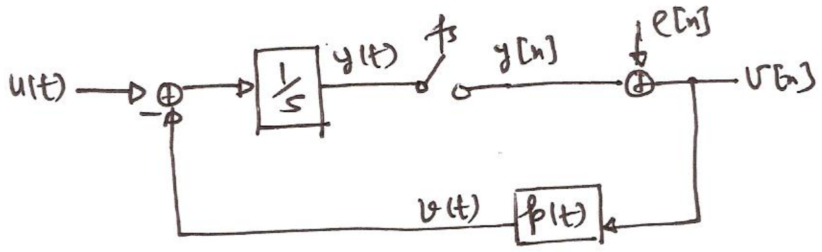
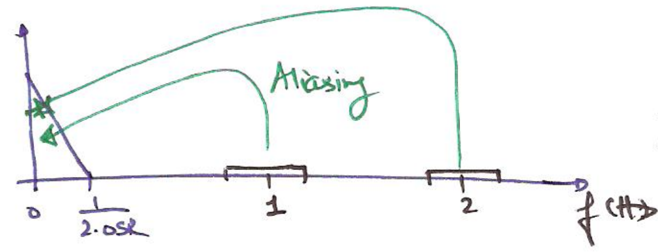
# Anti-alias filtering in CT-DSMs




$f_s = 1\,Hz$

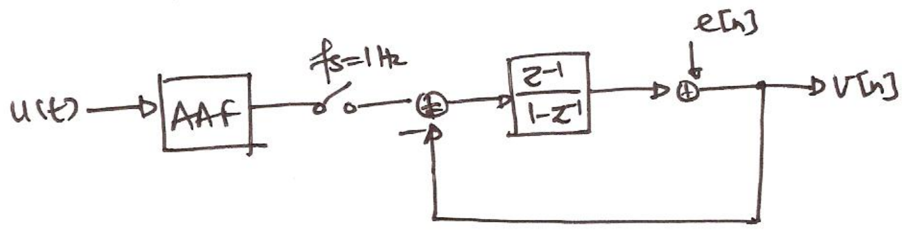


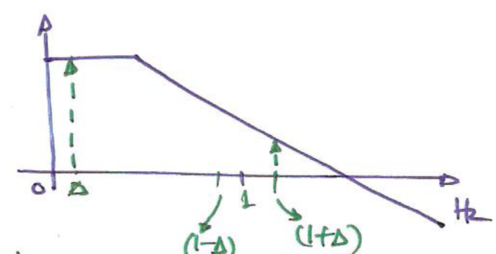

In-band frequency $\left[0, \frac{1}{2\cdot OSR}\right]$

Alias bands : $\left[k - \frac{1}{2\cdot OSR}, k + \frac{1}{2\cdot OSR}\right]$

$k = \pm 1, \pm 2$

- For DT-DSM, an external AAF filter is used.

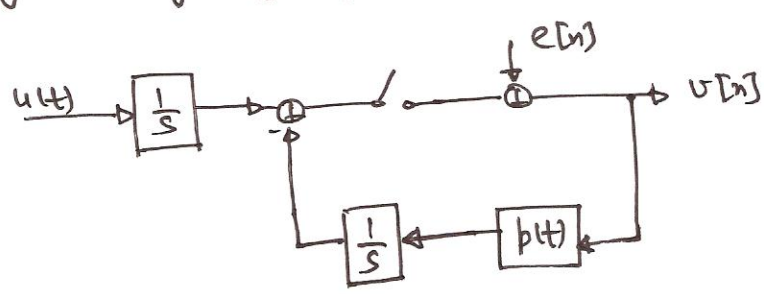


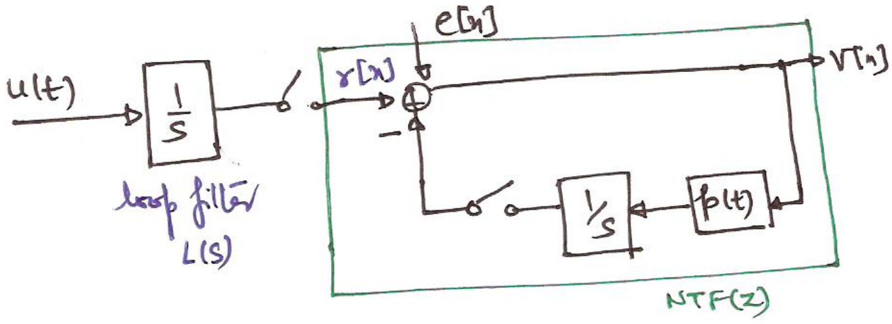

Need to understand how does the CT-DSM behave.

- intuitively, the continuous-time filter before the sampler should provide some alias suppression.

- Rearrange the signal flow graph for the CT-DSM :

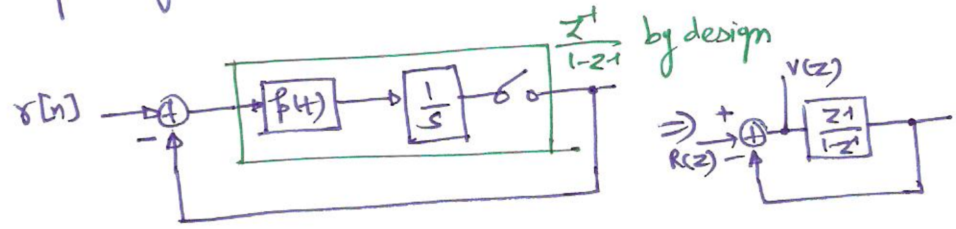



push $\frac{1}{s}$ out of the loop.



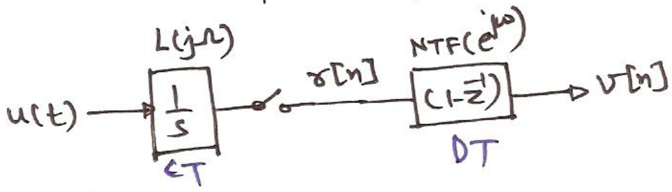

Cascade of a CT and a DT filter.

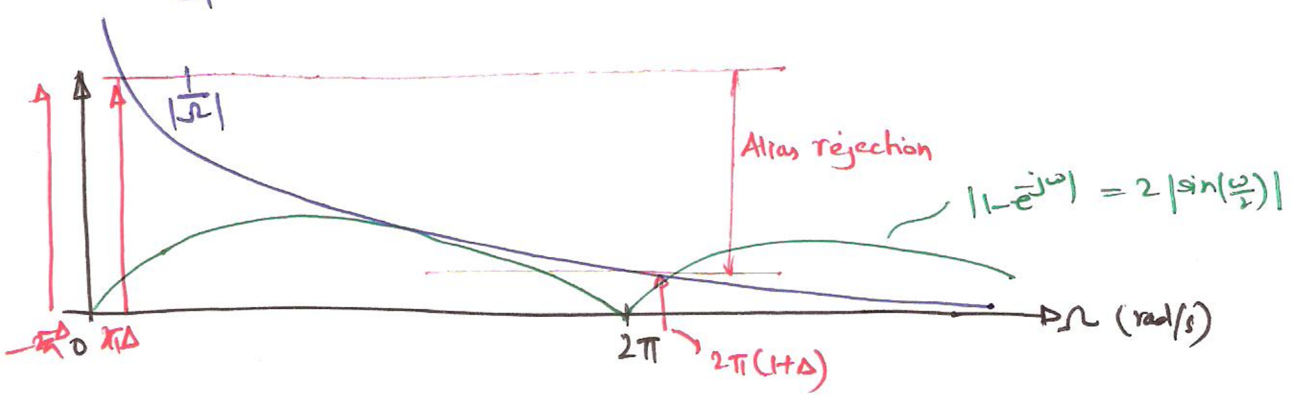
observe that the response of the DT block is same as that of the NTF.




$$\Rightarrow \quad R(z) \longrightarrow \boxed{(1-z^{-1})} \longrightarrow V(z)$$

$\Rightarrow$ Let $e[n]=0$, for the signal path




For sinusoidal input of frequency $2\pi\Delta$,

$$|V(e^{j\omega})| = \frac{1}{2\pi\Delta}\cdot\left|1-e^{-j2\pi\Delta}\right| \cong \frac{1}{2\pi\Delta}\cdot 2\pi\Delta = 1.$$

$\Rightarrow$ STF $=1$ for a low-frequency sinusoidal input.

$\Rightarrow$ Loop gain $\times$ NTF gain $\cong 1$ at low frequencies.

- A tone at the first alias band.

$$f_{in} = 2\pi(1+\Delta) \quad \text{Hz}$$

$\Rightarrow$

$$|V(e^{j\omega})| = \frac{1}{2\pi(1+\Delta)} \cdot |1 - e^{-j2\pi\Delta}| = \frac{2\pi\Delta}{2\pi(1+\Delta)} = \frac{\Delta}{1+\Delta} \simeq \Delta$$

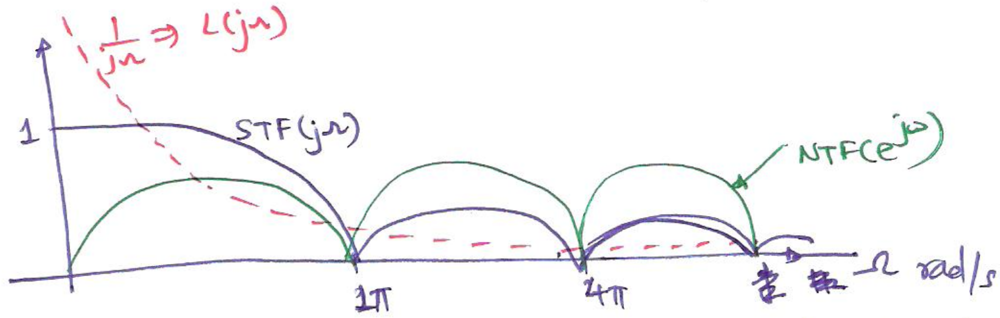
$\Rightarrow$ STF is very small for the first alias band.

$\Rightarrow$ Alias rejection $\Rightarrow \frac{\Delta}{1+\Delta}, \frac{\Delta}{2+\Delta}, \ldots, \frac{\Delta}{k+\Delta}$

$\Rightarrow$ STF at the frequencies around alias band is much smaller than in the signal band

↳ implicit anti-aliasing occurs in the CT-DSM

↳ AAF is much better for higher order loop-filters.




$\Rightarrow$ STF has notches at the alias-bands due to the NTF response in cascade with the loop-filter response

$\Rightarrow$ Double suppression of the interferers in the alias bands

↳ very lucrative feature of the CT-DSMs.

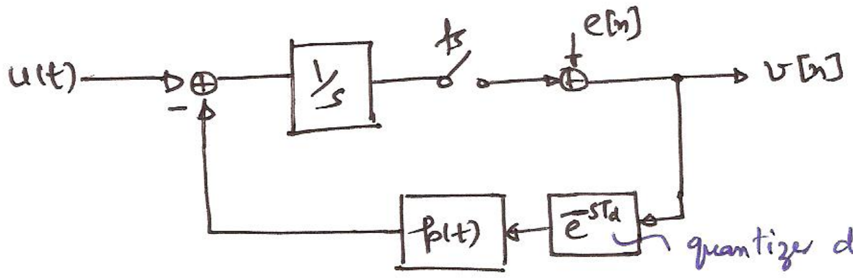
In general, the alias rejection is given by

$$\frac{|L[2\pi(1+\Delta)]|}{|L(2\pi\Delta)|}$$

- For generic, higher order topologies, we will revisit this topic later.

# Excess loop-delay in CT-DSMs

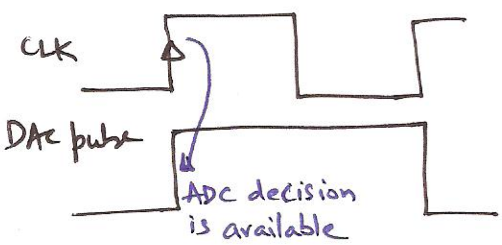
Non-idealities → Quantizer delay



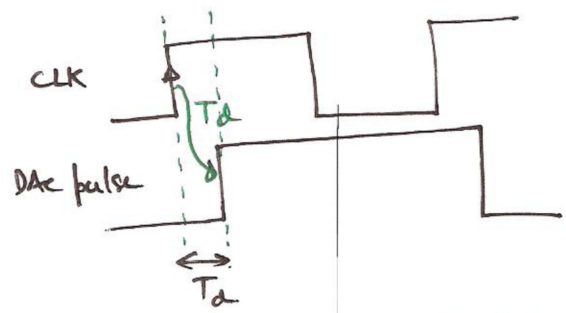

$f_s = 1\,Hz$

quantizer delay of $T_d$, $0 < T_d < 1$

Ideal (No delay) case

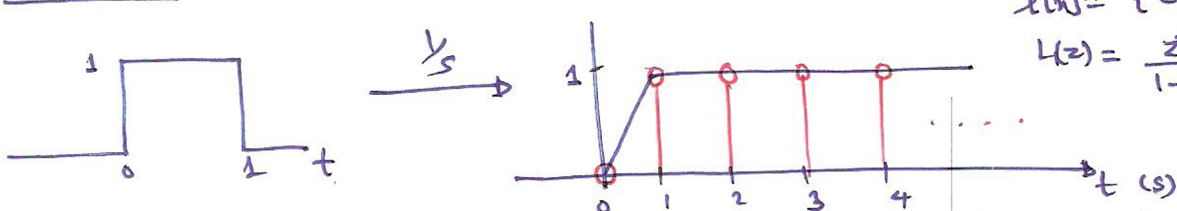



Real case (with quantizer delay)




⇒ More delay in the loop ⇒ NTF will be affected as $L_1(z)$ changes.

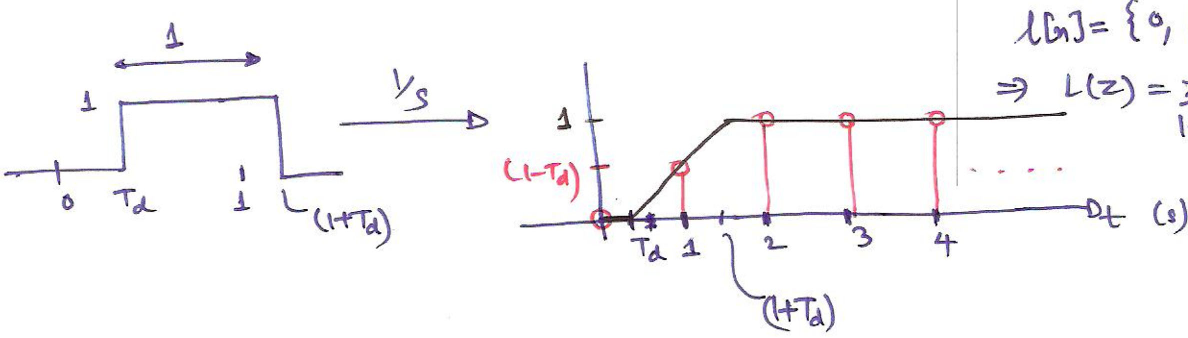
Ideal case:



$l[n] = \{0, 1, 1, \dots\}$

$L(z) = \frac{z^{-1}}{1-z^{-1}}$

with delay:



$l[n] = \{0, (1-T_d), 1, 1, \dots\}$

$\Rightarrow L(z) = \frac{z^{-1}}{1-z^{-1}} - T_d z^{-1}$

⇒ first sample after the unit-delay ($T_s$) is smaller than expected. ($l[0]$ must always be equal to 0).

with ELD:

$$NTF(z) = \frac{1}{1+L(z)} = \frac{1}{1+\frac{z^{-1}}{1-z^{-1}} - T_d z^{-1}}$$

$$= \frac{(1-z^{-1})}{1-T_d z^{-1}(1-z^{-1})} = \frac{(1-z^{-1})}{1-T_d z^{-1}+T_d z^{-2}}$$

for low frequency, $z \simeq 1$, $\Rightarrow NTF(z)\Big|_{z=1} \simeq (1-z^{-1})$
$\Rightarrow$ for DC input the NTF is unchanged.

- poles of the NTF are at $\frac{T_d \pm \sqrt{T_d^2 - 4T_d}}{2}$

$\Rightarrow$ As $T_d$ increases, the poles start moving out of the unit circle.
↳ modulator becomes unstable.
↳ too much delay $\Rightarrow$ phase of the loop-response is too high
↳ loop-becomes unstable.

$\Rightarrow$ As the order of the filter becomes higher, the excess loop delay (ELD) problem gets worse.
↳ amount of $T_d$ tolerable, as the loop-order increases, decreases.

$\Rightarrow$ only one NTF zero.
↳ noise-shaping is not enhanced
↳ but the order of the modulator is increased.