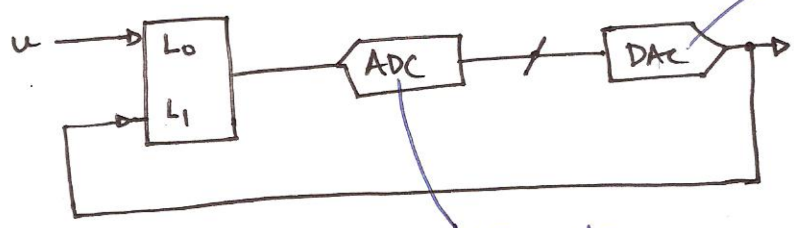
Continuous-time ΔΣ Modulators

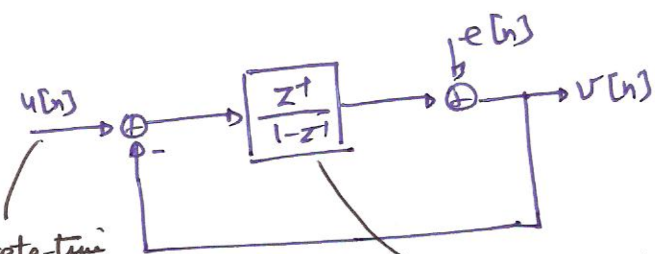


→ Capacitive DAC
Current steering DAC

→ fast ADC
↳ Flash
↳ maximum 4-5 bits.

recall that:
quantizer = ADC + DAC

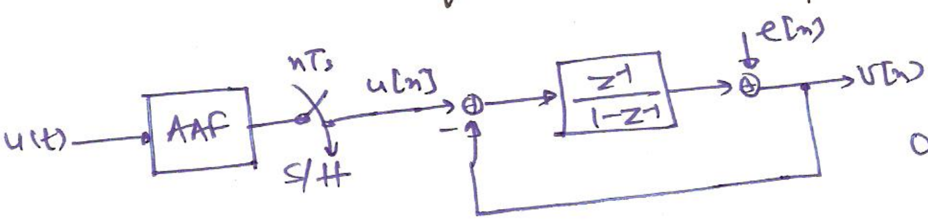
Discrete-time loop-filter:



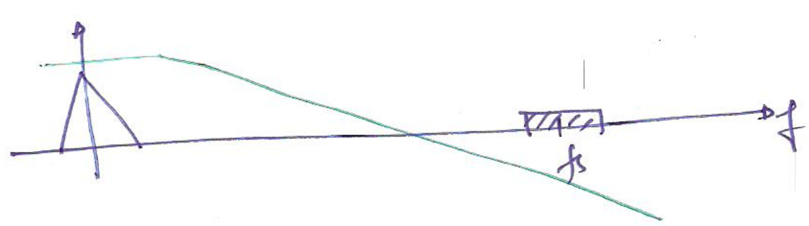
Discrete-time signal (already sampled)

→ Discrete-time accumulator
↳ implemented using switched capacitor circuits

DT-ΔΣ → well established technology
↳ initially used for audio recording applications (16-24 bits resolution).
↳ requires AAF in the front-end.

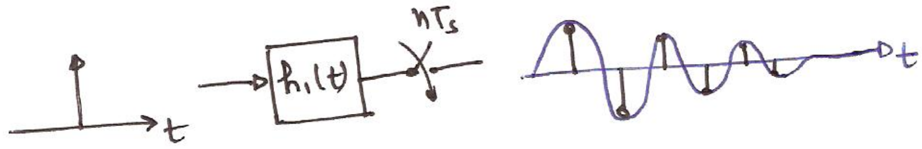


oversampling simplifies the AAF design.



Consider the DT impulse response, $h[n]$.

and.

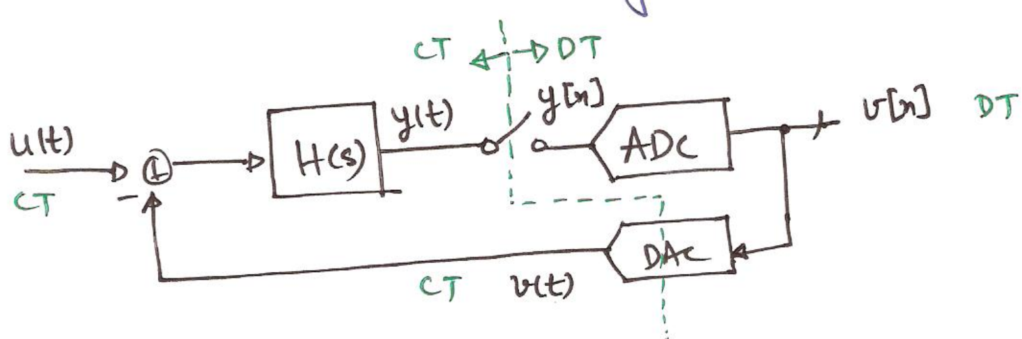


Can use a continuous-time (CT) impulse response $h_1(t)$ such that the sampled response is $h[n]$.

↳ many CT responses '$h_1(t)$' are possible.

↳ impulse invariance transformation

Basic idea ⇒ find a CT filter whose sampled impulse response is same as the DT filter.



The loop filter is realized using CT circuitry, but made to look like a discrete-time loop-filter.

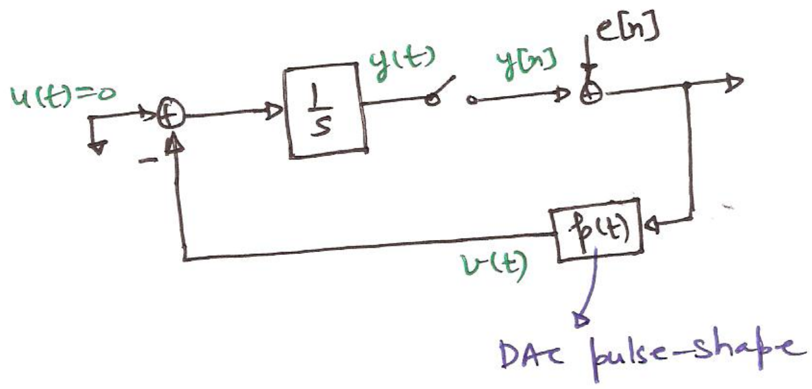
↳ the quantizer can not distinguish between the DT or CT loop-filter implementation.

CT-DSM has been gathering interest for past 10 years

↳ lower power dissipation

↳ being actively researched → requires deep understanding of signal and systems.

- We will initially study first-order CT-DSM and then later generalize to higher-order cases.

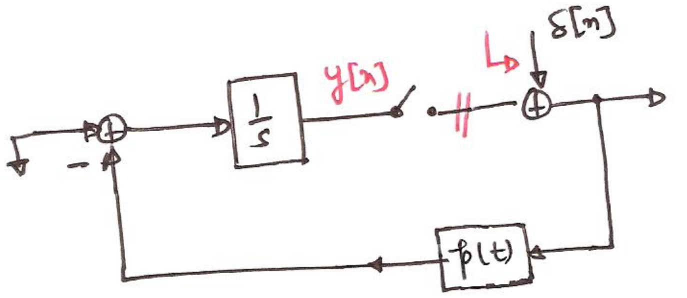


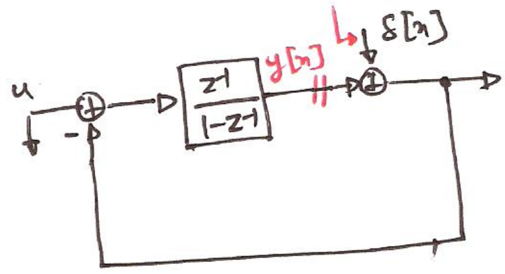
The DAC is converting a DT signal to a CT signal

↳ DAC pulse shape needs to be accounted for.

↳ The NTF should be same as the DT NTF.

Analysis: Break the loop after the sampler and inject a unit impulse.





- If a DT impulse is launched after the sampler, then the returned response after the sampler should be equal to the impulse response in DT modulator.

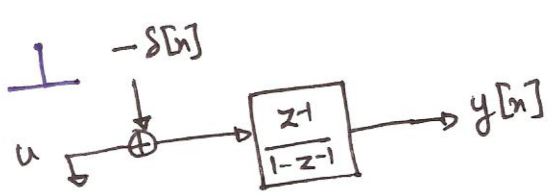
  ↳ This impulse response is the loop response seen by the quantization noise i.e. $L_1(z) \xleftrightarrow{z^{-1}} l_1[n]$.

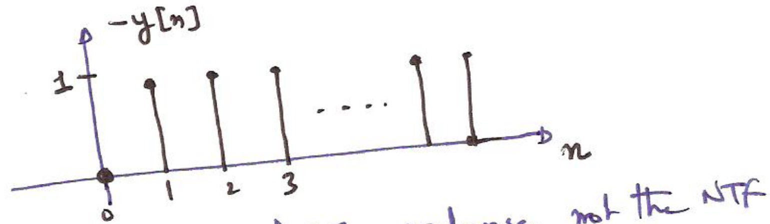
  - If $L_1(z)$ is same for the CT & the DT loops

    $\Rightarrow$ $NTF(z) = \frac{1}{1 - L_1(z)}$ is also guaranteed to be the same.

- Also the STF at low frequencies should be equal to 1.
  - ↳ low frequency components should be the same.
    - ↳ same DC gain.
  - ↳ set $u=0$ in both cases and just focus on NTF.

The DT impulse response

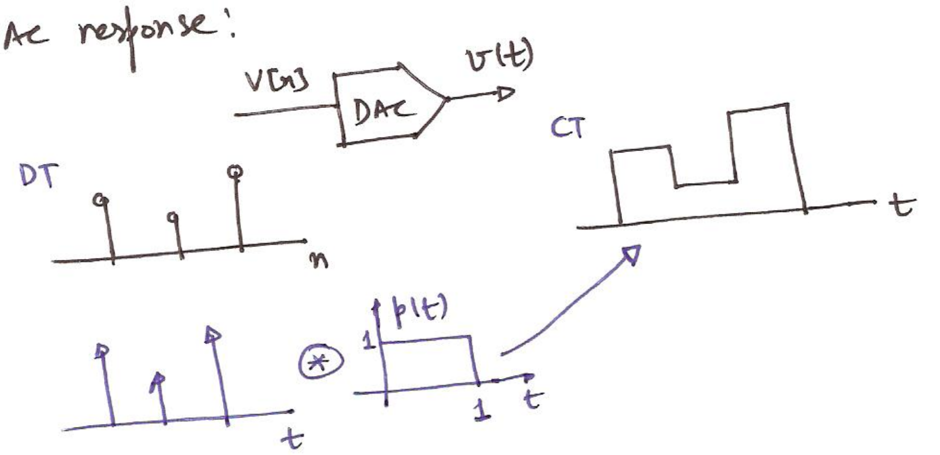


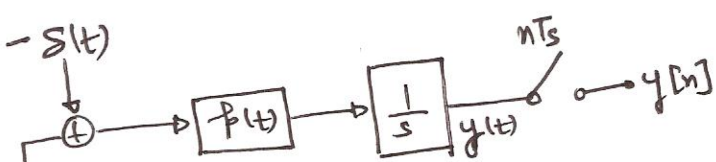


This is the $l_1[n]$ response not the NTF response.

⇒ Rather than working with $NTF(z) = \frac{1}{1-L_1(z)}$, deal with $L_1(z)$ and make it equal for DT as well as the CT ~~pass~~ loop.

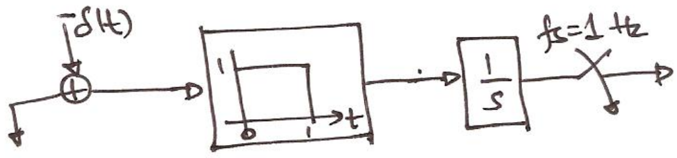
DAC response:





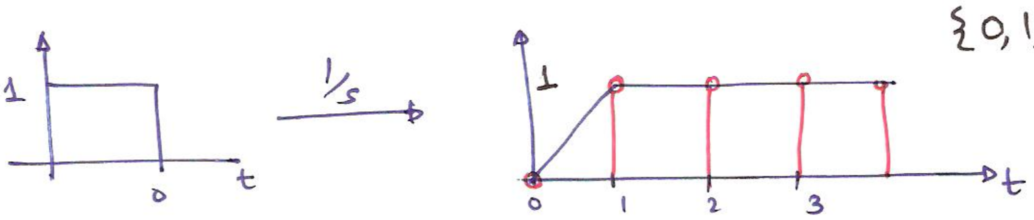
- Pulse shape of the feedback DAC influences the NTF.

NRZ Pulse-shape:



$T_s = 1s,\ f_s = 1\,\text{Hz}$

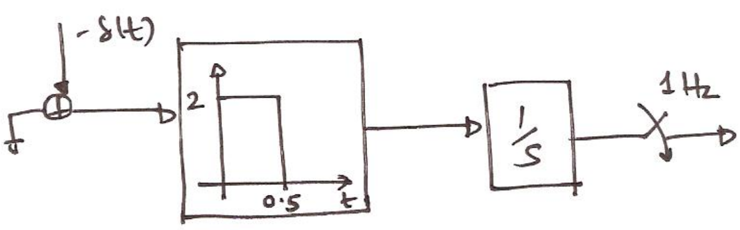
$\{0, 1, 1, \ldots\}$



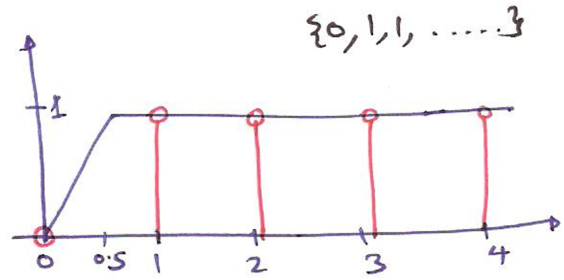
↳ L_1 response is same as in DT case

↳ NTF is also same for DT & CT.

RZ pulse-shape:



$\{0, 1, 1, \ldots\}$



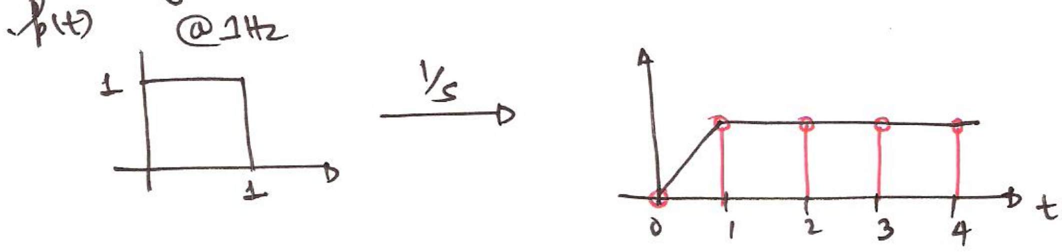
Need to scale the DAC pulse amplitude (×2) to obtain the same DC gain.

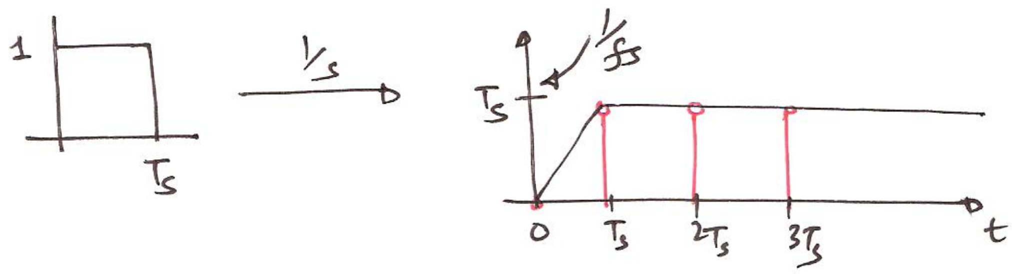
↳ same as DT impulse response

⇒ Can have any number of CT responses to result in the same DT impulse response.

⇒ same NTF(z) for many possibilities of the loop filter L(s).

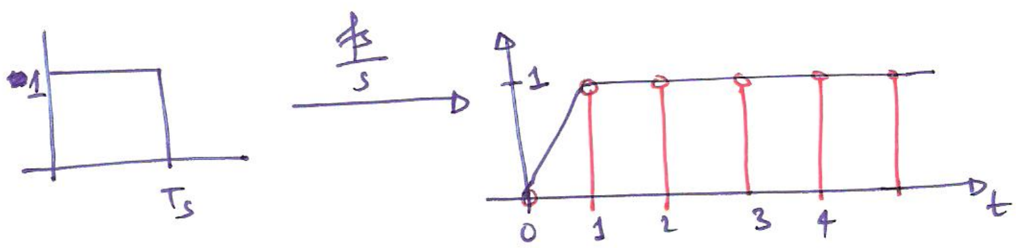
- Change the sampling rate





$\Rightarrow$ for sampling rate $f_s > 1$, the pulse shape width is reduced

↳ loop gain has changed.



$\Rightarrow$ $\boxed{\frac{1}{s}}$ is now $\boxed{\frac{f_s}{s}}$ or $\boxed{\frac{1}{sT_s}}$

↳ time-scaling of the loop filter

- Design for $f_s = 1$ Hz and at the last ~~st~~ step scale the loop filter to the appropriate response.

$$\left.\begin{array}{l} h(t) \xleftrightarrow{\mathcal{L}} H(S/\omega_0) \\ \Rightarrow h(\alpha t) \xleftrightarrow{\mathcal{L}} \frac{1}{|\alpha|} H\left(\frac{S}{\alpha\omega_0}\right) \end{array}\right\} \text{Time scaling of electrical networks.}$$

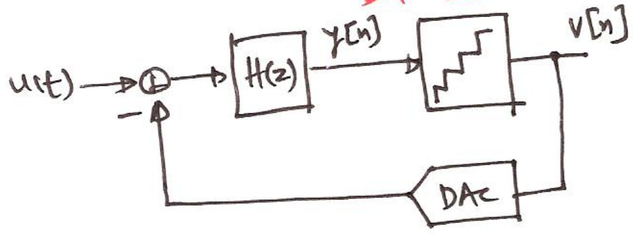
- Designing CT filters with $f_s = 1$ Hz also has advantages in MATLAB simulation where very large or very small quantities are avoided.

  ↳ truncation errors are avoided.

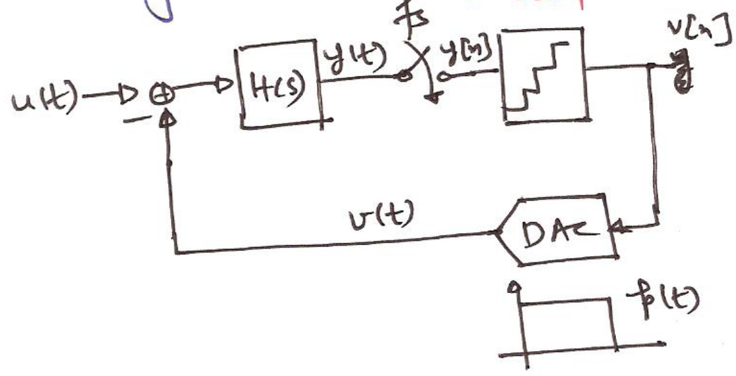
The Impulse-Invariant Transformation

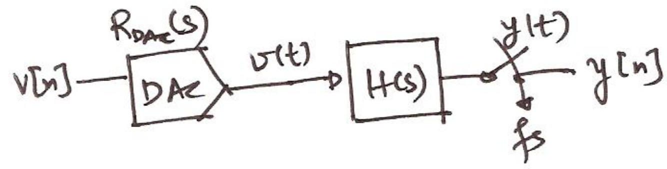
- Generalization of what we have done so far

DT-DSM



CT-DSM





- Both are discrete-time systems
- If the input to both the quantizers ($y[n]$) are same at the sampling instants, then for $v[n] = \delta[n]$,

$$y[n] \triangleq y(t)\Big|_{t=nT_s}$$

$$\Rightarrow \quad h[n] \triangleq y(t)\Big|_{t=nT_s,\ v[n]=\delta[n]}$$

$$\Rightarrow \quad \boxed{\mathcal{Z}^{-1}\{H(z)\} = \left(\mathcal{L}^{-1}\{R_{DAC}(s)\cdot H(s)\}\right)\Big|_{t=nT_s}}$$

* $p(t) \overset{\mathcal{L}}{\longleftrightarrow} R_{DAC}(s)$ is the impulse response of the specific DAC.

In time domain

$$h[n] = \left[p(t) \circledast h(t)\right]\Big|_{t=nT_s}$$

$$= \int_{-\infty}^{\infty} p(\tau)\cdot h(t-\tau)\, d\tau\Big|_{t=nT_s}$$

↳ This is called the impulse invariant transformation (IIT)

↳ impulse responses equal at sampling times.