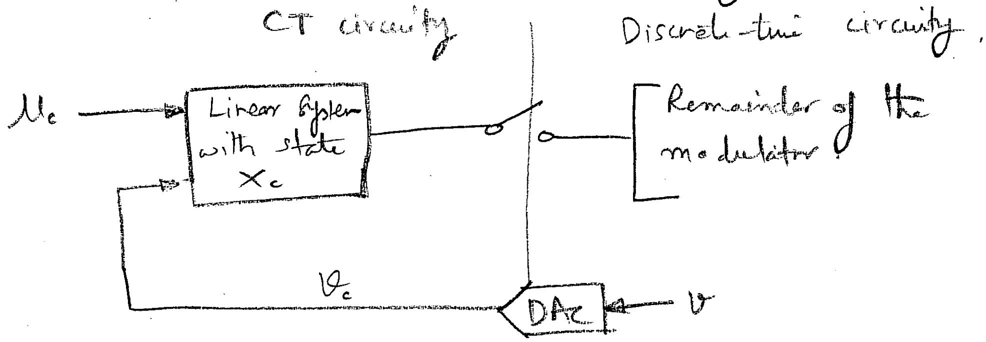
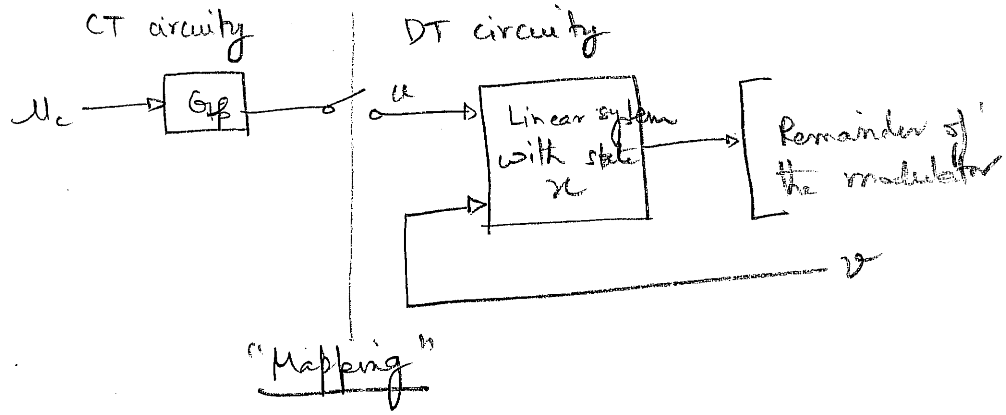
CT $\Delta\Sigma$ Design. **Lecture 22** Yellow book Chapter 4 Pg 158–162. (A1)

Exact transformation from a CT time to a DT system

↳ transform CT → DT for analysis

↳ map DT system to CT by inverse transformation

① 

② 

$f_s = 1\,Hz$:

State-space equations for the linear parts of the CT and DT modulator resp.

CT: $$\dot{x}_c = A_c x_c + B_c \begin{bmatrix} u_c \\ v_c \end{bmatrix}. \longrightarrow ①$$

DT: $$x[n+1] = A x[n] + B \begin{bmatrix} u[n] \\ v[n] \end{bmatrix}. \longrightarrow ②$$

① can be solved to yield the following equations — see ECE 560 linear Systems

$$x_c(t) = e^{A_c t} x_c(0) + e^{A_c t} \int_0^t e^{-A_c \tau} B_c \begin{bmatrix} u_c(\tau) \\ v_c(\tau) \end{bmatrix} d\tau$$

A sample of x_c may be found from previous sample and the linear system's inputs via, $t = (n+1)T_s = (n+1)$ as $T_s = 1s$

$$x_c[n+1] = e^{A_c(n+1)} x_c(0) + e^{A_c(n+1)} \int_0^{n+1} e^{-A_c\tau} B_c \begin{bmatrix} u_c(\tau) \\ v_c(\tau) \end{bmatrix} d\tau$$

$$= e^{A_c}\left[e^{A_c n} x_c(0) + e^{A_c n} \int_0^{n} e^{-A_c\tau} B_c \begin{bmatrix} u_c(\tau) \\ v_c(\tau) \end{bmatrix} d\tau \right]$$

$$+ e^{A_c(n+1)} \int_n^{n+1} e^{-A_c\tau} B_c \begin{bmatrix} u_c(\tau) \\ v_c(\tau) \end{bmatrix} d\tau$$

$$= e^{A_c} x_c[n] + \int_0^1 e^{A_c\tau} \cdot B_c \cdot \begin{bmatrix} u_c(n+1-\tau) \\ v_c(n+1-\tau) \end{bmatrix} d\tau$$

previous state

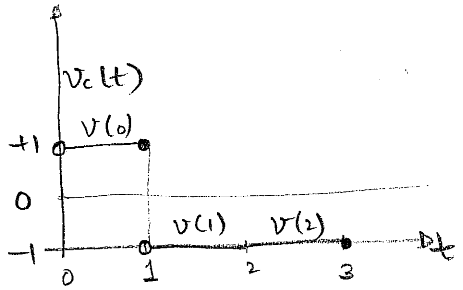
$$= e^{A_c} x_c[n] + \int_0^1 e^{A_c\tau} B_{c_1} u_c(n+1-\tau)\, d\tau + \underbrace{\int_0^1 e^{A_c\tau} B_{c_2} v_c(n+1-\tau)\, d\tau}_{A_c^{-1}(e^{A_c}-I)B_{c_2}} \longrightarrow (3)$$

where $B_c = [B_{c_1} \cdot B_{c_2}]$.

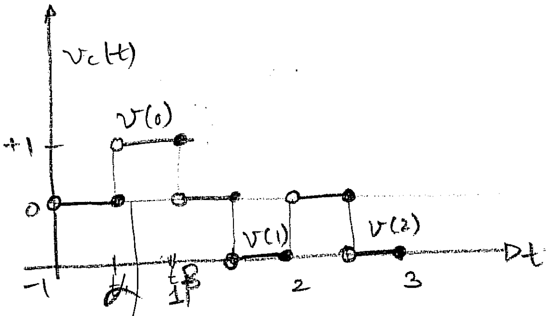
- The first integral in (3) represents the filtering operation on u_c, which precedes the sampling operation.
  - ↳ this filtering doesn't impact the stability of the modulator and can be neglected for our purpose.

- For the second integral we must know the DAC pulse shape (v_c).
  If v_c has NRZ pulse shape, then
  $$v_c(t) = v(n) \quad \text{for} \quad n < t < n+1.$$

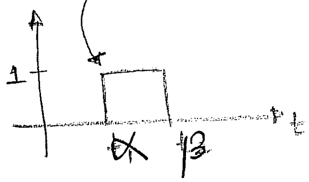
$$\Rightarrow \int_0^1 e^{A_c\tau} B_{c_2} v_c(n+1-\tau)\, d\tau = A_c^{-1}(e^{A_c} - I) B_{c_2} \cdot v[n]$$



(a) NRZ DAC



(b) RZ DAC



⇒ Systems given by equations ① , ② are identical if

$$A = e^{A_c},$$

and

$$B_2 = A_c^{-1}(A-I)B_{c_2} \longrightarrow ④$$

the inverse transformation

$$A_c = \ln A$$
$$B_{c_2} = (A-I)^{-1} A_c B_2 \longrightarrow ⑤$$

* If the DAC waveform is of the form $0 \le \alpha < \beta \le 1$, using same analysis

• $A = e^{A_c}$ & $B_2 = A_c^{-1}\left(e^{A_c(1-\alpha)} - e^{A_c(1-\beta)}\right)B_{c_2}$

The value of B_2 here is different from ④ by a factor of

$$\left[e^{A_c(1-\alpha)} - e^{A_c(1-\beta)}\right](A-I)^{-1} = \left(e^{-A_c t_2} - e^{-A_c t_1}\right)\left(e^{-A_c} - I\right)^{-1}.$$

⇒ d2cm function can be used with this correction factor applied for the DAC pulse shape.