SynthesizeNTF procedure (Algorithm).

* Textbook page 114

① Choose the order N of the modulator → Based upon the specified SQNR and OSR.

② Choose the NTF high-pass filter type (Butterworth, inv chebyshev) etc.

③ Place the 3-dB cuttoff frequency ω_{3dB} of the NTF slightly above the edge of the signal band ⇒ $\omega_{3dB} > \frac{\pi}{OSR}$.

④ Based on the choices made in Steps ① and ②, find the NTF zeros z_i's and the poles p_i of the NTF. Also to satisfy the realizability condition $H(\infty) = 1$, the NTF is of the form

$$H(z) = \prod_{i=1}^{N} \frac{z - z_i}{z - p_i} \quad \text{s.t.} \quad H(\infty) = 1$$

⑤ Predict the stability of the modulator. For multi-bit quantization, ~~the~~ may use the theorem for guaranteed stability:

$$MSA = \|u\|_\infty \leq M + 2 - \|h\|_1 \triangleq \frac{\Delta}{2}\left(M + 2 - \|h\|_1\right) \quad \} \text{ Toolbox uses } \Delta = 2$$

• For single-bit quantization use Lee's rule.

$$|H(-1)| = \prod_{i=1}^{N} \frac{1 + z_i}{1 + p_i} < 1.5$$

⑥ Confirm the stability estimation with extensive simulations.

⑦ If the predicted stability is unsatisfactory, shift the poles away from $z = -1$ point (i.e. reduce the OBG), while maintaining flat loop-filter gain in the signal band. ⇒ Done by reducing the ω_{3dB}.

⇒ reduce OBG and enhance stability

⑧ If the stability is robust, but the SQNR doesn't reach the specified limit, make the design more aggressive by increasing ω_{3dB}.

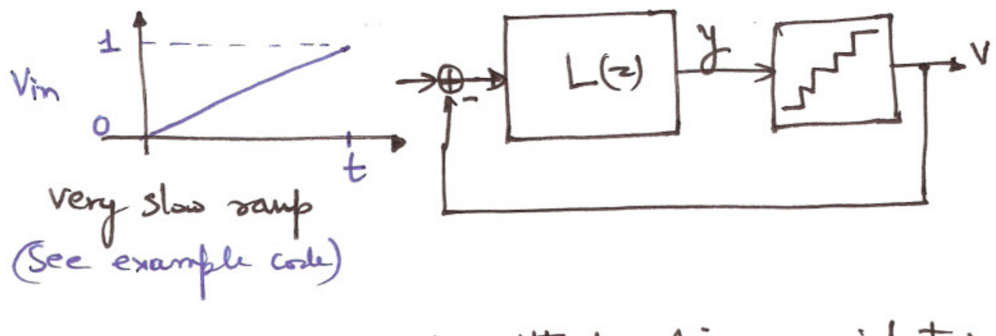
⑨ Goto Step ⑥ till ~~the~~ all the specs are met.

Estimating MSA (Maximum Stable Amplitude)

* Use simulation.
* Simulate for sinewave inputs for all possible frequencies in the signal band.
* for each frequency step up the input amplitude and compute in-band SNR.

- Beyond MSA, the NTF poles will move out of the unit circle
- ↳ Noise shaping is destroyed and the SNR falls.
  ↳ At this point the quantizer input ~~~~ $y[n]$ blows up.
- Simulate SNR function in the toolbox does the same.
  ↳ uses $f_{in} = \frac{f_s}{4 OSR}$.
- Time consuming and slow designing procedure.

Better Method

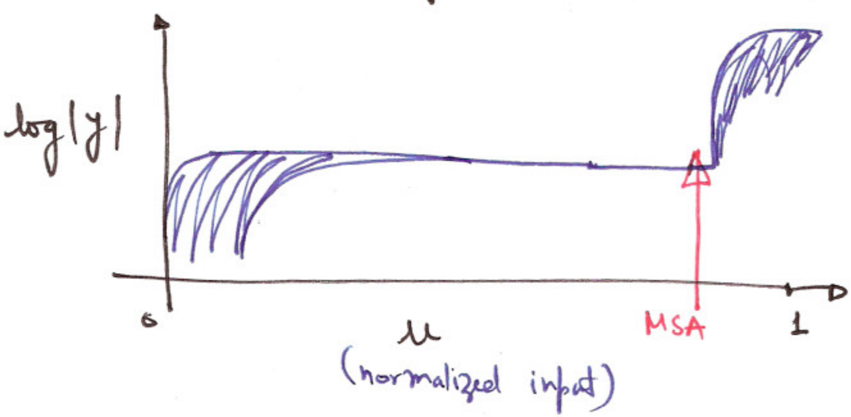


Estimating MSA without sinewave inputs:

- suggested by Lars Risbo.
- Use a slow ramp input with increasing value from 0 to full scale input.

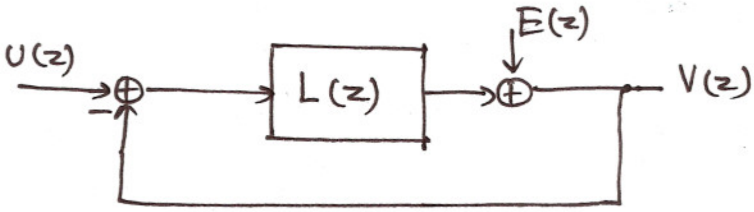
- Beyond the MSA, the NTF poles move out of the unit circle
- Observe $\log_{10}|y[n]|$. Beyond MSA, this value blows up to infinity rapidly. ($y[n] \rightarrow \infty$).
- Use 90% of this value where $y[n]$ blows up as a conservative estimate for the MSA.
- Results in an MSA close to the one predicted by sine wave inputs.
- Much quicker than simulateSNR function.

  ↳ write your own toolbox function to do this!



See example code "MSA_Risbo_Method.m"

Senstivity of a feedback loop



$$V(z) = U(z)\cdot\frac{L(z)}{1+L(z)} + E(z)\cdot\frac{1}{1+L(z)}$$

⇒ The loop rejects the disturbance E at frequencies where the loop-gain is high.

⇒ The senstivity of the loop is $\frac{1}{1+L(e^{j\omega})}$.

↳ how effectively the disturbance is suppressed is called the senstivity of the loop.

↳ The loop is insensitive to the disturbance at low frequencies. ∵ $L(e^{j\omega})$ is high at low frequencies

- Senstivity is same as the NTF.

- $h(0) = NTF(\infty) = 1$

The NTF can be expanded as

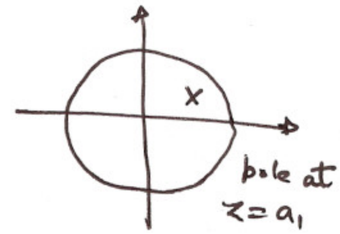
$$NTF(z) = \frac{(1+a_1z^{-1})(1+a_2z^{-1}+a_3z^{-2})\cdots}{(1+b_1z^{-1})(1+b_2z^{-1}+b_3z^{-2})\cdots}$$

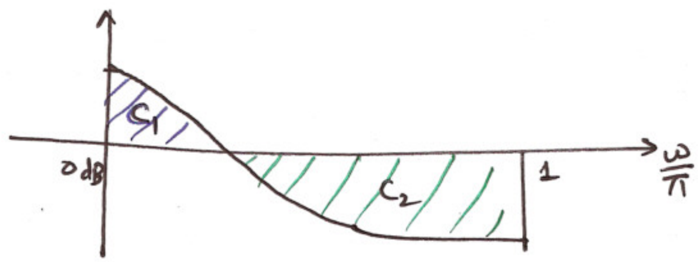
(complex zeros pair) (complex pole pair)

- poles must be within the unit circle.
- zeros are inside (on) the unit circle.

① It can be shown that

$$\int_0^{\pi} \log\left(\left|1 + a_1 e^{-j\omega}\right|\right) d\omega = 0 \quad \text{if} \quad |a_1| \le 1$$





$\Rightarrow$ $C_1 = C_2$

$\Rightarrow$ Area above the 0 dB line = Area below the 0-dB line

② Using ① we can show that

$$\int_0^{\pi} \log\left(\left|1 + a_2 e^{-j\omega} + a_3 e^{-j2\omega}\right|\right) d\omega = 0$$

if the roots of $(1 + a_2 z^{-1} + a_3 z^{-2})$ lie within (or on) the unit circle.

Using ① and ②, we ~~get~~ have

$$\int_0^{\pi} \log\left|NTF(e^{j\omega})\right| d\omega = \int_0^{\pi} \log\left|\frac{(1 + a_1 e^{-j\omega})(1 + a_2 e^{-j\omega} + a_3 e^{-j2\omega})\ldots}{(1 + b_1 e^{-j\omega})(1 + b_2 e^{-j\omega} + b_3 e^{-j2\omega})\ldots}\right| d\omega$$

$$= \int_0^{\pi} \log\left|1 + a_1 e^{-j\omega}\right| d\omega + \int_0^{\pi} \log\left|1 + a_2 e^{-j\omega} + a_3 e^{-j2\omega}\right| d\omega + \ldots$$

$$- \int_0^{\pi} \log\left|1 + b_1 e^{-j\omega}\right| d\omega - \int_0^{\pi} \log\left|1 + b_2 e^{-j\omega} + b_3 e^{-j2\omega}\right| d\omega + \ldots$$

$$= 0$$

$$\boxed{\int_0^{\pi} \log\left|NTF(e^{j\omega})\right| d\omega = 0}$$

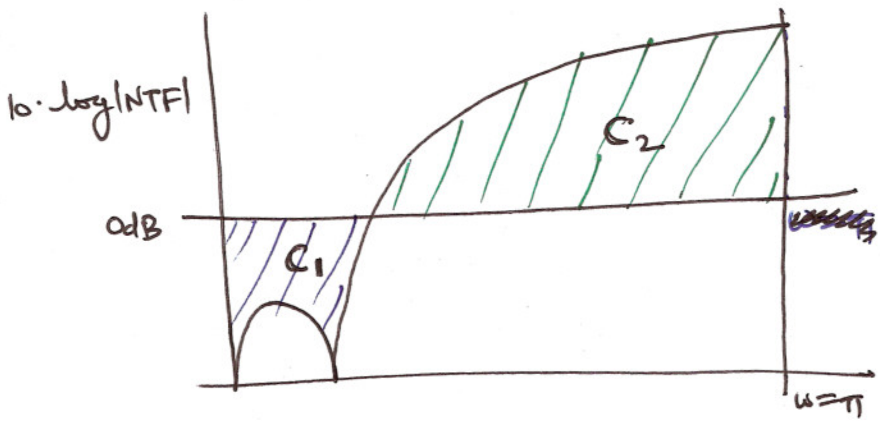
← Bode's Sensitivity Theorem

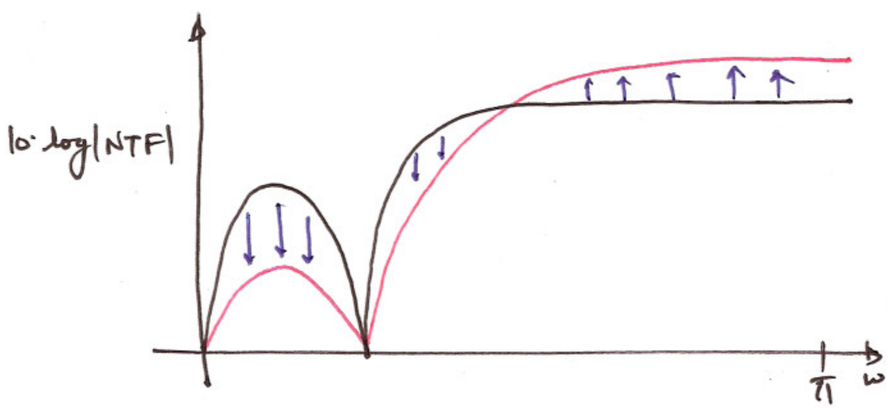
$\Rightarrow$ The integral of the log-magnitude of a 'stable' NTF is 0

⇒ Loop cannot be insensitive to the disturbance at all the frequencies.

↳ Loop is highly sensitive to the disturbance at high frequencies and very less sensitive ~~to~~ at low frequencies.

↳ maximum senstivity at $\omega = \pi$.





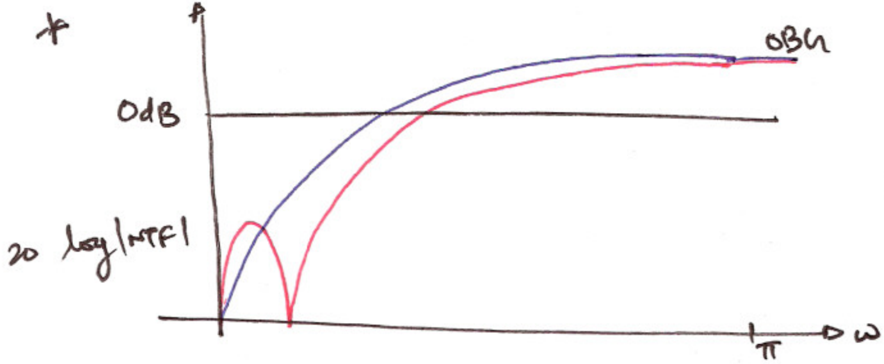
To keep $C_1 = C_2$, if the in band performance is improved, the OBG increases.

↳ Like a waterbed example.

⇒ "Good in-band performance comes at the expense of poor out-of band performance".

⇒ trade-off between IBN and OBG

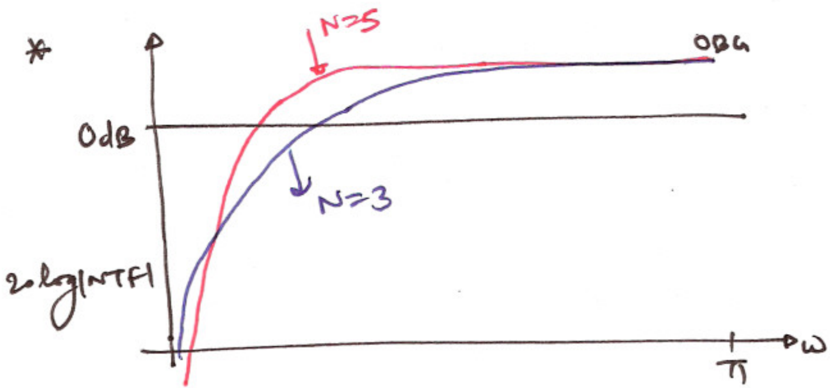
*



Complex NTF zeros better than choosing all NTF zeros at z=1.

⇒ lower IBN for same OBG.

*



Higher order NTFs exhibit lower inband noise (IBN) for the same OBG.

ECE 697 Delta-Sigma Converters Design

Lecture#13 Slides

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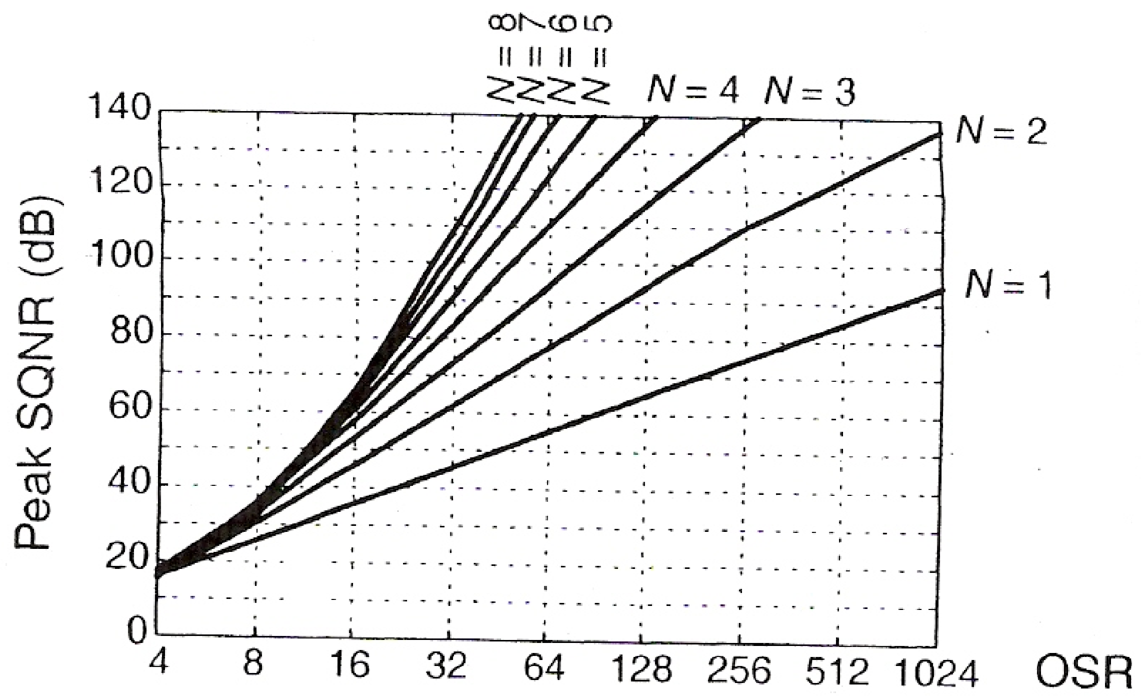
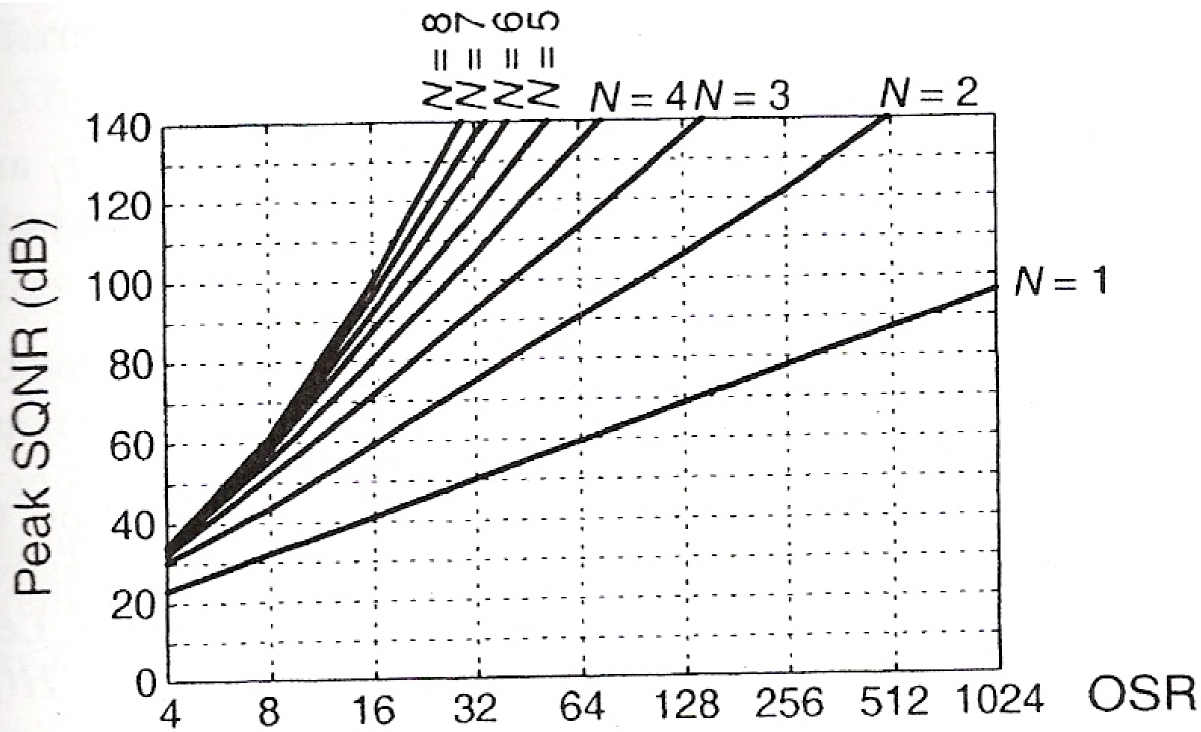
SQNR Limit for DSMs with 1-bit Quantizers



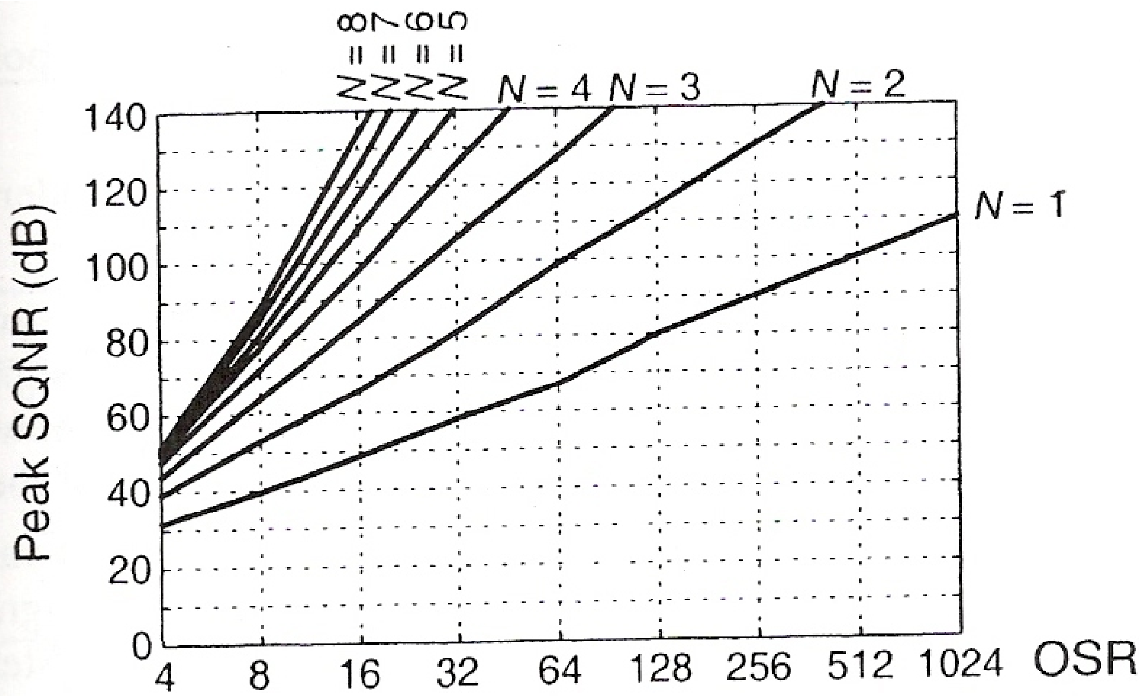
4.14: Empirical SQNR limit for 1-bit modulators of order *N*.

SQNR Limit for DSMs with 2-bit Quantizers



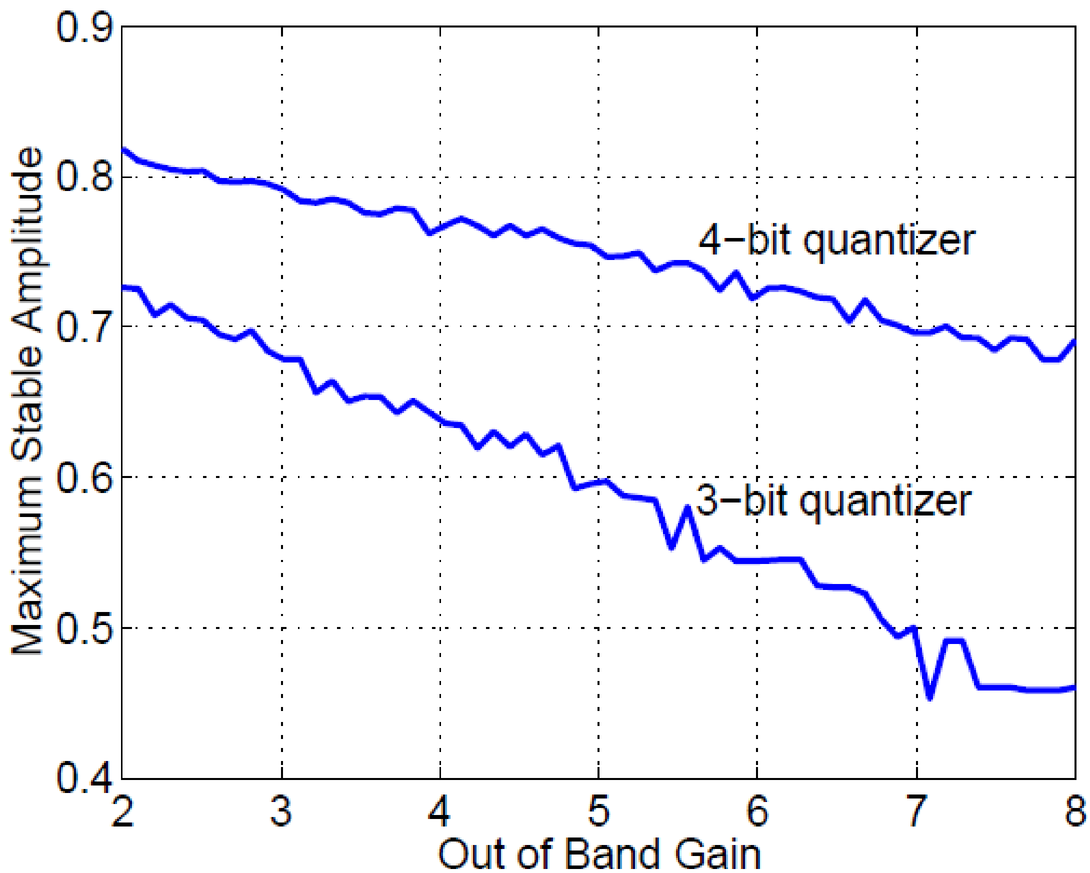
re 4.15: Empirical SQNR limit for modulators with 2-bit quantizers.

SQNR Limit for DSMs with 3-bit Quantizers



4.16: Empirical SQNR limit for modulators with 3-bit quantization.

MSA vs OBG for a Third-Order NTF



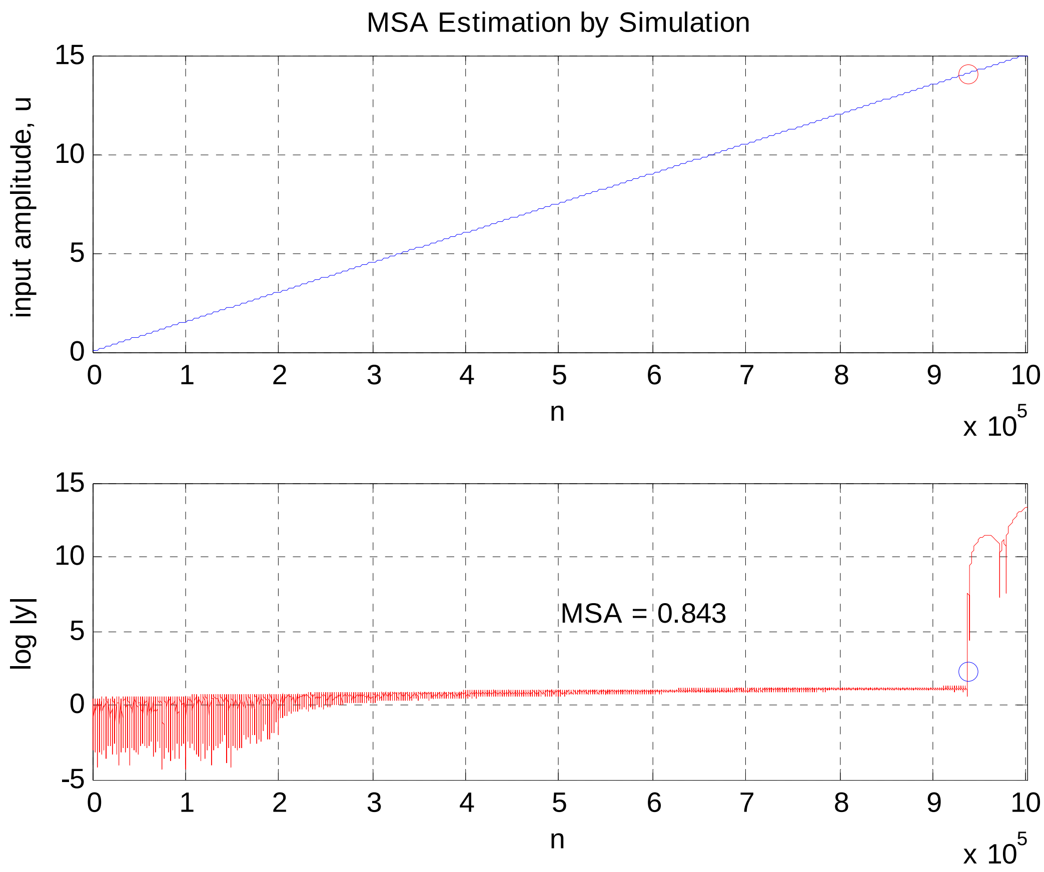
Estimating MSA (Maximum Stable Amplitude)

- MSA is found through extensive simulation.
- Simulate for input sinusoids of varying amplitudes for all possible signal frequencies in the signal band.
  - ✓ For every input amplitude compute in-band SNR.
  - ✓ Beyond the MSA, the NTF poles move out of the unit circle.
  - ✓ Noise shaping is disrupted and the in-band SNR drops.
  - ✓ At this point the quantizer input (y[n]) blows up.
- `simulateSNR` function in the toolbox does exactly the same.
- Time consuming and often impractical for iterative design.

Estimating MSA using Risbo's Method

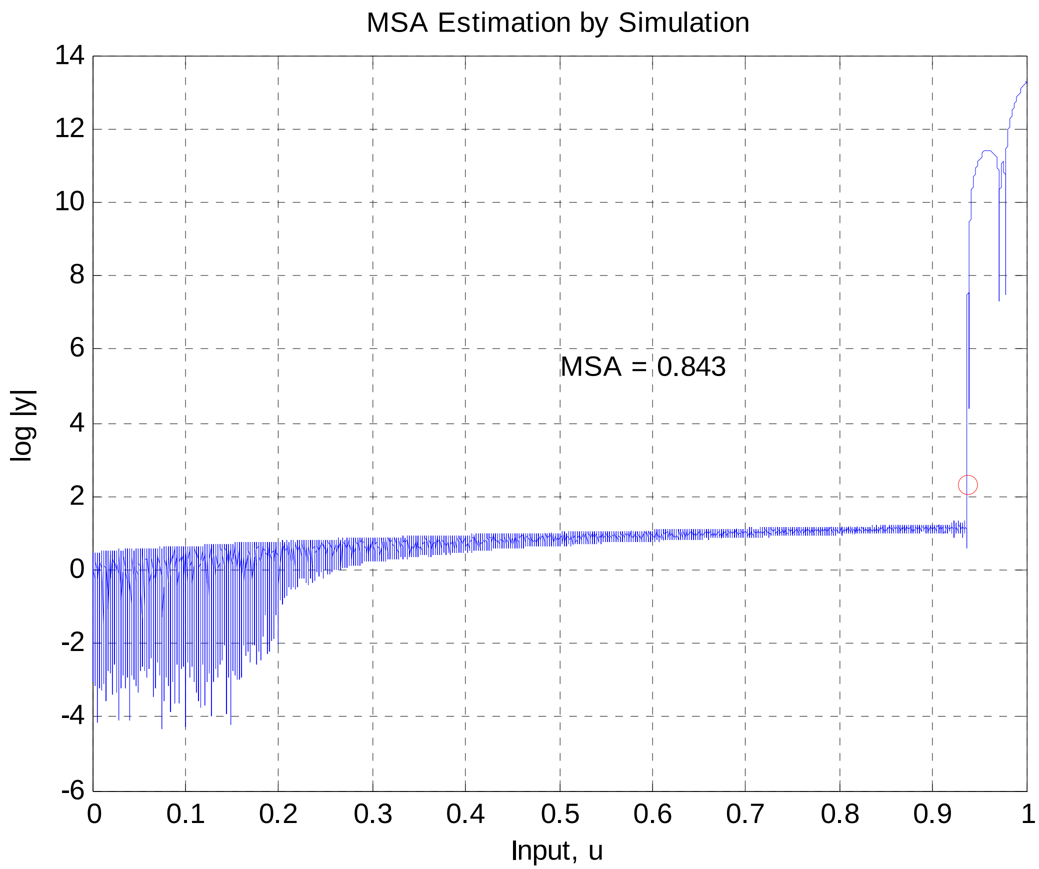
- Lars Risbo suggested a method for estimating MSA without sinewave inputs.
- Use a slow ramp input from 0 to FS value.
  - Plot $\log_{10}|y[n]|$. Observe where this plot blows up.
  - Take 90% of the input amplitude where $\log_{10}|y[n]|$ blows up as a conservative estimate for MSA.
  - Estimated MSA is close to that predicted by the sinewave input method.
- Much quicker than the sinewave technique (simulateSNR function).
- Write your own toolbox function generalizing this method !

Estimating MSA using Risbo's Method contd.



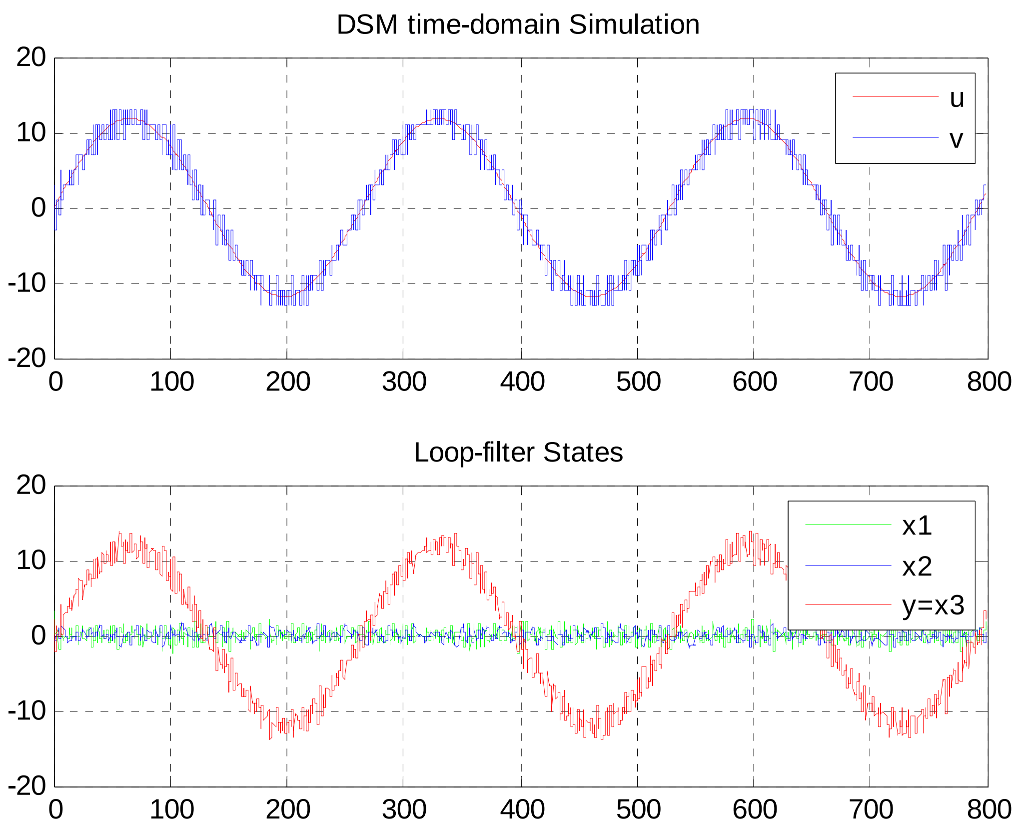
File: MSA_Risbo_Method.m

Estimating MSA using Risbo's Method contd.



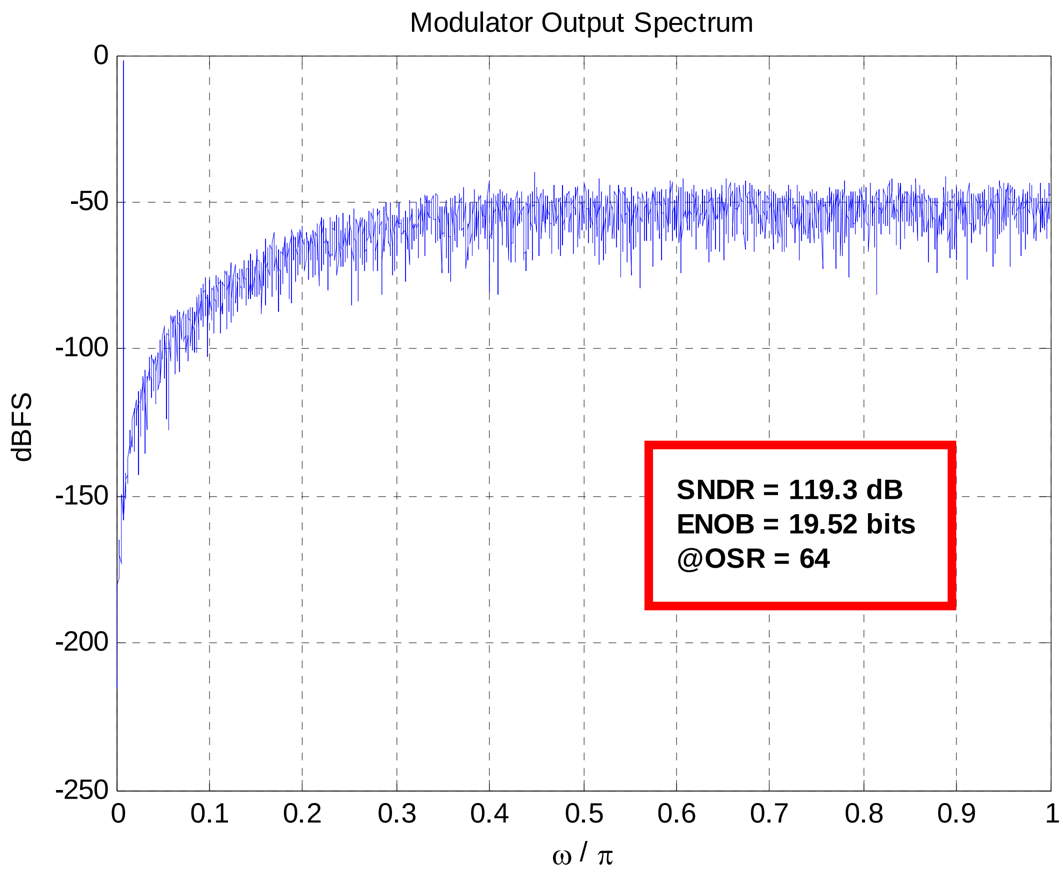
File: MSA_Risbo_Method.m

Simulation with input with MSA



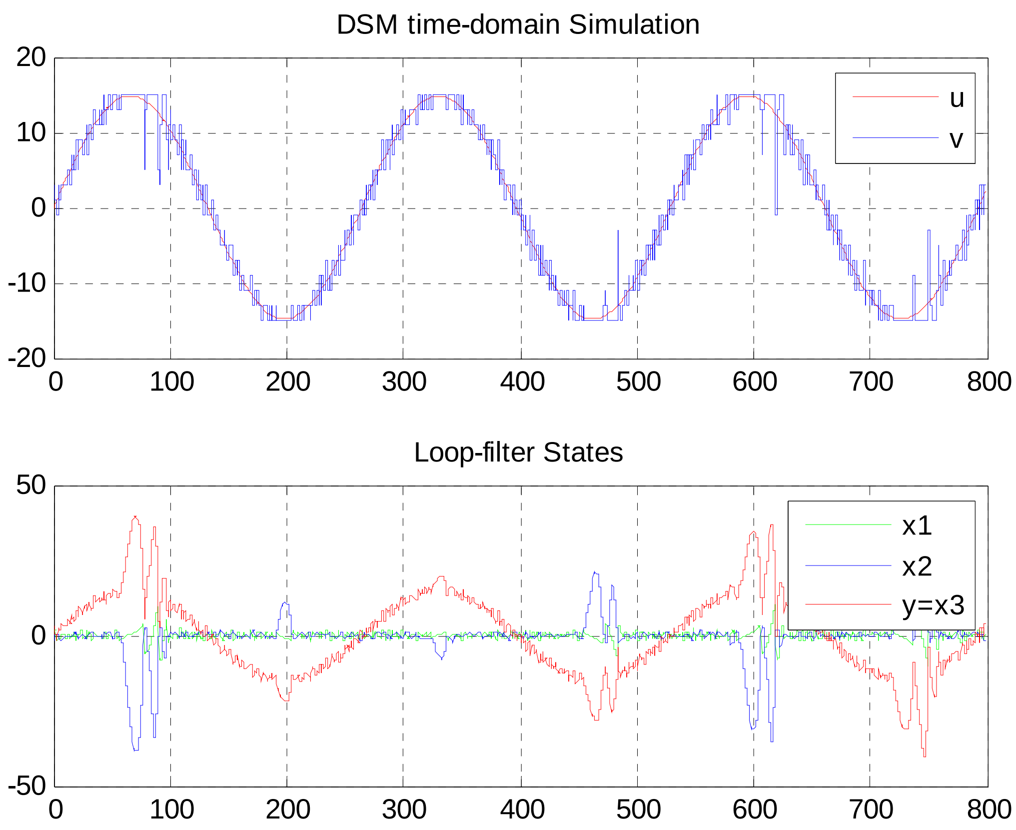
File: MSA_Risbo_Method.m

Simulated SNR with input with MSA



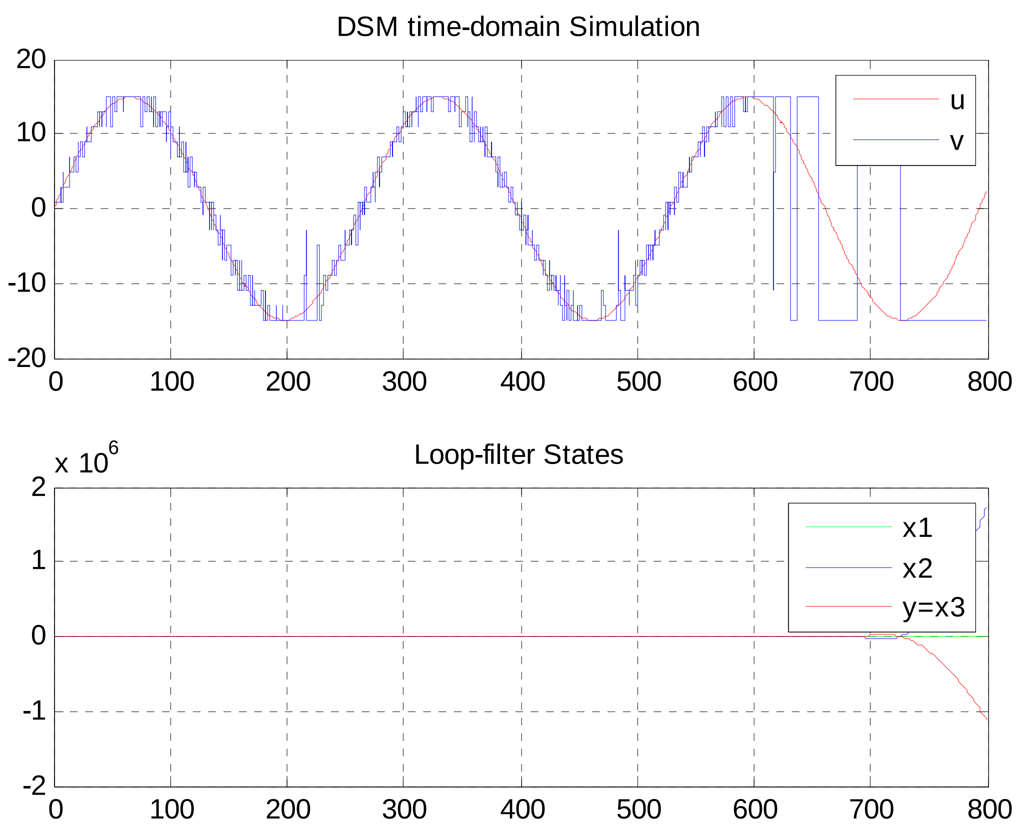
File: MSA_Risbo_Method.m

Simulation with input with 1.2*MSA



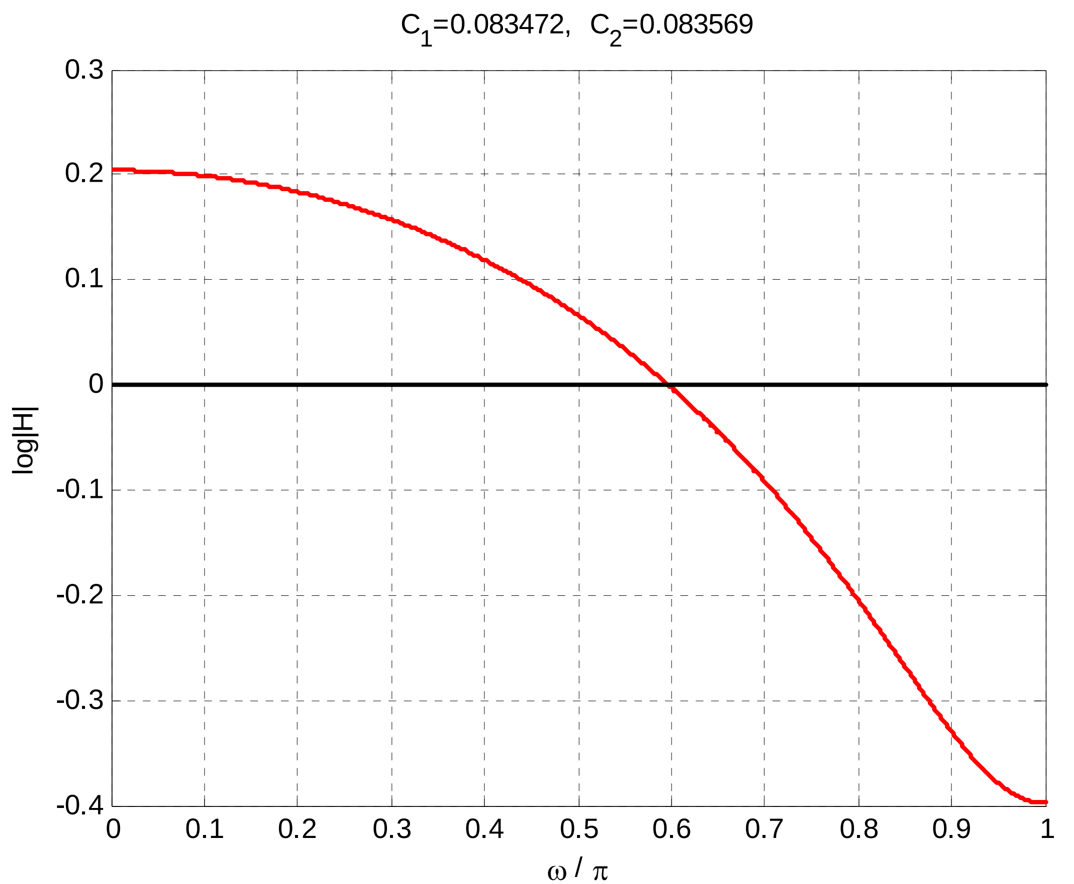
File: MSA_Risbo_Method.m

Simulation with input with 1.2*MSA



File: MSA_Risbo_Method.m

Bode Sensitivity Integral

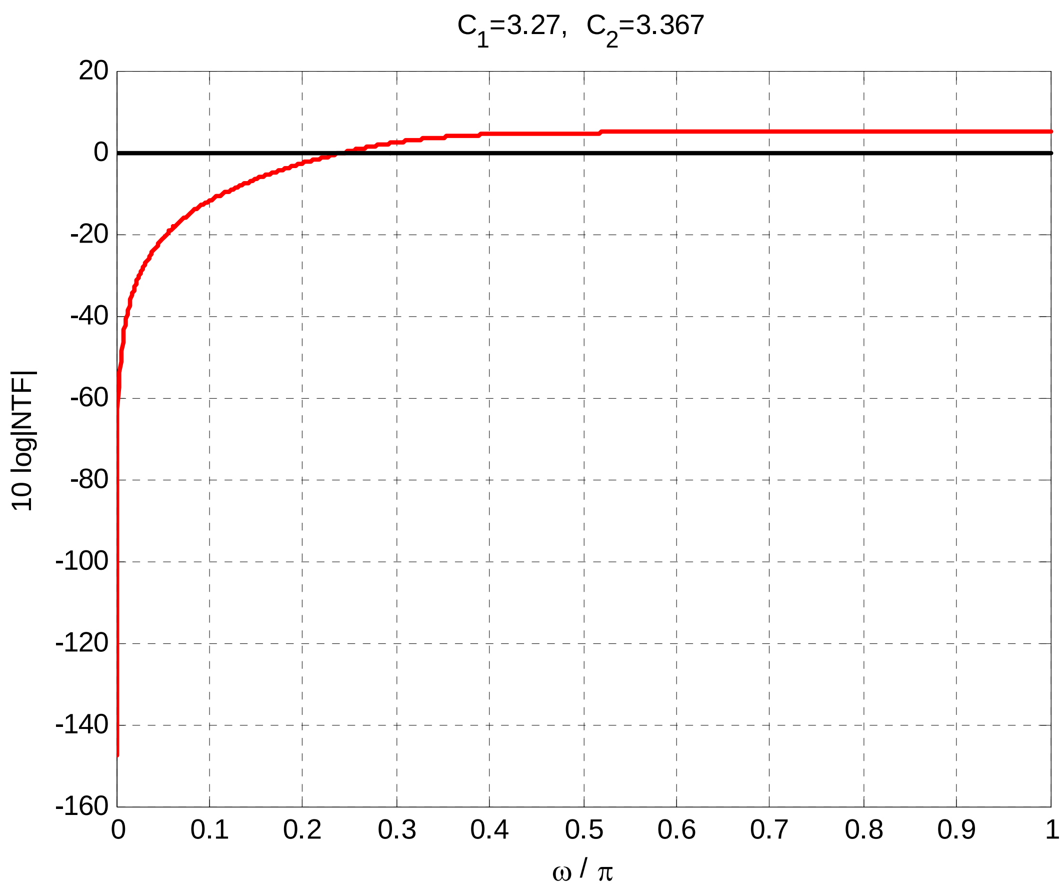


Single pole/zero transfer function with pole/zero inside the unit circle.

Area above and below the 0-dB axis are equal.

File: BodeSensitivity1.m

Bode Sensitivity Integral

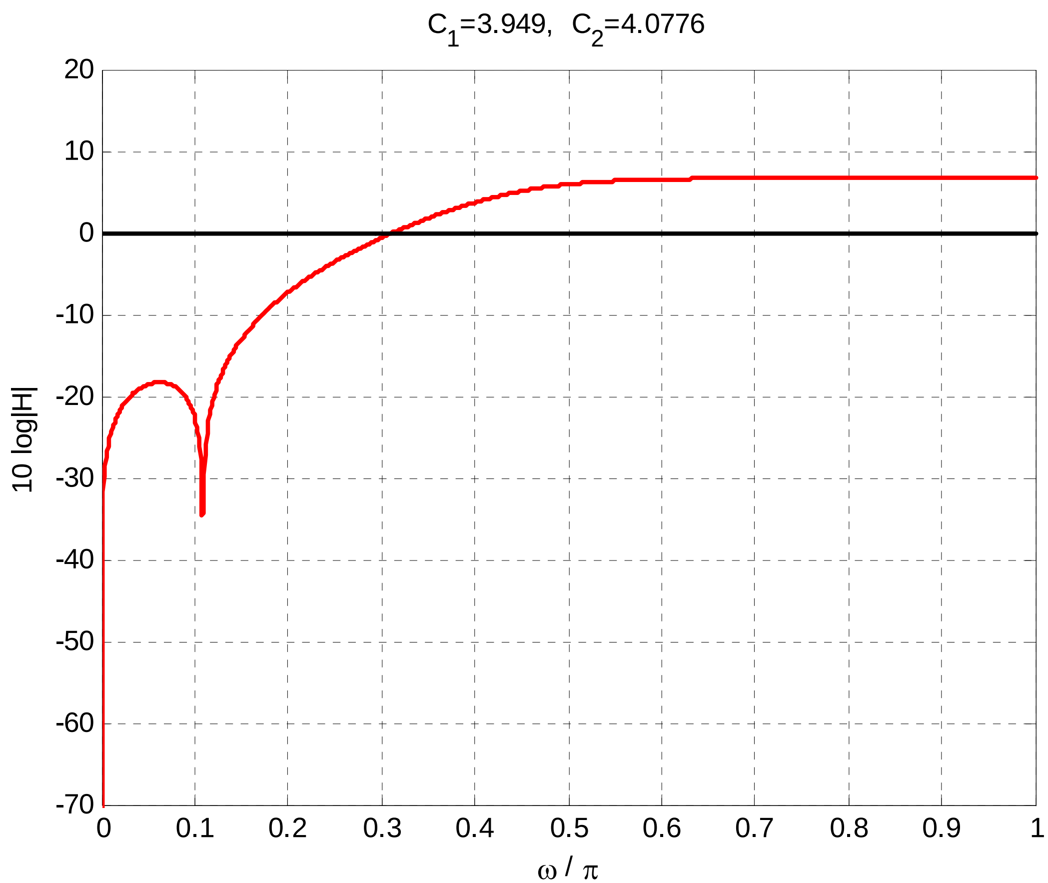


Butterworth NTF.

Area above and below the 0-dB axis are equal.

File: BodeSensitivity2.m

Bode Sensitivity Integral

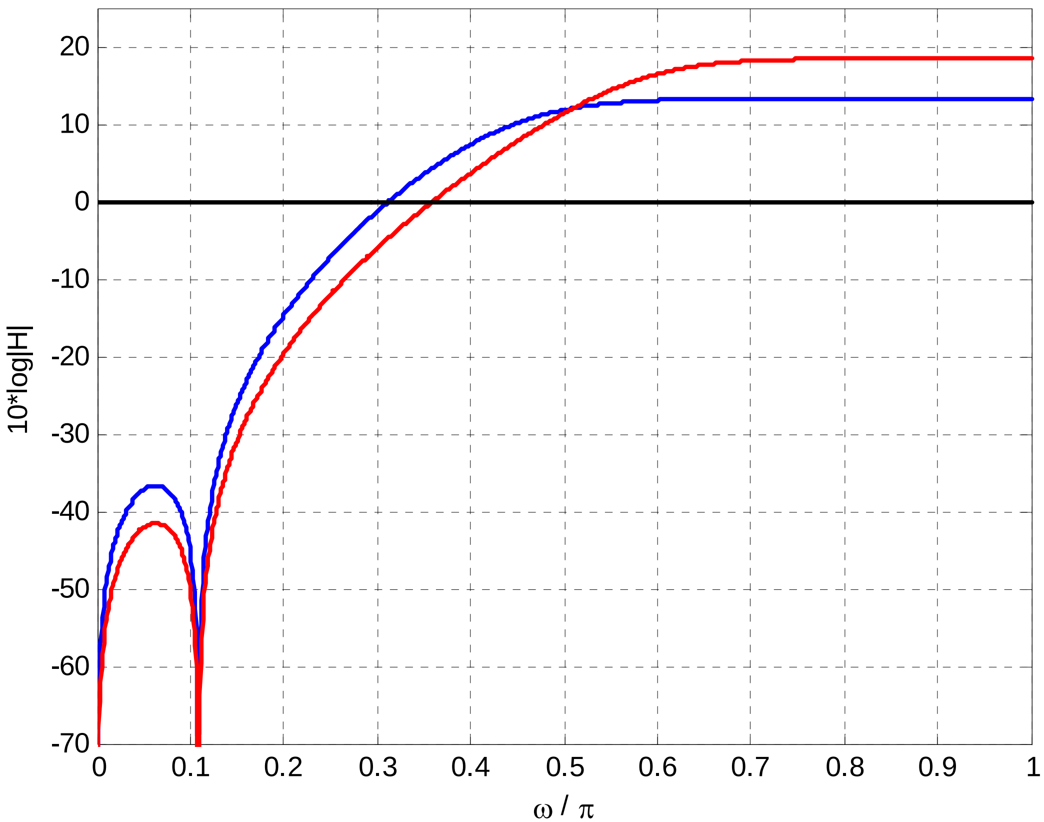


Inverse Chebyshev NTF.

Area above and below the 0-dB axis are equal.

File: BodeSensitivity3.m

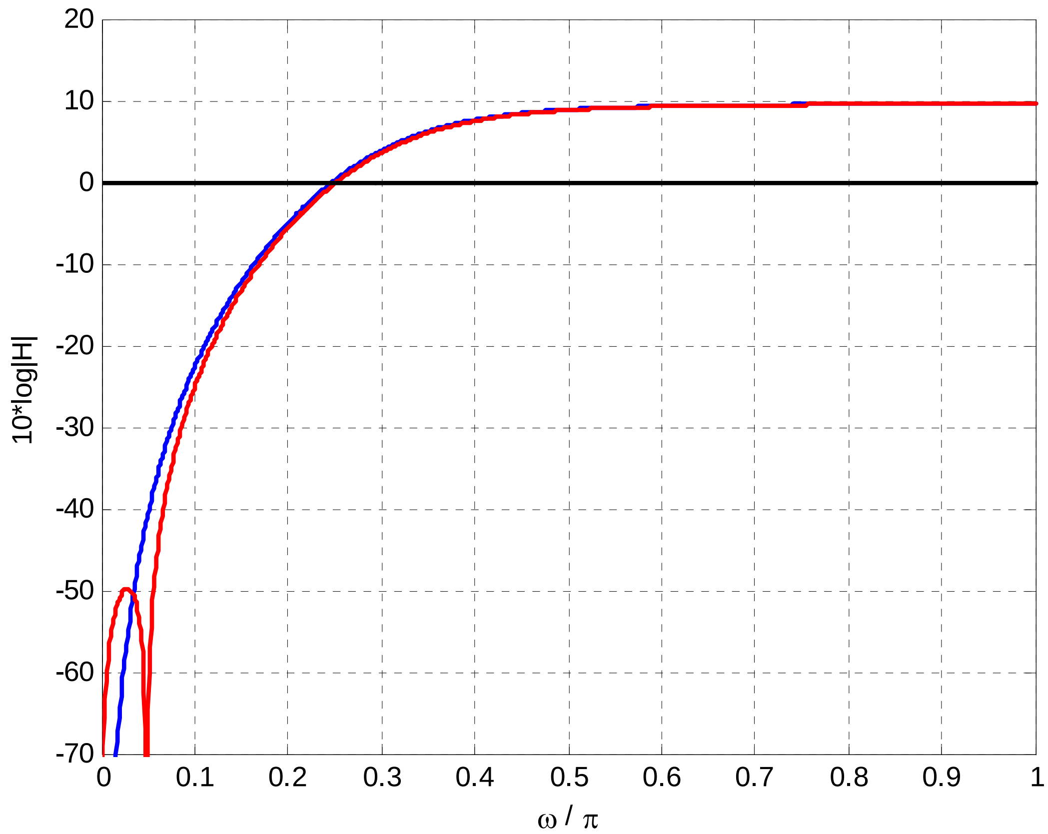
Bode Sensitivity Integral



Better in-band performance results in worse out-of-band performance.

File: BodeSensitivity4.m

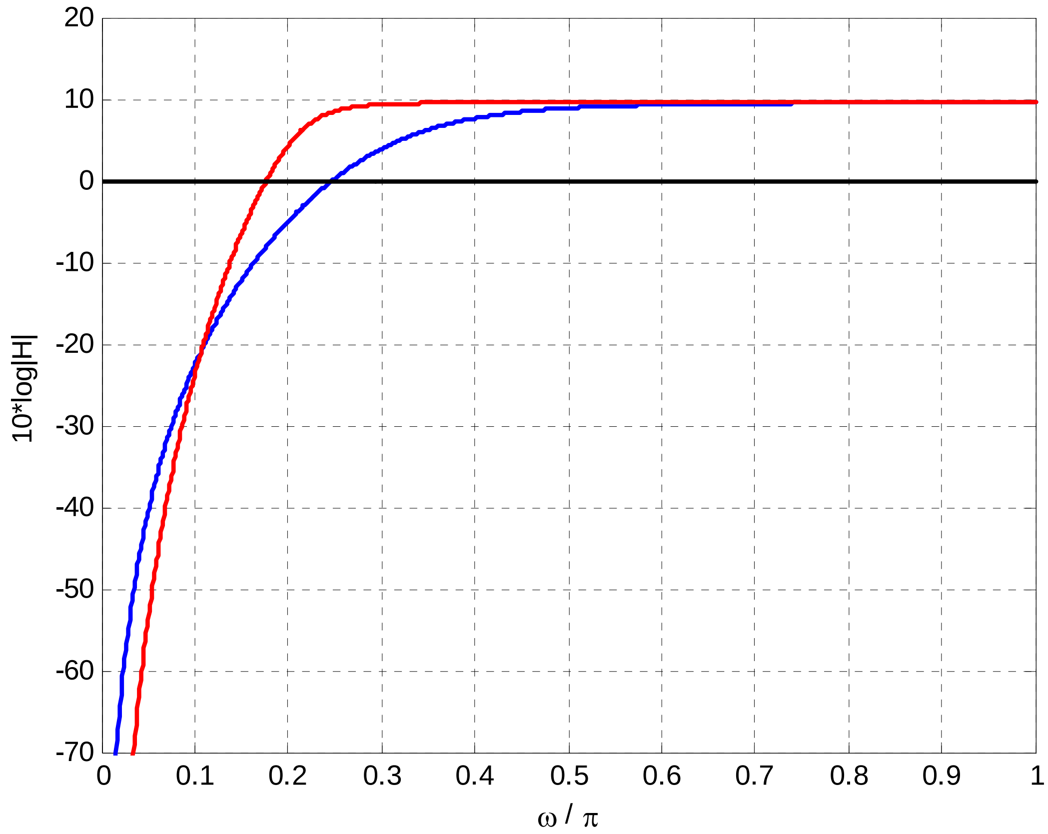
Bode Sensitivity Integral



Complex NTF zeros result in better in-band performance for the same OBG.

File: BodeSensitivity5.m

Bode Sensitivity Integral



Higher-order NTF results in better in-band performance for the same OBG.

File: BodeSensitivity6.m

References

[1] R. Schreier, Understanding Delta-Sigma Data Converters, Wiley, 2005.

[2] S. Pavan, N. Krishnapura, "Tutorial: Oversampling Analog to Digital Converters," *21st International Conference on VLSI Design*, Jan. 4, 2008. [Online]:http://www.ee.iitm.ac.in/~nagendra/presentations/20080104vlsiconf/20080104vlsiconf.pdf