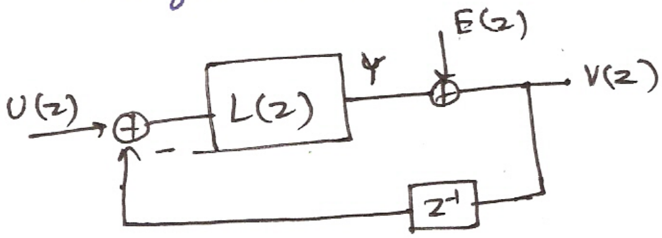
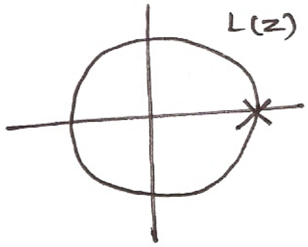
"Linearized model"



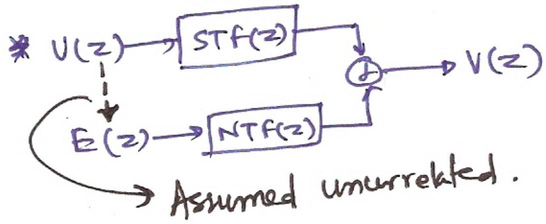
$|L(z)| \to \infty$ at $z=1$.



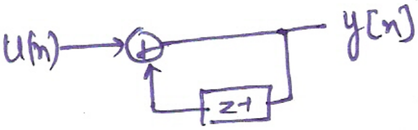
- $L(z) = \frac{1}{1-z^{-1}}$ Accumulator (first order system)

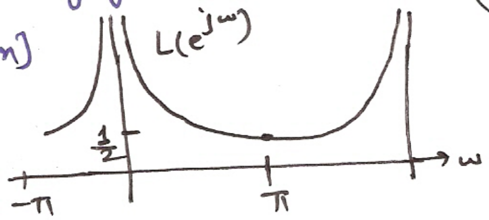
$*$ $[U(z) - z^{-1}V(z)]L(z) + E(z) = V(z)$

$$\Rightarrow V(z) = \underbrace{\boxed{\frac{L(z)}{1+z^{-1}L(z)}}}_{STF(z)} U(z) + \underbrace{\boxed{\frac{1}{1+z^{-1}L(z)}}}_{NTF(z)} E(z)$$



for $L(z) = \frac{1}{1-z^{-1}}$, Accumulator (Non-delaying integrator)

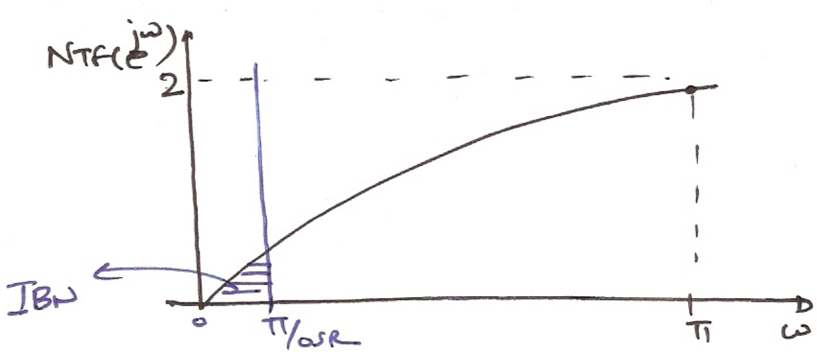




$NTF(z) = (1-z^{-1})$

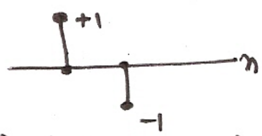
$STF(z) = 1$.

High gain at low frequencies $\Rightarrow$ $|u-v| \to 0$ at $\omega < \frac{\pi}{OSR}$ for $OSR \gg 1$.



$NTF(z) = (1 - z^{-1})$

$\xrightarrow{\mathcal{Z}^{-1}} h[n] = [1, -1]$



high-pass response

* $h[n]$ is the impulse response of $NTF(z)$.

o/p quantization noise $= e[n] \circledast h[n]$

$\Rightarrow$ PSD of the o/p quantization noise $= S_{no}(\omega) = S_e(\omega) \cdot |NTF(e^{j\omega})|^2$

$$= \frac{\Delta^2}{12\pi} \cdot |1 - e^{-j\omega}|^2$$

$$S_{n}(\omega) \simeq \frac{\Delta^2}{12\pi}\omega^2 \quad \text{at low frequencies}$$

In-band signal noise

$$IBN = \int_0^{\pi/OSR} \frac{\Delta^2}{12\pi}\omega^2 d\omega$$

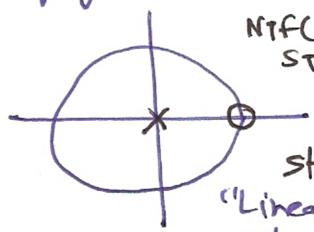
$$= \frac{\Delta^2}{12\pi}\left.\frac{\omega^3}{3}\right|_0^{\pi/OSR} = \frac{\Delta^2}{12\pi}\left(\frac{\pi}{OSR}\right)^3 \cdot \frac{1}{3}$$

$$= \frac{\Delta^2 \cdot \pi^2}{36 \cdot OSR^3}$$

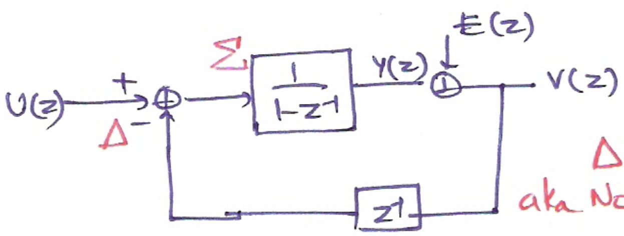
$$\boxed{IBN = \frac{\Delta^2\pi^2}{36} \cdot OSR^{-3}}$$

Stability of the linearized ΔΣ.

$NTF(z) = 1 - z^{-1}$
$STF(z) = 1$



stable
"Linearized Model only."



ΔΣ Modulator
aka Noise-shaping Modulator.

- Dominant (low frequency) pole due to the $L(z)$ ← accumulator
  - ⇒ accumulates the error $(u-v)$ and makes decision on the long-term average
    - ↳ impulse response $l(n)$ is long.
  - ⇒ The feedback loop doesn't act instantaneously but acts based upon the integrated (averaged) error. $(u-v)$.

Single-bit first-order DSM

Time-domain:

$$v[n] = \mathrm{Sgn}[y[n]] \longrightarrow (1)$$

$$y[n] = y[n-1] + u[n] - v[n-1]. \longrightarrow (2)$$

for $n = 1, 2, \cdots, N$

$$y[N] - y[0] = \sum_{n=0}^{N} (u[n] - v[n-1]).$$

Assuming the loop is stable

$\Rightarrow$ $y[n]$ is bounded

$$\Rightarrow \lim_{N\to\infty} \frac{y[N]-y[0]}{N} \Rightarrow 0 \longrightarrow (3)$$

from (2) & (3)

$$\lim_{N\to\infty} \frac{y(N)-y[0]}{N} = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N} \left(u(n) - v[n-1]\right)$$

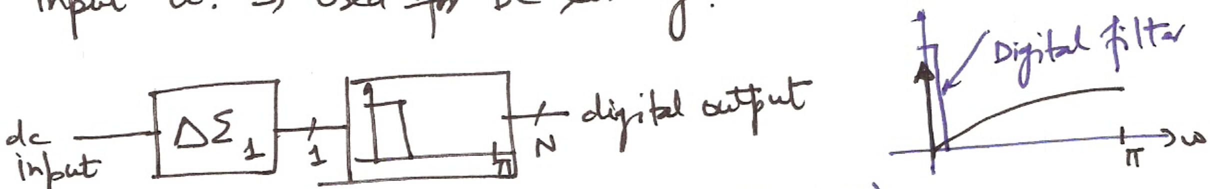
$$0 = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N} u(n) - \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N} v[n-1]$$

$$\Rightarrow \frac{1}{N} \sum_{n} u(n) = \frac{1}{N} \sum_{n} v(n)$$

$$\Rightarrow \boxed{\lim_{N\to\infty} \frac{1}{N} \sum_{n} v[n] = \bar{u}}$$

the average of the input equals the average of the output $v[n]$.

$\Rightarrow$ for a dc input, $\bar{u}$, the long-term average value of the digital output v will be a good approximation of the dc input $\bar{u}$. $\Rightarrow$ Used for DC sensing.



$\Rightarrow$ Simplest digital filter = counter (single-bit ΔΣ).

- Read on idle-tones in first order ΔΣ from textbook section 2.6.
- Stability to be discussed later.

Oversampling w/o noise-shaping

- In-band noise?

$$IBN = \frac{\Delta^2}{12\pi} \cdot \frac{\pi}{OSR} = \frac{\Delta^2}{12} OSR^{-1}$$

$\Rightarrow$ Noise $\downarrow$ 3dB for 2x OSR

$\Rightarrow$ SQNR $\uparrow$ 3dB $\Rightarrow$ 0.5 bit increase in resolution

- Total quantization noise in the band?

$$\boxed{\frac{\Delta^2}{12}}$$

Parseval's Theorem

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

- Out of band gain (OBG) = NTF gain at $\omega = \pi$?

1

first-order noise shaping

$$IBN = \frac{\Delta^2 \pi^2}{36} \cdot OSR^{-3}$$

$\Rightarrow$ for 2x OSR

Noise $\downarrow$ 9 dB

$\Rightarrow$ SQNR $\uparrow$ 9 dB

$\Rightarrow$ 1.5 bit increase in resolution

Parseval's Theorem

$$\frac{\Delta^2}{12\pi}\int_0^{\pi} |NTF(e^{j\omega})|^2 d\omega$$

$$= \frac{\Delta^2}{12\pi} \cdot \left(\sum_n |h(n)|^2\right)$$

$$= \boxed{\frac{\Delta^2}{12} \cdot 2}$$

total noise is increased but IBN is much lower.

$$|NTF(e^{j\pi})|_{\omega=\pi} = 2$$

$$NTF(e^{j\omega})\Big|_{\omega=\pi} = \sum_n h[n] e^{-j\omega n}\Big|_{\omega=\pi}$$

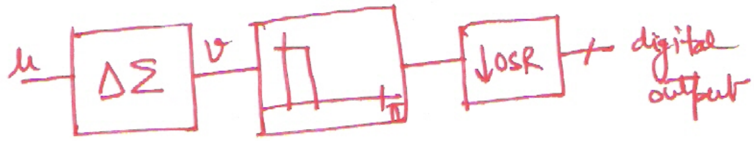
$$\Rightarrow \boxed{OBG = \sum_n (-1)^n h[n].}$$

- NTF gain at $\omega = \pi$

$\Rightarrow$ a high frequency component wiggling with a gain of 2

Intuition $\Rightarrow$ $e^{-j\omega n}\Big|_{\omega=\pi} = e^{-j\pi n} = (-1)^n$

Sinc Decimation filter:



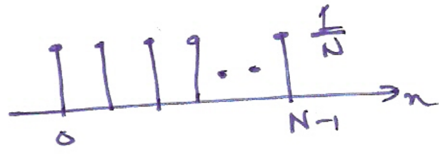
An FIR filter with $(N-1)$ delays and N ~~tap~~ equal valued weights.

$\Rightarrow$ computes a running average of the ΔΣ output $v[n]$.

$$\Rightarrow \quad w[n] = \frac{1}{N}\sum_{i=0}^{N-1} v[n-i]$$

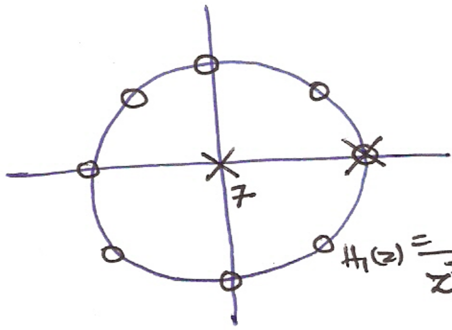
Impulse-response:

$$\Rightarrow \quad h_1[n] = \begin{cases} 1/N, & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$$

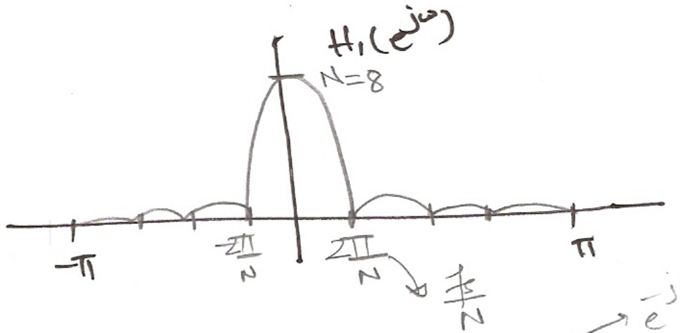


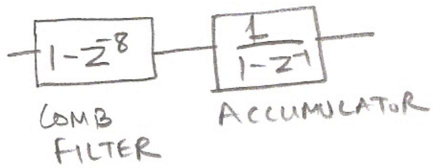
$$\Rightarrow \quad H_1(z) = \frac{1}{N}\cdot\left(\frac{1-z^{-N}}{1-z^{-1}}\right).$$

N=8



$$H_1(z) = \frac{(z^8-1)}{z^7(z-1)}$$



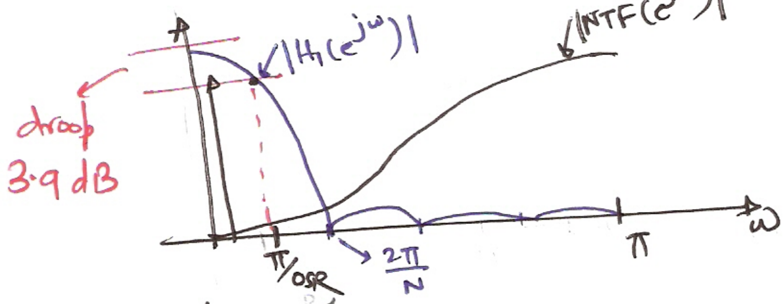


$$H_1(e^{j\omega}) = \underbrace{e^{-j\frac{\omega N}{2}}}_{} \cdot \frac{N\sin\left(\frac{N\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}$$

$\omega = 2\pi f$

$$\Rightarrow \quad \boxed{|H_1(e^{j\omega})| = \frac{\operatorname{sinc}\left(\frac{N\omega}{2}\right)}{\operatorname{sinc}(\omega/2)}} = \frac{\operatorname{sinc}(Nf)}{\operatorname{sinc}(f)}.$$

- choose $N = OSR$.



$$Q_1(z) = H_1(z)\cdot NTF(z)\,E(z)$$

$$= \frac{1}{N}(1-z^{-N})\,E(z)$$

All these noise will fold into the baseband after decimation total output quantization ~~floor~~ noise

$$\sigma_{q_1}^2 = \int_0^{\pi} \frac{1}{N^2}\cdot\left|1-e^{-j\omega N}\right|^2\cdot S_e(\omega)\, d\omega$$

$$= \frac{\Delta^2}{12\pi}\cdot\frac{1}{N^2}\int_0^{\pi}\left|1-e^{-j\omega N}\right|^2 d\omega$$

$$= \frac{\Delta^2}{12}\cdot\frac{1}{N^2}\cdot 2$$

$$\boxed{\sigma_{q_1}^2 = \sigma_e^2\cdot\frac{2}{N^2}} = \frac{\Delta^2}{12}\cdot 2\cdot OSR^{-2}$$

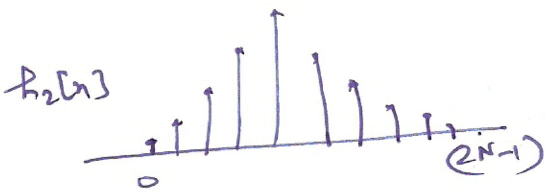
$\longrightarrow$ total system will give only 6 dB↑ in SNR for 2× OSR
⇒ 1-bit ↑

- If we used an ideal LPF after the first-order DSM.

$$\sigma_q^2 = \frac{\pi^2\Delta^2}{3\cdot 12}\cdot OSR^{-3} = \sigma_e^2\cdot\frac{\pi^2}{3}\cdot N^{-3}$$

⇒ sinc filter is nearly 'N' times less effective than the ideal LPF.

- Sinc² decimation filter:



$$H_2(z) = \left(\frac{1}{N}\cdot\frac{1-z^{-N}}{1-z^{-1}}\right)^2$$

$$H_2(e^{j\omega}) = \left|\frac{\mathrm{sinc}\left(\frac{N\omega}{2}\right)}{\mathrm{sinc}\left(\frac{\omega}{2}\right)}\right|^2$$

$$Q_2(z) = H_2(z)\, E(z)\cdot NTF(z)$$

$$= \frac{1}{N^2}\cdot\left(\frac{1-z^{-N}}{1-z^{-1}}\right)\left(1-z^{-N}\right)E(z)$$

$$= \frac{1}{N}\cdot H_1(z)\,\left(\left(1-z^{-N}\right)\right)E(z)$$

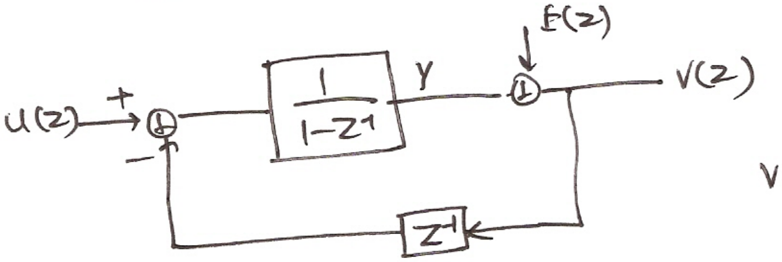
$$\sigma_{q_2}^2 = \int_0^{\pi}\left|H_2(e^{j\omega})\right|^2\cdot\frac{\Delta^2}{12\pi}\cdot\left|\left(1-e^{-j\omega}\right)\right|^2 d\omega = \frac{2N\sigma_e^2}{N^4} = \frac{2\sigma_e^2}{N^3}$$

⇒ $\sigma_{q_2}^2$ is still lower than with the ideal LPF but still better than the sinc filter

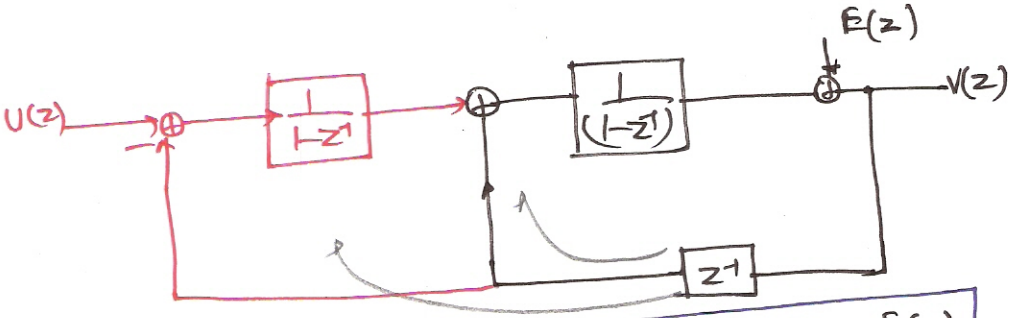
↳ sufficient for first-order DSM.

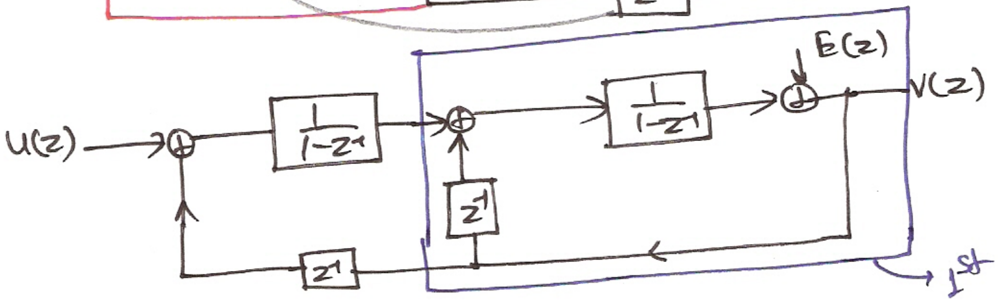
↳ signal droop is larger

2nd order DSM

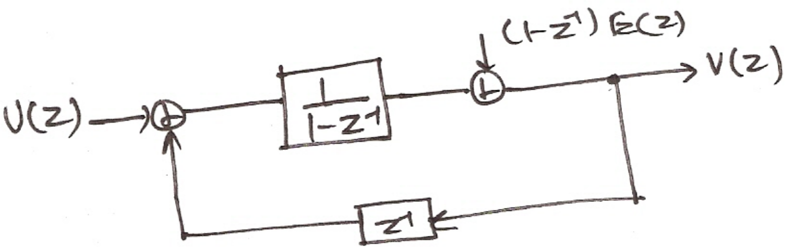


$$V(z) = U(z) + (1-z^{-1})E(z)$$





→ 1st order DSM



"Double-differentiation of quantization noise"

$$V(z) = X(z) + (1-z^{-1})(1-z^{-1})E(z)$$
$$= X(z) + \underbrace{\boxed{(1-z^{-1})^2}}_{NTF(z)} E(z)$$

$$\Rightarrow \quad NTF(z) = (1-z^{-2})$$

→ In band quantization noise

$$IBN = \frac{\Delta^2}{12\pi}\int_0^{\pi/OSR} \left|1-e^{-j\omega}\right|^2 d\omega = \frac{\Delta^2}{12\pi}\cdot\int_0^{\pi/OSR} \omega^4 d\omega$$

$$= \frac{\Delta^2}{12\pi}\cdot\frac{\omega^5}{5}\Big|_0^{\pi/OSR} = \frac{\Delta^2}{12\pi}\left(\frac{\pi}{OSR}\right)^5\cdot\frac{1}{5} = \boxed{\frac{\Delta^2}{60}\pi^4 \, OSR^{-5}}$$

2× OSR ⇒ 15 dB ↑ SQNR ⇒ 2.5 bits ↑ in ~~resolution~~ resolution.

ECE 697 Delta-Sigma Converters Design

Lecture#7 Slides

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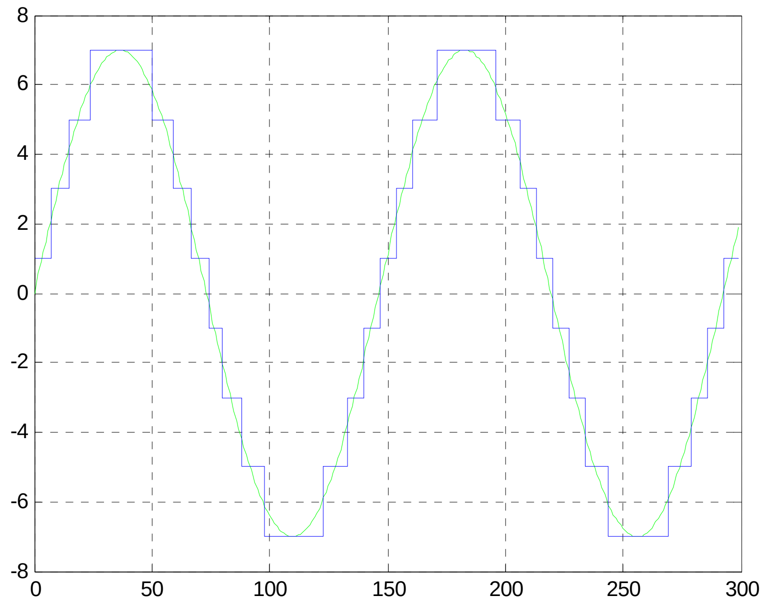
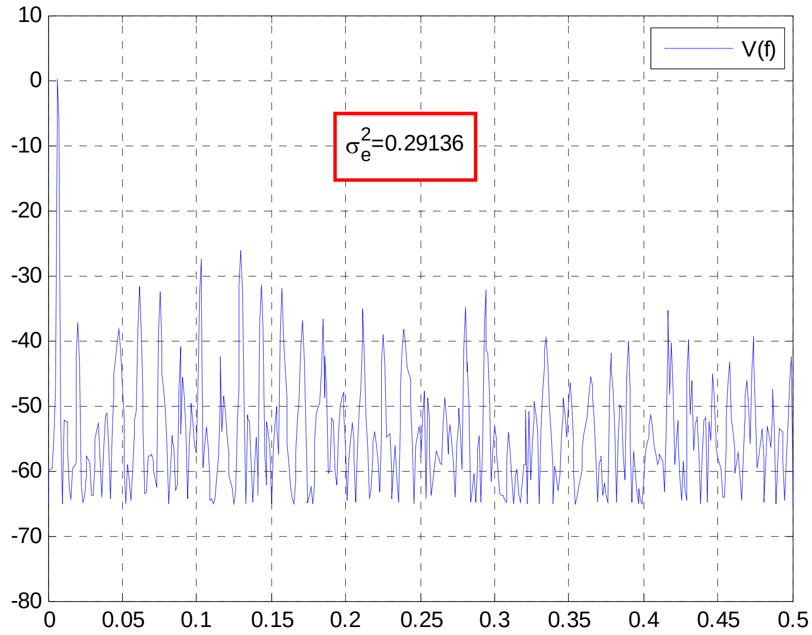
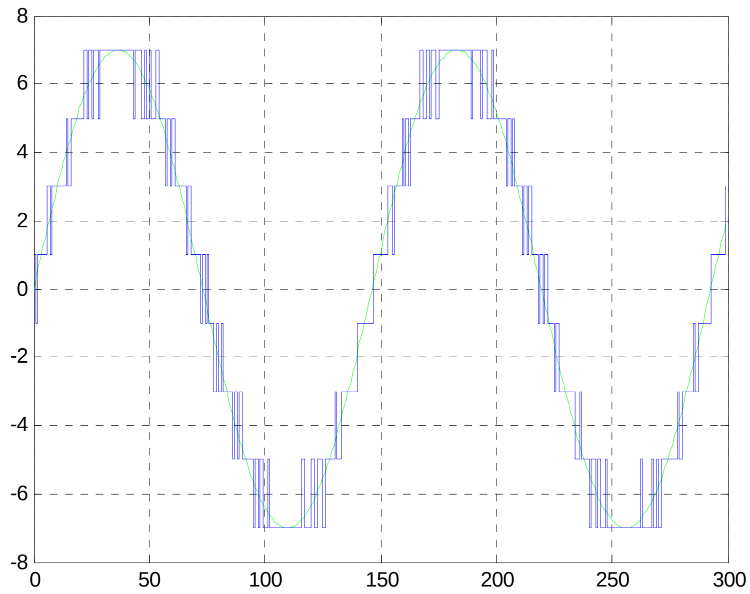
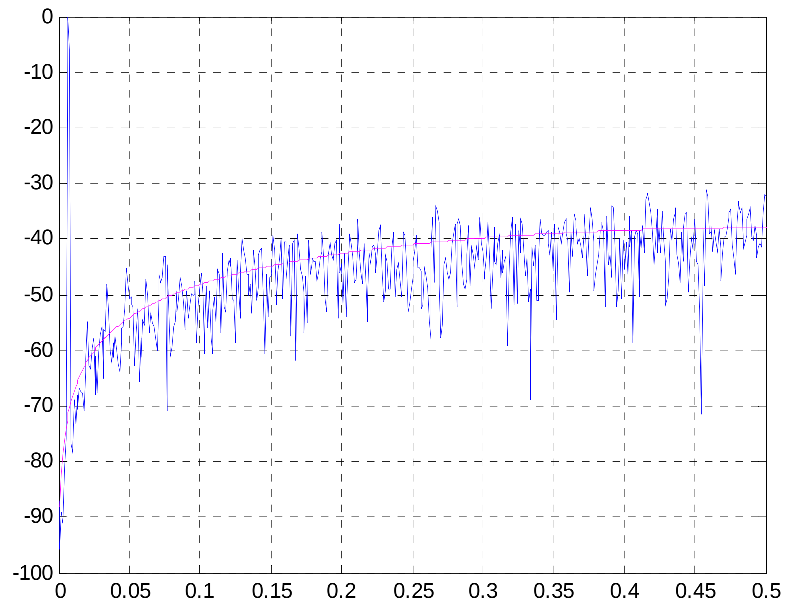
Oversampling



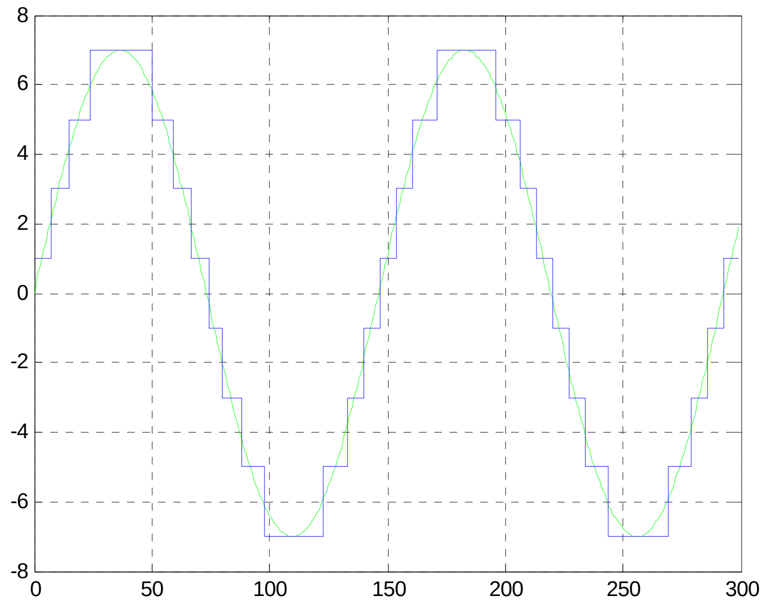


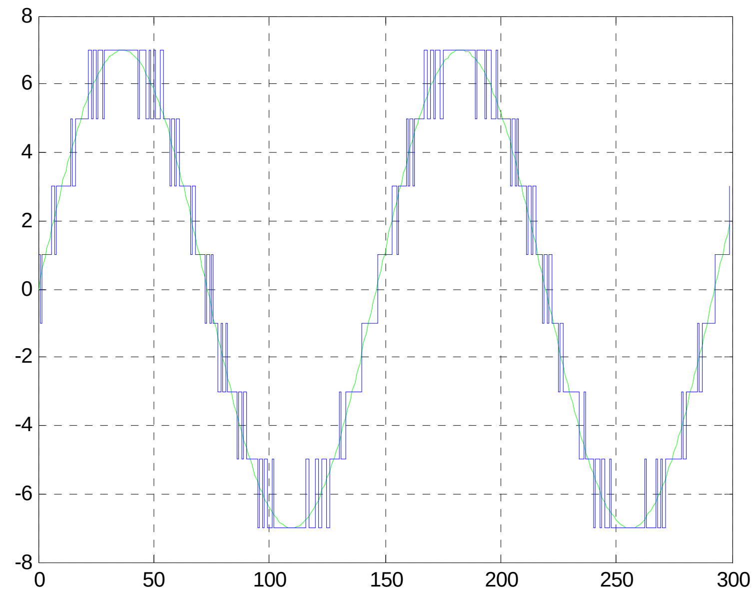
First-order Noise Shaping





Comparison:


8
6
4
2
0
-2
-4
-6
-8
0
50
100
150
200
250
300


8
6
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-2
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-8
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