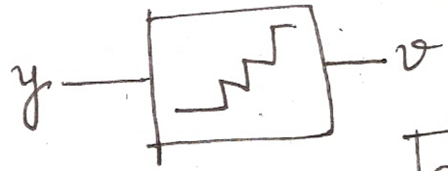
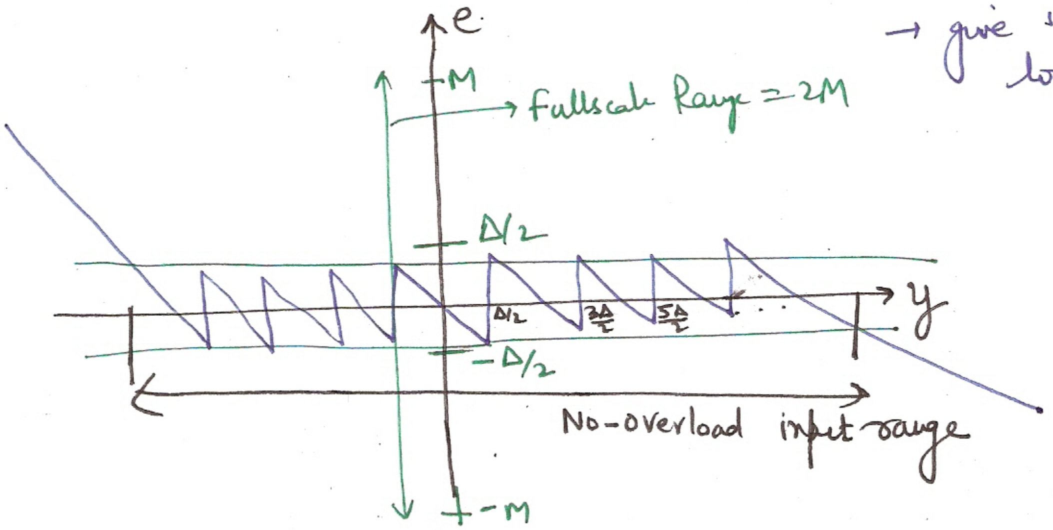
Quantizer Noise Modeling

Hard non-linearity. (memoryless)



$$e = v - y.$$

- Characterization
  → give slow ramp input and look at the steps



LSB = Δ

Nature of the quantization Noise? → Mathematically challenging to analyze!

Conditions:

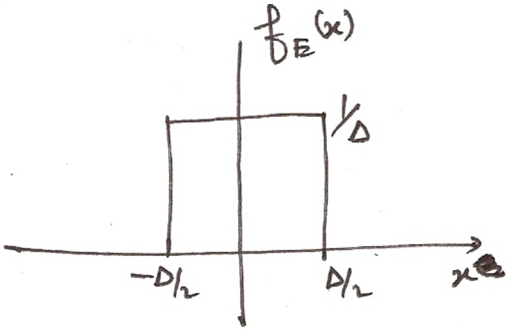
① y stays within the no-overload input range (no-overloading of the quantizer)

② $e[n]$ is uncorrelated with the input $y[n]$. (input is busy)

③ spectrum of $e[n]$ is "white" $\Rightarrow$ $E[e[n]\, e[n+m]] = \delta(m)\cdot\sigma^2$

④ quantization error is uniformly distributed $\Rightarrow$ $e \sim U[-\frac{\Delta}{2}, \frac{\Delta}{2}]$.
↳ makes calculations easy.

PDF of $e[n] \sim U[-\Delta/2, \Delta/2]$.



$\mu = E(e) = 0 \leftarrow$ zero mean

$$\sigma_e^2 = E[(e-\mu)^2] = E(e^2)$$

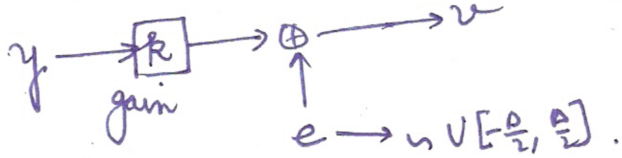
$$= \int_{-\infty}^{\infty} x^2 f_E(x)\,dx$$

$$= \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} x^2\,dx = \frac{2}{\Delta}\left.\frac{x^3}{3}\right|_0^{\Delta/2}$$

$$= \frac{2}{\Delta}\cdot\left(\frac{\Delta}{2}\right)^3\cdot\frac{1}{3} = \frac{\Delta^2}{12}$$

$\Rightarrow$ Quantization noise power $= \boxed{\sigma_e^2 = \frac{\Delta^2}{12}} = \frac{(LSB)^2}{12}$.

Linearized Quantizer Model.

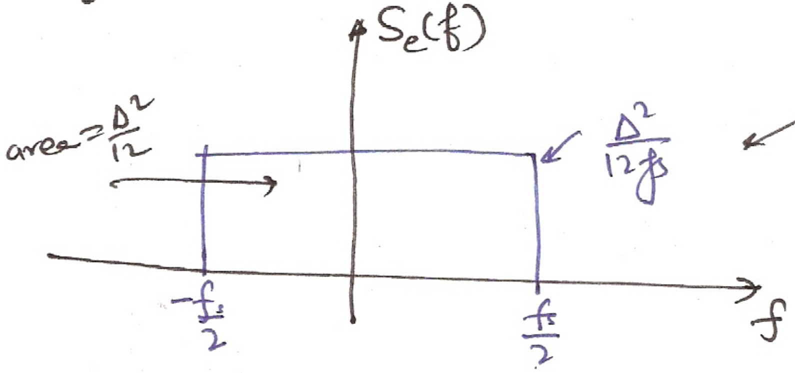


$\Rightarrow$ Hard non-linearity is converted to an additive noise model.

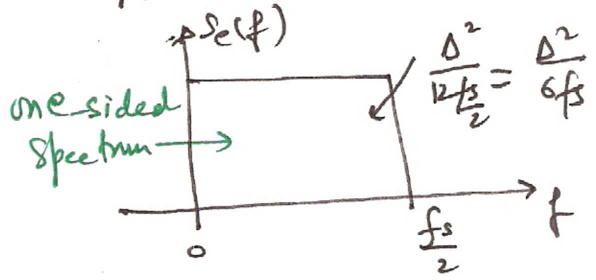
- When does this model break down?
  - ↳ y is not varying fast → constant y.
  - ↳ periodic y with a frequency harmonically related to f_s. ?
  - ↳ quantizer overload

- $\frac{f_{in}}{f_s} = \begin{cases} \frac{p}{q}, & p \ll q \Rightarrow q \text{ closely spaced tones} \Rightarrow \text{quantization noise} \\ \text{irrational}, & \text{continuous spectrum.} \end{cases}$

Quantization Noise PSD



All an approximation.



- $y = A\sin(\omega_{in} t)$ is quantized with a quantizer with LSB $= \Delta$.

$$\Rightarrow SQNR = \frac{\text{signal power}}{\text{quantization noise power}} = \frac{P_s}{\Delta^2/12}$$

for an N-bit ADC:

full scale range $= 2^N \Delta$

maximum amplitude $= A_{max} = \frac{2^N \Delta}{2} = 2^{N-1}\Delta.$

maximum signal power $= \frac{(2^{N-1}\Delta)^2}{2}$

$$\Rightarrow \text{Peak } SQNR = \frac{(2^{N-1}\Delta)^2}{2 \cdot \frac{\Delta^2}{12}} = \frac{2^{2N-2}}{2} \cdot 12 = \frac{3}{2} \cdot 2^{2N}$$

$$SQNR_{dB} = 10 \log_{10}\left(\frac{3}{2} \cdot 2^{2N}\right)$$

$$\boxed{SQNR = 6.02N + 1.76 \text{ dB}}$$

Use Matlab to show ~~tones of periodic~~ tones in spectrum
↳ intuition on SFDR

$\Rightarrow \quad \Delta_1 \rightarrow \frac{1}{2}\Delta \quad \Rightarrow$ noise power $\downarrow \times 4 \Rightarrow SQNR \uparrow$ by 6dB

~~6dB~~ 1 bit increase $\Rightarrow$ 6 bit increase in SQNR.

Quantization Noise Simulation

①
- for $\frac{f_{in}}{f_s} = \frac{m}{N}$, for m to be much smaller than N $\simeq 9$, the quantization "noise" has mean square value $\simeq \frac{\Delta^2}{12}$
- for $\frac{f_{in}}{f_s} = \frac{9.01}{256} \Rightarrow E(e^2) \simeq \frac{\Delta^2}{12}$, with FFT leakage
- for $\frac{f_{in}}{f_s} = \frac{1}{8} \Rightarrow$ the quantization error samples are correlated with adjacent samples and $E(e^2) < \frac{\Delta^2}{12}$
  $\Rightarrow$ Approximation no longer valid

"QuantizationNoise1.m"

② See "QuantizationNoise2a1b.m".

$\frac{f_{in}}{f_s} = \frac{1}{1024}$ ← slow input to observe quantization noise periodicity.

$\Delta = 0.1$ and 0.2

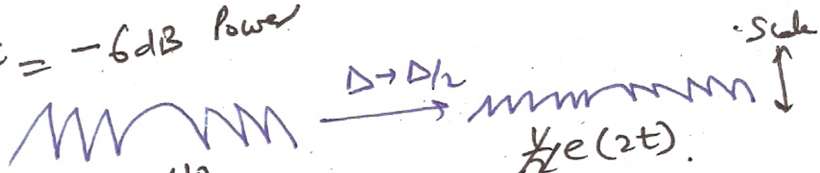
e[n] look like an FM waveform ⇒ local periodicity and a global periodicity
↳ fourier spectrum given by Bessel functions.
↳ Interesting mathematical analysis.
↳ chapter 2nd of the "Yellow-Book".

- for $\Delta = 0.1$
  Number of local periods = 44
  ⇒ in FFT spectrum, most of the tones lie around 44^{th} bin

- for $\Delta = 0.2$
  Number of local periods = 20
  ⇒ In the FFT spectrum, most tones around 20^{th} bin.

⇒ when Δ is halved.

tone power $= \left(\frac{1}{2}\right)^2 = \frac{1}{4}^{th} = -6\,dB$ Power



$\frac{1}{2}e(2t)$.
$\Rightarrow \frac{1}{2}E(f/2)$
⇒ tones spread out by 2.

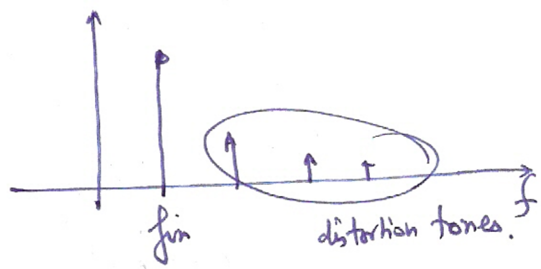
Also the tones spread out in frequency by 2
⇒ another tone power reduction by 3 dB

⇒ total tone power reduction = 9 dB with halving of Δ.

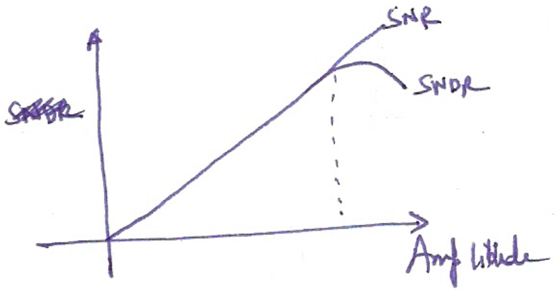
⇒ If Δ is halved ⇒ SFDR increases by $\simeq$ 9 dB.
- Quantization noise decreases by 6 dB.

Frequency domain measures :

- $SNDR = 10 \log_{10}\left(\frac{P_s}{P_N + P_{distortion}}\right)$.

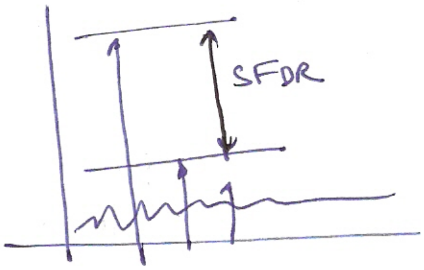


- $ENOB \triangleq \frac{SNDR - 1.76}{6.02}$



- SFDR ⇒ Spurious free dynamic range

- $SFDR_{dB} = 10 \log\left(\frac{\text{signal power}}{\text{largest spurious power}}\right)$



- **Harmonic Distortion :**

HD_k ⇒ Harmonic distortion w.r.t. the k^{th} harmonic

$HD_k = 10 \log\left(\frac{X_k^2}{X_1^2}\right)$

X_k^2 = rms of k^{th} component
X_1^2 = rms of the fundamental

THD = Total harmonic distortion

$$= 10 \log\left(\frac{\sum_{k=2}^{\infty} X_k^2}{X_1^2}\right).$$

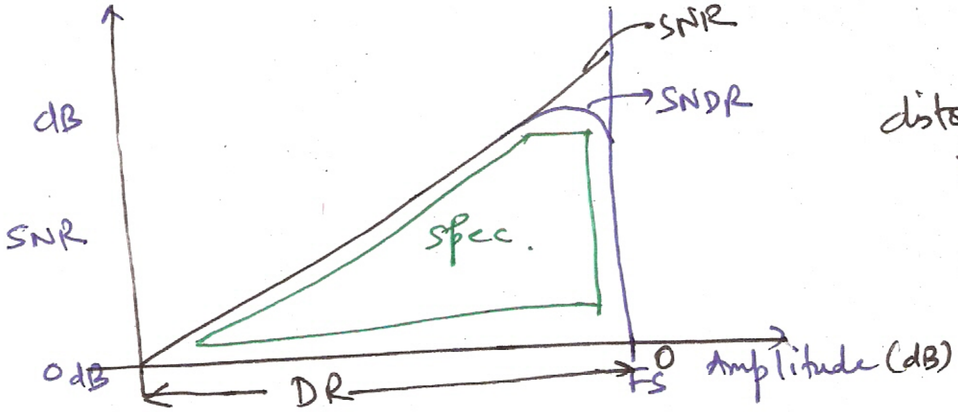
- Dynamic Range :(DR):

  - $DR = 10 \log\left(\frac{\text{Maximum signal power detected}}{\text{Smallest signal power detected}}\right)$

  - ~~the range of the~~

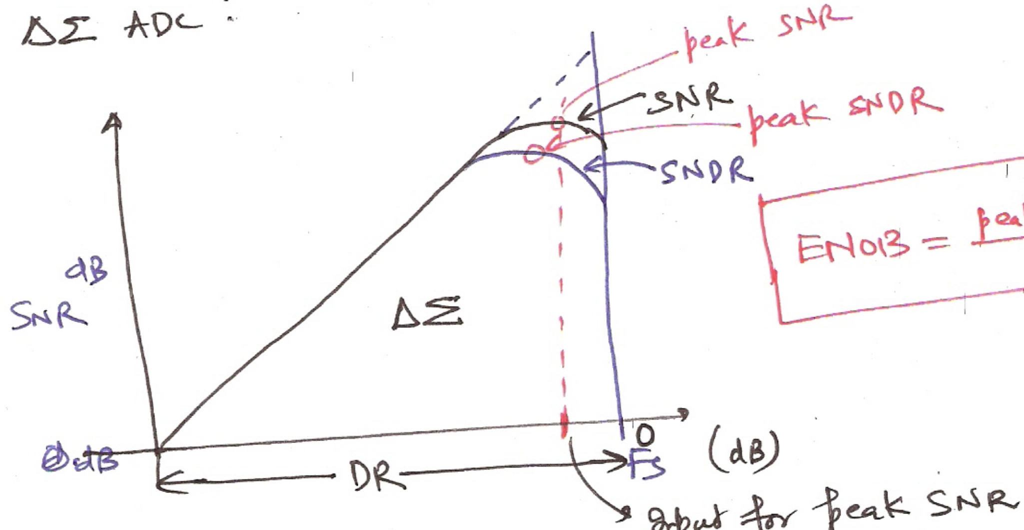
The range from the full scale (FS) to the smallest detectable signal is the dynamic range.

For a Nyquist rate ADC : $SNR \sim 10 \log \left(\frac{A^2/2}{\sigma_N^2} \right)$



distortion at higher signal amplitude

for a ΔΣ ADC :



$$ENOB = \frac{\text{peak SNDR} - 1.76}{6.02}$$

- SNR drops beyond maximum-~~input~~ stable input (MSA).
- See example slides.

ECE 697 Delta-Sigma Converters Design

Lecture#5 Slides

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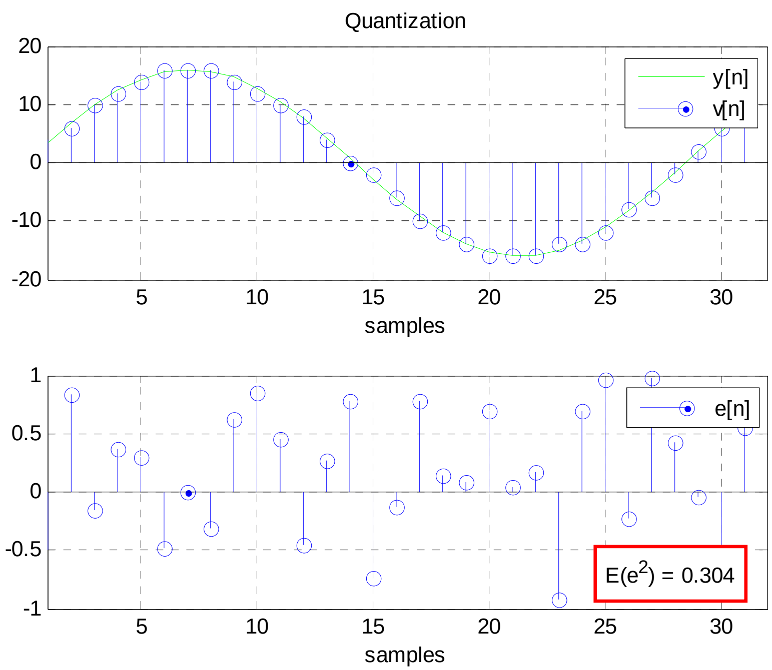
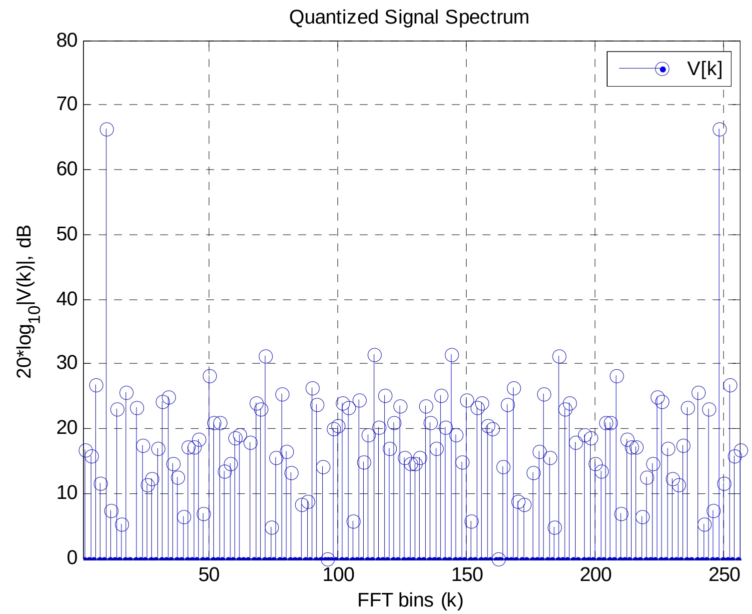
Quantization Noise Spectrum

Quantization Noise : Example 1



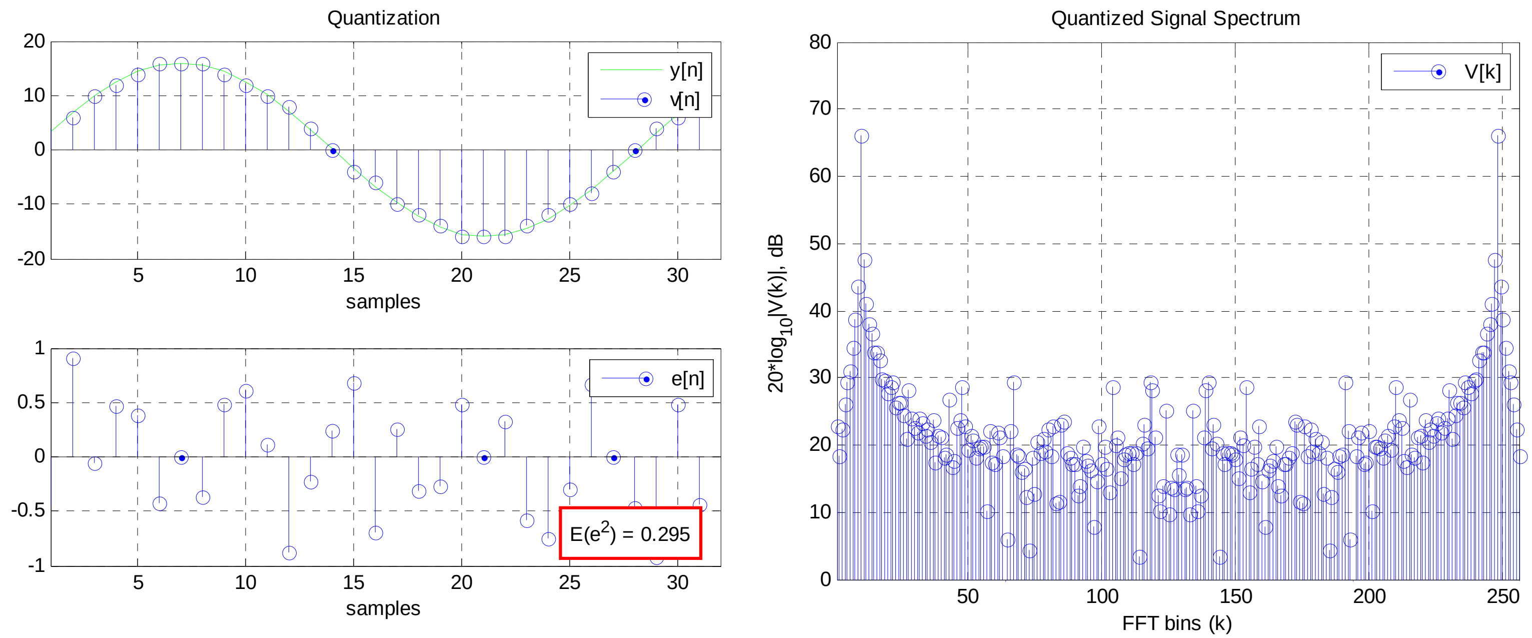


nLev=17, Δ=2, $f_{in}/f_s = 9/256$:

- $E(e^2) = 0.304 \approx \Delta^2/12$

file:Quantization_Noise1.m

Quantization Noise : Example 1 contd.



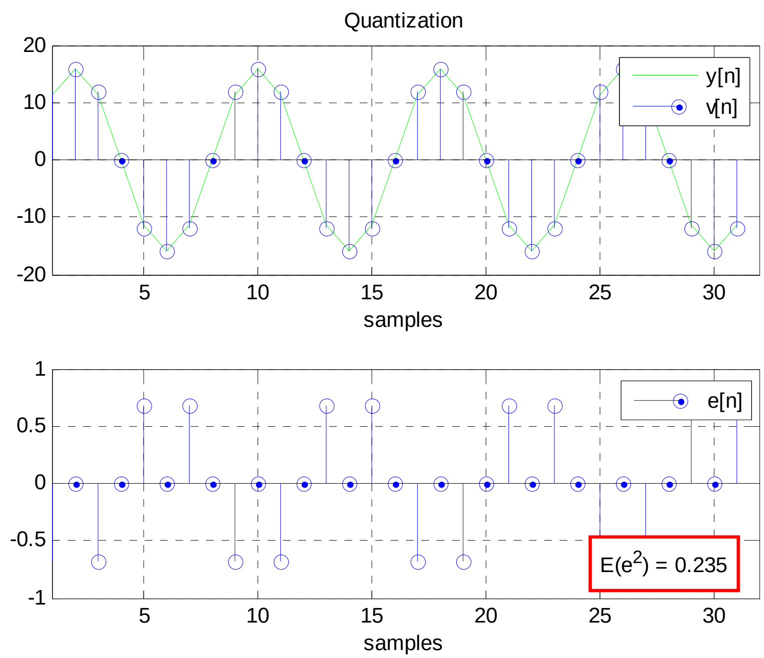
nLev=17, Δ=2, $f_{in}/f_s = 9.1/256$:

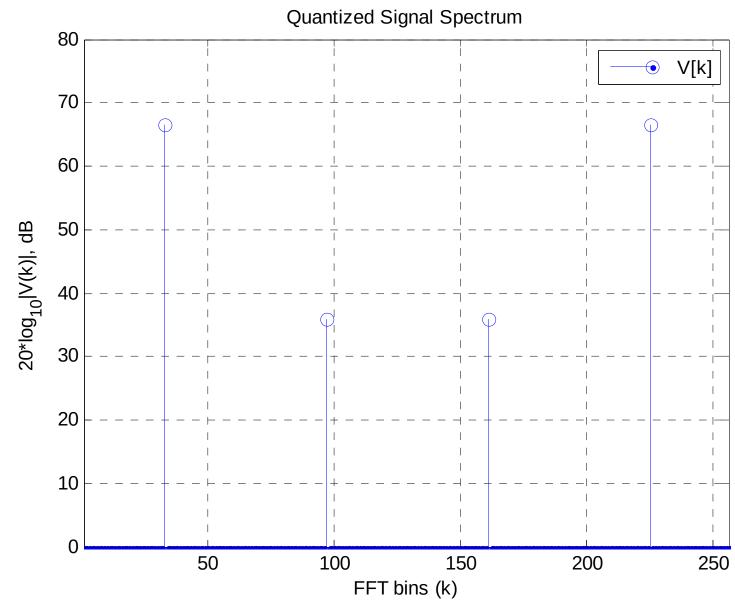
•$E(e^2) = 0.295 \approx \Delta^2/12$

•Notice the FFT leakage.

file:Quantization_Noise1.m

Quantization Noise : Example 1 contd.



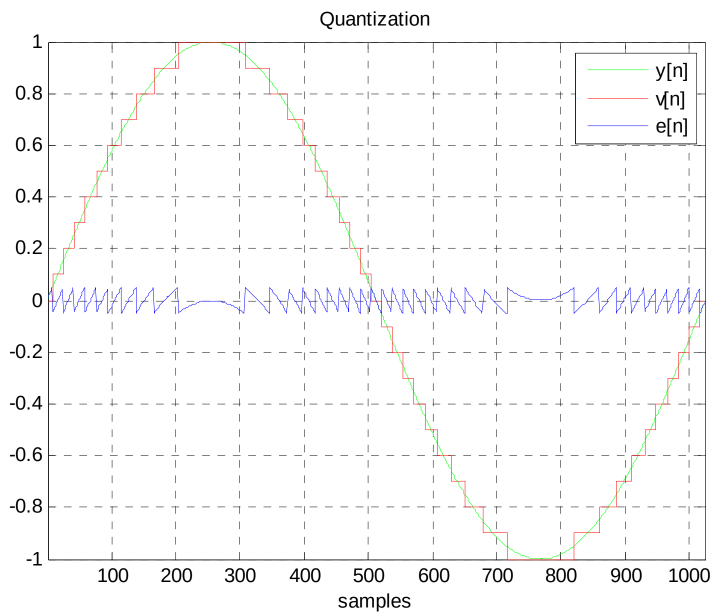


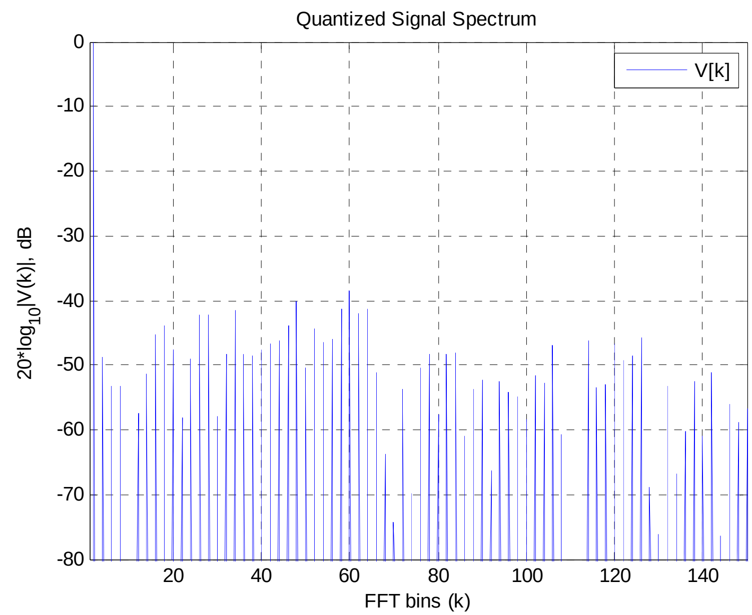
nLev=17, Δ=2, $f_{in}/f_s = 32/256 = 1/8$:

- $E(e^2) = 0.235 < \Delta^2/12$
  - Quantization *noise* approximation not valid

file:Quantization_Noise1.m

Quantization Noise : Example 2





A=1, Δ=0.1, $f_{in}/f_s = 1/1024$:

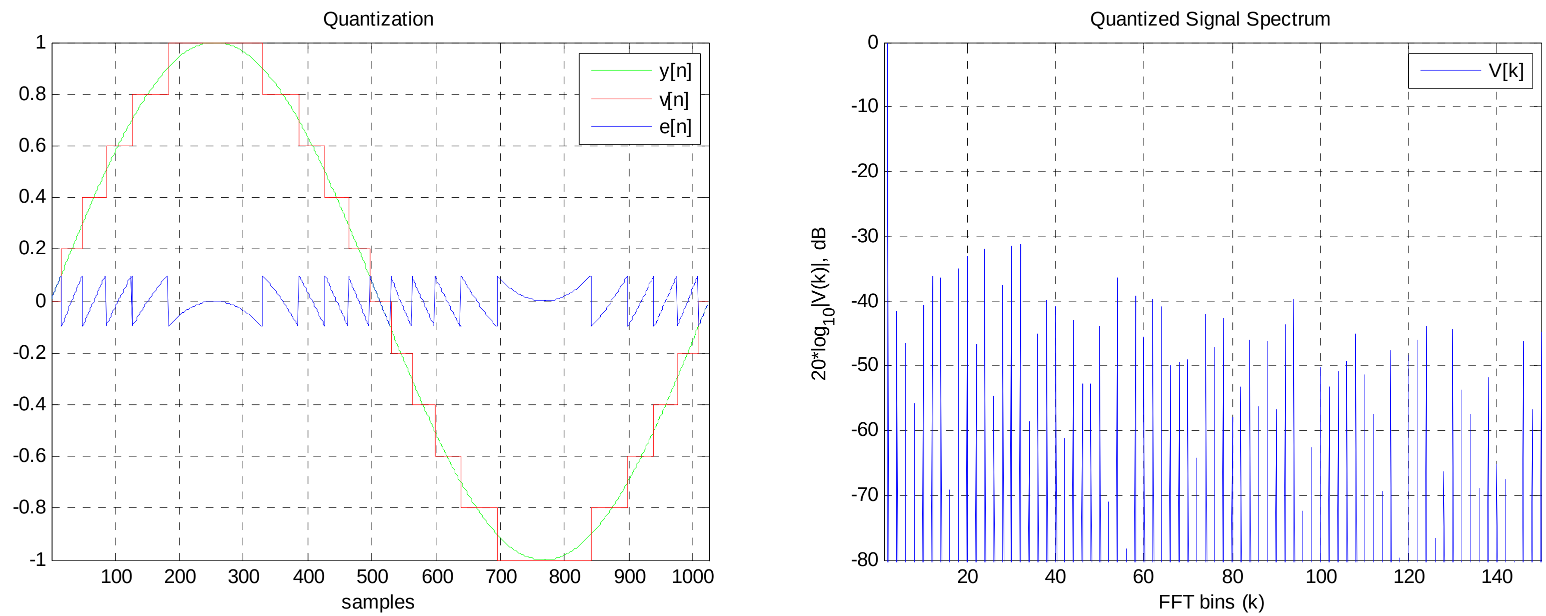
•Most of the tones around the 44th bin

•Average quantization noise floor lowers by 6 dB

•SFDR = -39 dB (SFDR increases by 9 dB if LSB size is halved)

file:Quantization_Noise2a.m

Quantization Noise : Example 2 contd.



A=1, Δ=0.2, $f_{in}/f_s = 1/1024$:

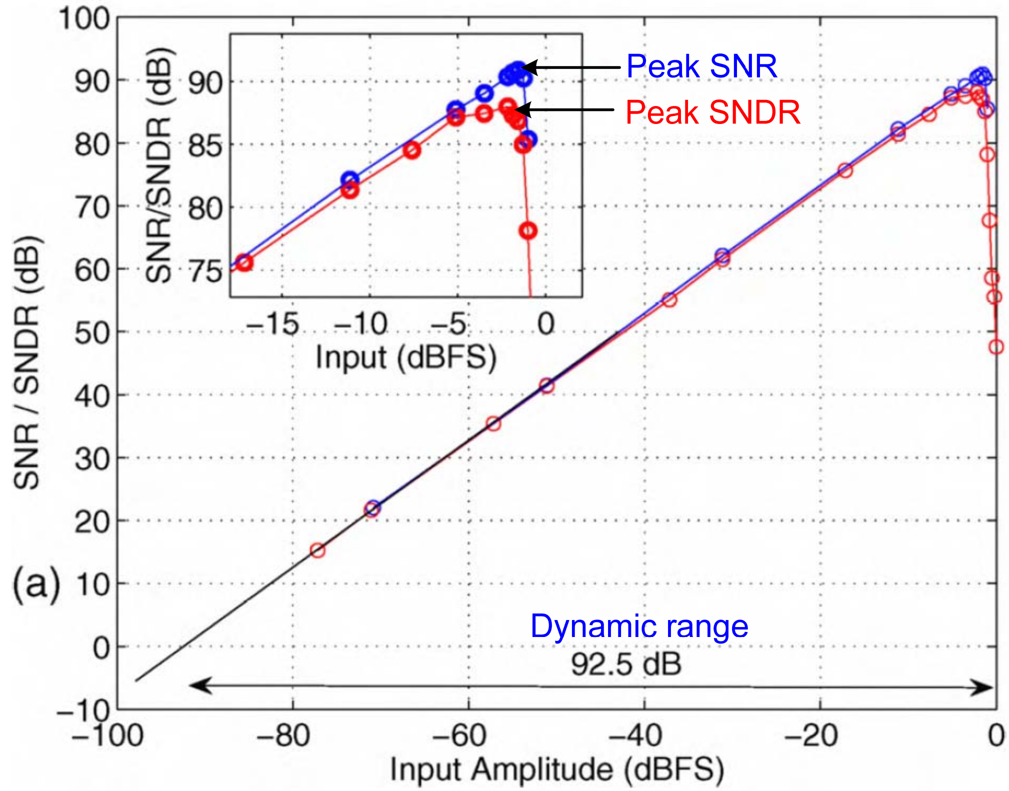
•Most of the tones around the 20th bin

•SFDR = -30 dB

•Quantizer spectrum not white and the error (e) is correlated with the input (y).

file:Quantization_Noise2b.m

Frequency Domain Measurements

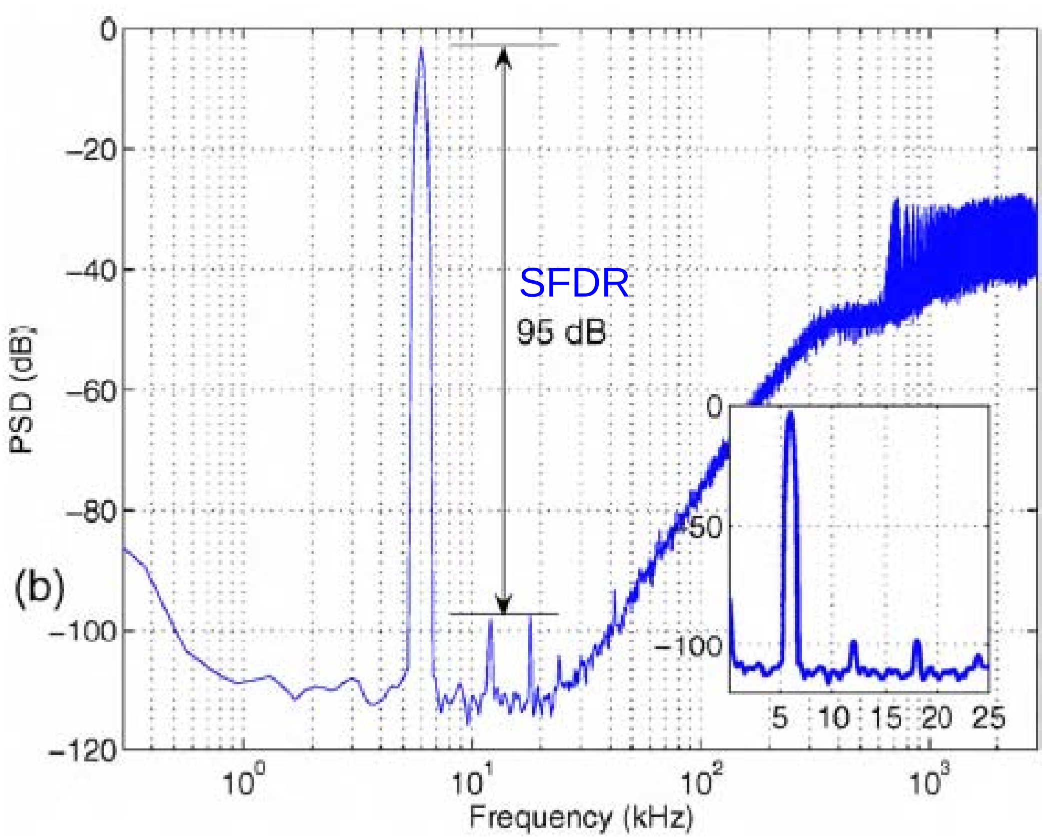


SUMMARY OF MEASURED ADC PERFORMANCE.

Signal Bandwidth/Clock Rate	24 kHz / 6.144 MHz
Quantizer Range	3.6 $V_{pp,diff}$
Input Swing for peak SNR	-1.6 dBFS
Dynamic Range/SNR/SNDR	92.5 dB/91 dB/88 dB
Active Area	0.24 mm^2
Process/Supply Voltage	0.18 μm CMOS/1.8 V
Power Dissipation (Modulator + References)	110 μW
Figure of Merit	0.0665 pJ/level

Reference [2]

Spurious (tone) **F**ree **D**ynamic **R**ange (SFDR)



Reference [2]

References

[1] M. Gustavsson, J. Wikner, N. Tan, *CMOS Data Converters for Communications*, Kluwer Academic Publishers, 2000.

[2] S.Pavan and P.Sankar, "A 110 μW Single Bit Audio Continuous-time Oversampled Converter with 92.5 dB Dynamic Range", *Proceedings of the European Solid State Circuits Conference (ESSCIRC), Athens, Greece, September 2009.*

[3] S. Pavan, N. Krishnapura, "EE658 VLSI Data Conversion Circuits Course," 2008, [Online]: http://www.ee.iitm.ac.in/~nagendra/videolectures/doku.php?id=start