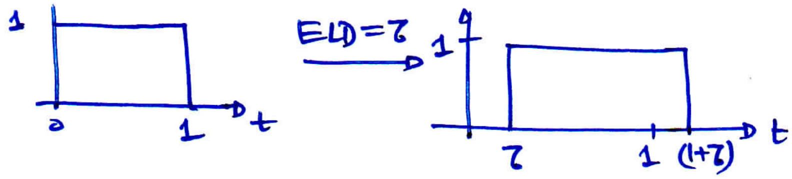
Effects of Excess Loop Delay and it's Compensation

NRZ DAC

$T_s = 1$

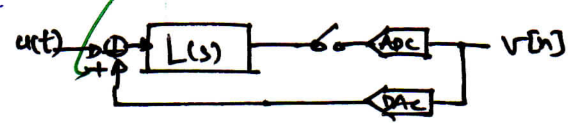


Refer to [Cherry 1999]

- Excess loop-delay alters α and β (DAC pulse shape)
  - ↳ affects the equivalence between $L(z)$ and $L(s)$.

Example: 2^{nd}-order CT $\Delta\Sigma$ with NRZ DAC

Note the +ve sign convention here



$$\Rightarrow L(s) = -\frac{1+1.5s}{s^2}, \qquad L(z) = \frac{-2z+1}{(z-1)^2} \text{ with } \hat{r}_s(t) = (0,1)$$

↳ DAC pulse-shape

$$\triangleq -\left(\frac{k_1 s + k_2}{s^2}\right)$$

- Assuming an excess loop-delay of τ
  - ↳ NRZ DAC pulse shape is delayed by τ

$\Rightarrow (\alpha, \beta) = (\tau, 1+\tau)$

- The formulae in the [Cherry 1999, Table III] work only for $\beta < 1$.

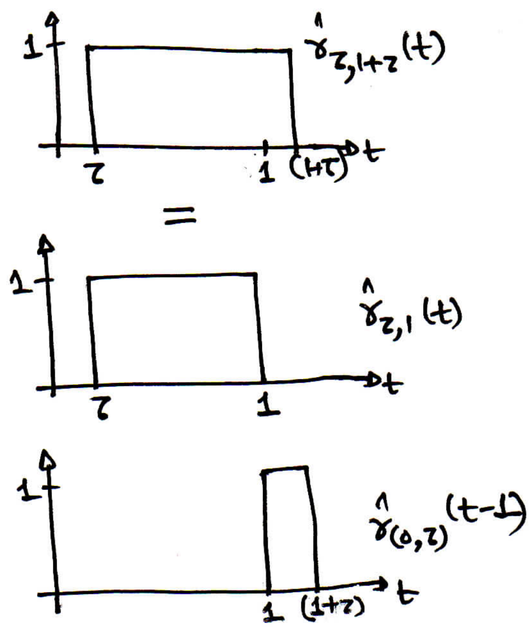
$\Rightarrow$ Modify the τ-delayed NRZ pulse as

$$\hat{r}_{(\tau,1+\tau)}(t) = \hat{r}_{(\tau,1)}(t) + \hat{r}_{(0,\tau)}(t-1)$$

$\Rightarrow$ linear combination of

a DAC pulse from (τ to 1) and

one sample delayed pulse from (0 to τ)



Since $L(s) = \frac{-1.5}{s^2} + \frac{-1}{s^2}$, apply s-domain to z-domain pole mapping Table (Cherry, Table III). with the modified DAC pulse.

$$\frac{1}{s-s_k} \longleftrightarrow \frac{y_0}{z-z_k}, \quad z_k = e^{s_k}$$

$$\Rightarrow \frac{1}{s} \longleftrightarrow \frac{(\beta-\alpha)}{z-1}$$

$$\& \quad \frac{1}{s^2} \longleftrightarrow \frac{y_1 z + y_0}{(z-1)^2}, \quad y_1 = \tfrac{1}{2}\left[\beta(2-\beta) - \alpha(2-\alpha)\right], \quad y_0 = \tfrac{1}{2}(\beta^2-\alpha^2).$$

for ELD of τ,

$$\frac{1}{s} \longleftrightarrow \frac{1-\tau}{(z-1)} + z^{-1}\frac{\tau}{(z-1)} \longrightarrow (A)$$

$$\frac{1}{s^2} \longleftrightarrow \frac{\frac{1}{2}(1-2\tau+\tau^2)z + \frac{1}{2}(1-\tau^2)}{(z-1)^2} + z^{-1}\cdot\frac{\frac{1}{2}(2\tau-\tau^2)z + \frac{1}{2}\tau^2}{(z-1)^2} \longrightarrow (B)$$

$1 \longleftrightarrow z^{-1} \leftarrow$ if we have a direct path around the quantizer.

The equivalent discrete-time loop response $L(z,\tau)$ with an ELD of τ.

No direct term here

$$\Rightarrow L(z,\tau) = \underbrace{-1.5}_{k_1} \times \left\{ \text{RHS of } (A) \right\} \underbrace{-1}_{k_2} \left\{ \text{RHS of } (B) \right\}$$

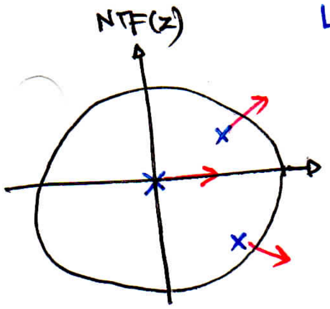
$$= \frac{(-2+2.5\tau-0.5\tau^2)z^2 + (1-4\tau+\tau^2)z + (1.5\tau-0.5\tau^2)}{z(z-1)^2}$$

- Verify: for $\tau=0$, we get back $L(z) = \frac{-2z+1}{(z-1)^2}$
- Note the increase in order by 1.
  - $\hookrightarrow$ 3rd-order system

$\Rightarrow$ NTF poles start moving towards unit circle as τ is increased.

$\hookrightarrow$ instability (IBN stays roughly constant but increases for higher τ)

$\hookrightarrow$ MSA decreases as $\tau \uparrow$



↳ DR ↓ as τ↑

↳ effect on performance is severe for $f_s \geq \frac{f_T}{10}$

↳ for higher-order NTFs, a lower OBG can provide some immunity against ELD.

↳ Take 'τ' into account in the design process.

- With the NRZ DAC pulse extending to the next clock period, the numerator order of $L(z, \tau)$ increases by 1

  ↳ system is not controllable. (?) ← refer to linear systems course

  ↳ need to introduce one more degree of freedom to make it controllable.

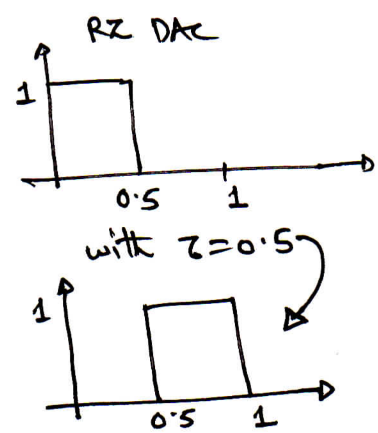
Excess-Loop Delay Compensation:

① DAC pulse selection

- When the DAC pulse extends beyond $t=1$, it creates additional pole in $L(z)$ and increases the system order by 1.

- If we used DAC pulses with $\beta < 1$, then the delayed DAC pulse would extend past 1 only if $\tau > 1-\beta$.

  ⇒ If we could use RZ DAC instead of NRZ, then $L(z)$ will remain 2^{nd}-order for $\tau \leq 0.5$.



⇒ For RZ DAC and $\tau \leq 0.5$:

If we know exactly what τ was, we could appropriately select the feedback coefficients $\{k_1, k_2\}$ to get exactly the same DT loop-response as $L(z) = \frac{-2z+1}{(z-1)^2}$.

$\Rightarrow$ Let $L(s) = \frac{k_1}{s} + \frac{k_2}{s^2}$, using $(\alpha, \beta) = (\tau, \tau + 1/2)$, $\tau < 1/2$

Using tables we obtain:

$$\frac{1}{s} \leftrightarrow \frac{1/2}{(z-1)}$$

$$\frac{1}{s^2} \longleftrightarrow \frac{y_1 z + y_0}{(z-1)^2}, \quad y_1 = \tfrac{1}{2}\left[(\tau+\tfrac{1}{2})(2-\tau-\tfrac{1}{2}) - \tau(2-\tau)\right] = (-\tau + 3/4)$$

$$y_0 = \tfrac{1}{2}\left[(\tau+\tfrac{1}{2})^2 - \tau^2\right] = \tfrac{1}{2}(2\tau + \tfrac{1}{2})\tau = \tau(\tau + 1/4).$$

$$\Rightarrow L(z,\tau) = \frac{[4k_1 + k_2(3-4\tau)]z + [-4k_1 + k_2(1+4\tau)]}{8(z-1)^2}$$

for $L(z,\tau) = \frac{-2z+1}{(z-1)^2}$, we need to select

$$\{k_1', k_2'\} = \{-5/2 - 2\tau, -2\} = \{k_1 + k_2\tau, k_2\}$$

Recap: initially with $\tau = 0$ we had $\{k_1, k_2\} = \{-5/2, -2\}$

After ELD compensation with $\tau > 0$ we get the modified loop-filter coefficients $\{k_1', k_2'\} = \{k_1 + k_2\tau, k_2\} = \{-\frac{5}{2} - 2\tau, -2\}$.

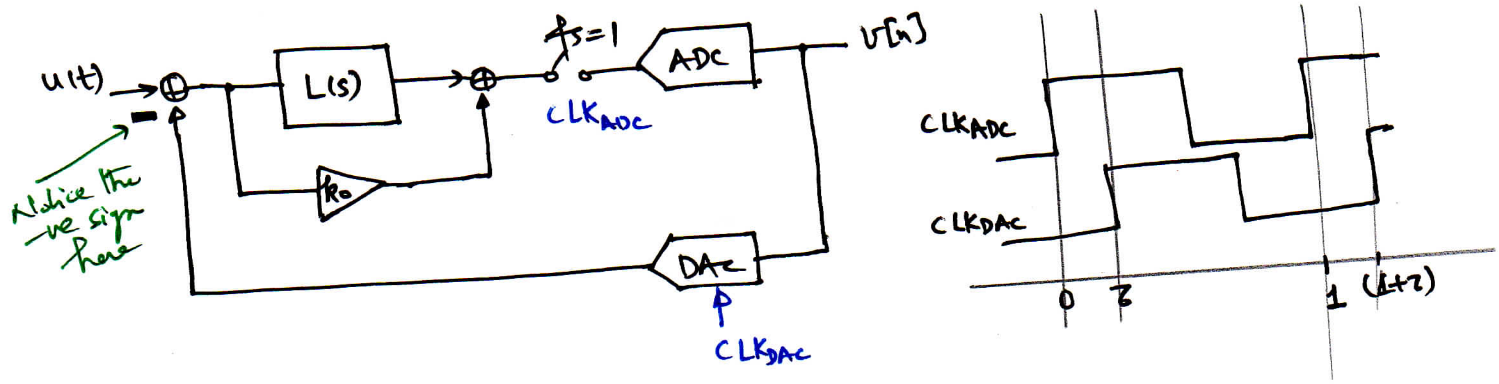
↳ this is termed as "coefficient tuning".

$\Rightarrow$ For a given $\tau \leq 1/2$ and with RZ DAC pulses, we can make the DT open loop-response exactly match $L(z)$ by "tuning" the coefficients k_1 and k_2.

↳ No extra path is required.

3 Direct feedback path around the quantizer

aka → the short loop.



⇒ The modified loop-filter response with the direct path "k_0" is

$$L'(s) = k_0 + \frac{k_1}{s} + \frac{k_2}{s^2}$$

. Extra feedback path provides the additional control parameter in the loop response.

we need the DT loop-response

$$L(z) = \frac{-2z+1}{(z-1)^2}.$$

Now, when the ELD is completely compensation :

$$\Rightarrow \quad k_0 z^{-1} + k_1 \times (\text{RHS of (A)}) + k_2 \times (\text{RHS of (B)}) = -\left(\frac{-2z+1}{(z-1)^2}\right) = \frac{2z-1}{(z-1)^2}$$

Note the sign change due to our convention

Going through the algebra, we get :

$$\left.\begin{aligned} 0.5\tau^2 k_2 - \tau k_1 + k_0 &= 0 \\ (0.5-\tau+0.5\tau^2)k_2 + (1-\tau)k_1 + k_0 &= 2 \\ -(0.5+\tau-\tau^2)k_2 + (1-2\tau)k_1 + 2k_0 &= 1 \end{aligned}\right\} \longrightarrow (T)$$

Solving this set of equation we get

$$\{k_0', k_1', k_2'\} = \{1.5\tau + 0.5\tau^2,\ 1.5+\tau,\ 1\}$$

Verify for $\tau = 0$, $\{k_0, k_1, k_2\} = \{0, 1.5, 1\}$ ← same as before