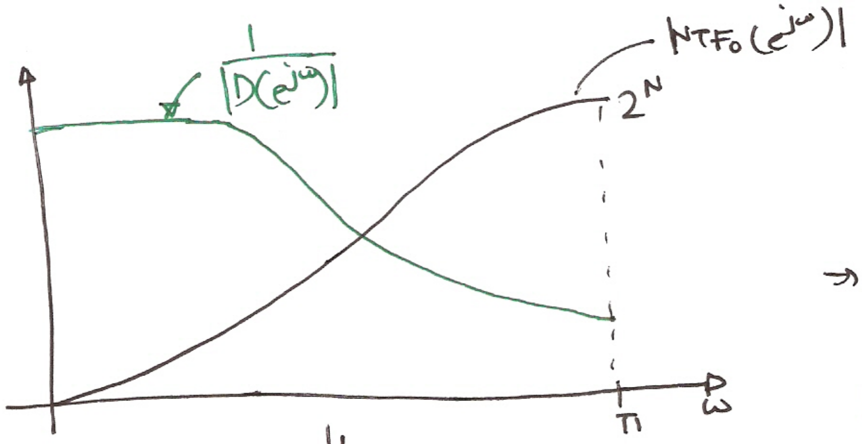
Systematic NTF Design Procedure

- For higher-order NTF, MSA reduces drastically as $OBG = 2^N$, increases with order.
- But we wish to limit the MSA to 75% to 80% of the full scale amplitude/range.

$NTF_0(z) = (1 - z^{-1})^N$



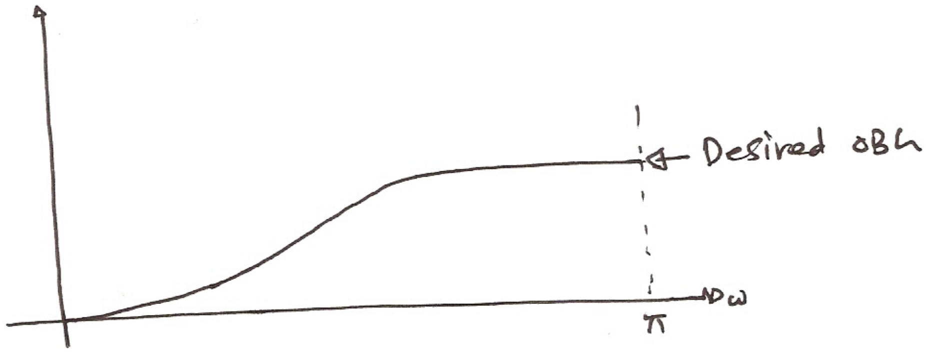
⇒ Introduce poles in the signal band to reduce the OBG.

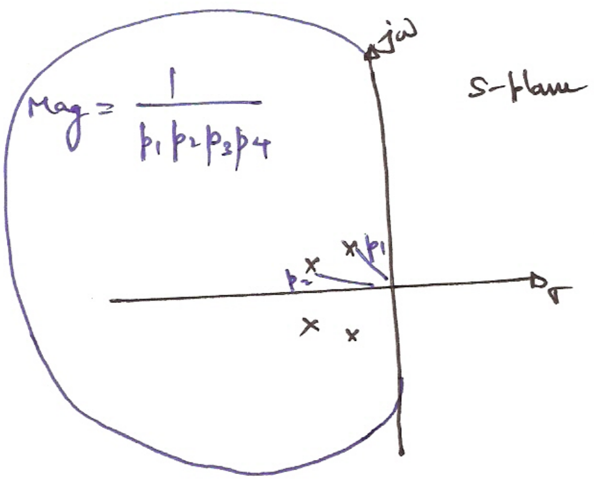
↳ Try to keep the same IBN

↳ Variance of the noise is reduced

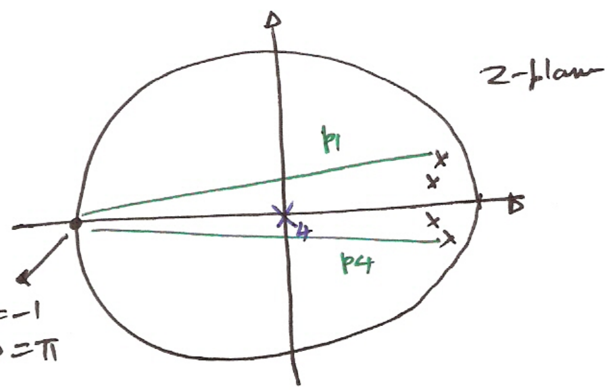
↳ Allows larger MSA.

⇓





⇒



Gain at $\omega = \pi$ is lower as the poles are farther away from $z = -1$.

Recall the realizability condition (delay-free loops)

$h[n] = NTF(\infty) = 1$

If

$$NTF(z) = \frac{(1-z^{-1})^4}{a_0 + a_1 z^{-1} + \cdots + a_4 z^{-4}} = \frac{N(z)}{D(z)}$$

$N(z)$ → high pass noise-shaping function

$D(z)$ → low pass denominator

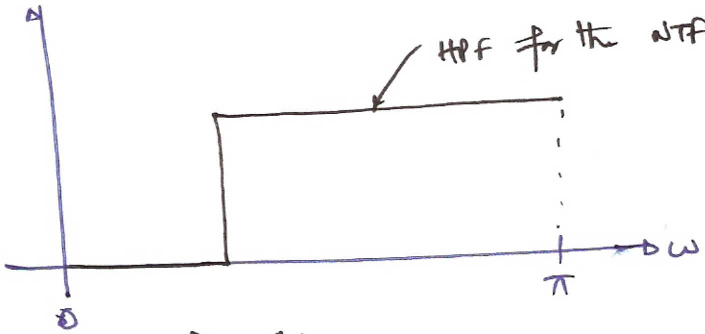
$\Rightarrow \boxed{a_0 = 1}$ or $D(z=\infty) = 1$

* Overall the NTF is high-pass response so that the OBG is reduced by controlling the pole locations.

$$OBG = \frac{2^N}{D(-1)}$$

which pole configuration to use?

Approximation problem.

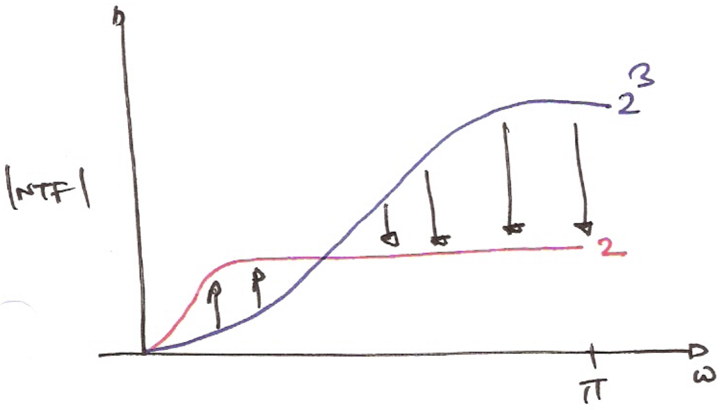


Approximate the ideal brickwall with a polynomial response.

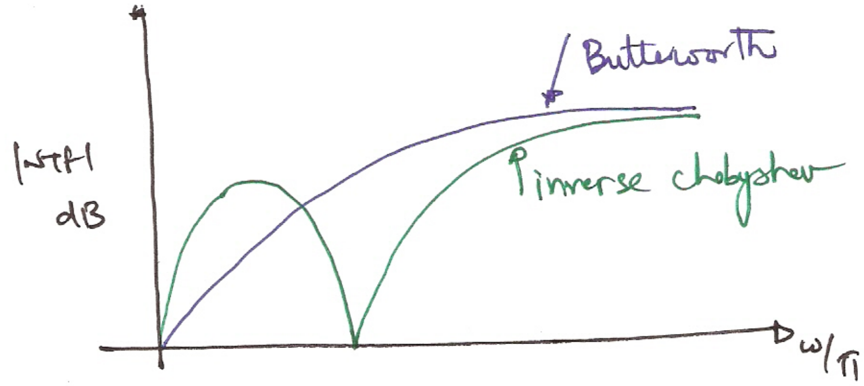
↳ use MATLAB to find polynomial coefficients

Many responses are possible!

1. Butterworth
2. Inverse chebyshev
3. Maximally flat all-pole transfer f^n, etc.



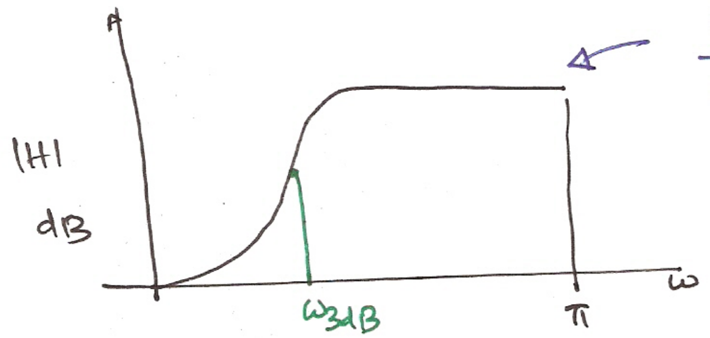
- properly chosen poles reduce OBG
  - ↳ MSA ↑
  - ↳ stability ++
- Stability comes at the expense of higher in-band noise (IBN)



MATLAB FUNCTIONS:

butter (n, Wn)
cheby2 (n, R, Wst)
maxflat (n, m, Wn).

Butterworth Example:



$$\frac{(1-z^{-1})^4}{a_0 + a_1 z^{-1} + \dots + a_4 z^{-4}}$$

4th-order Butterworth high-pass response, all zeros at $z=1$.

↳ maximally flat at $\omega = \pi$

Can this be an NTF?

$h[0] = NTF(\infty) = 1 \Rightarrow a_0 = 1$ (Recap)

- Many modulator implementation are possible for the same transfer function.
- Specs will not specify the transfer functions.

Example specifications:

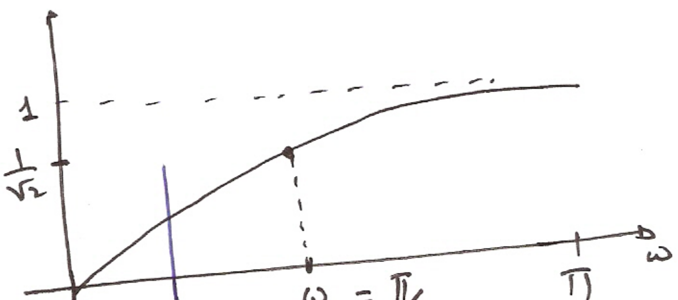
- Signal Bandwidth
- OSR ← study fs for the process, quantizer design etc.

required SQNR ≥ 120 dB

Start with:
order (L) = 3, OSR = 64, n = 16 (num of levels), SQNR ≥ ~~...~~ 115 dB

signal band edge = $\frac{\pi}{OSR} = \frac{\pi}{64}$

- Start with $\omega_{3dB} = \pi/8$ ('slightly' larger than the signal band edge).



MATLAB sets $|H(e^{j\omega})|_{\omega=\pi} = |H(e^{j\pi})| = 1$

see file: SystematicNTFDesign.m

① • get H(z) from MATLAB
[b, a] = butter(3, 1/8, 'high')

↳ $$H(z) = \frac{0.6735 - 2.0204 z^{-1} + 2.0204 z^{-2} - 0.6735 z^{-3}}{1 - 2.219 z^{-1} + 1.715 z^{-2} - 0.454 z^{-3}}$$

But we want NTF(∞) = 1

⇒ Scale H(z) by $\frac{1}{0.6735}$ to obtain NTF

⇒ $NTF(z) = \frac{H(z)}{b(1)}$ ← b_0

⇒ $$NTF(z) = \frac{(1 - 3z^{-1} + 3z^{-2} - z^{-3})}{(1 - 2.22 z^{-1} + 1.715 z^{-2} - 0.4535 z^{-3})}$$

② Find loop filter using $\frac{1}{1+L(z)} = NTF(z)$.

③ Simulate the modulator using the loop filter

- Compute peak SNR.
  ↳ gives MSA

④ If SNR is not high enough, use higher ω_{3dB} for the Butterworth HPF. and repeat the procedure

↳ this will increase the OBG

↳ MSA will be reduced

⑤ If SNR is too high use lower ω_{3dB} and repeat the procedure

↳ OBG ↓

↳ MSA ↑

with $\omega_{3dB} = \pi/4$, $SNR_{pk} = 116dB$, $OBG = 2.25$, $MSA = 0.8$

⇒ we are done!

* This iterative algorithm is generalized and implemented in synthesizeNTF function in the toolbox.

NTF-zero optimization

Textbook pages 107–111

- Spread zeros in the signal band to minimize the in-band noise (IBN). $\omega_B \Rightarrow$ signal bandwidth

N	zero locations normalized to ω_B	SQNR increase
1	0	0 dB
2	$\pm\frac{1}{\sqrt{3}}$	3.5 dB
3	$0, \pm\sqrt{\frac{3}{5}}$	8 dB
4	$\pm\sqrt{\frac{3}{7} \pm \sqrt{\left(\frac{3}{7}\right)^2 - \frac{3}{35}}}$	13 dB
5	$0, \pm\sqrt{\frac{5}{9} \pm \sqrt{\left(5/9\right)^2 - 5/21}}$	18 dB

- zero locations obtained by minimizing the noise integral numerically.
- MATLAB ΔΣ Toolbox function: ds_optzeros(order, 1)

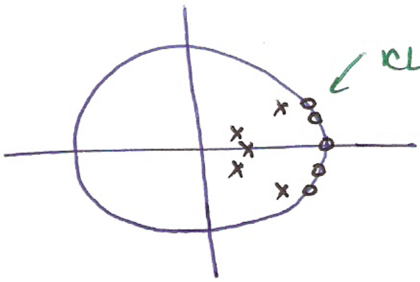
* Note: Use quantizer gain value $k = \frac{E(v \cdot y)}{E(y^2)}$ obtained from simulations to find the actual NTF(z).

* for even order NTFs, might want to place double zeros at $z=1$ for better DC suppression.

NTF-pole optimization

- Stability considerations govern the pole placement
- Must satisfy $H(\infty)=1$, and $\|H\|_\infty \triangleq OBG$ constraints
- Exhaustive search done using MATLAB ΔΣ Toolbox (synthesizeNTF() function).
- Better optimization using the CLANS (closed-loop Analysis of noise-shapers) algorithm. → Yellow book ch. 14.

clans() function in ΔΣ Toolbox.



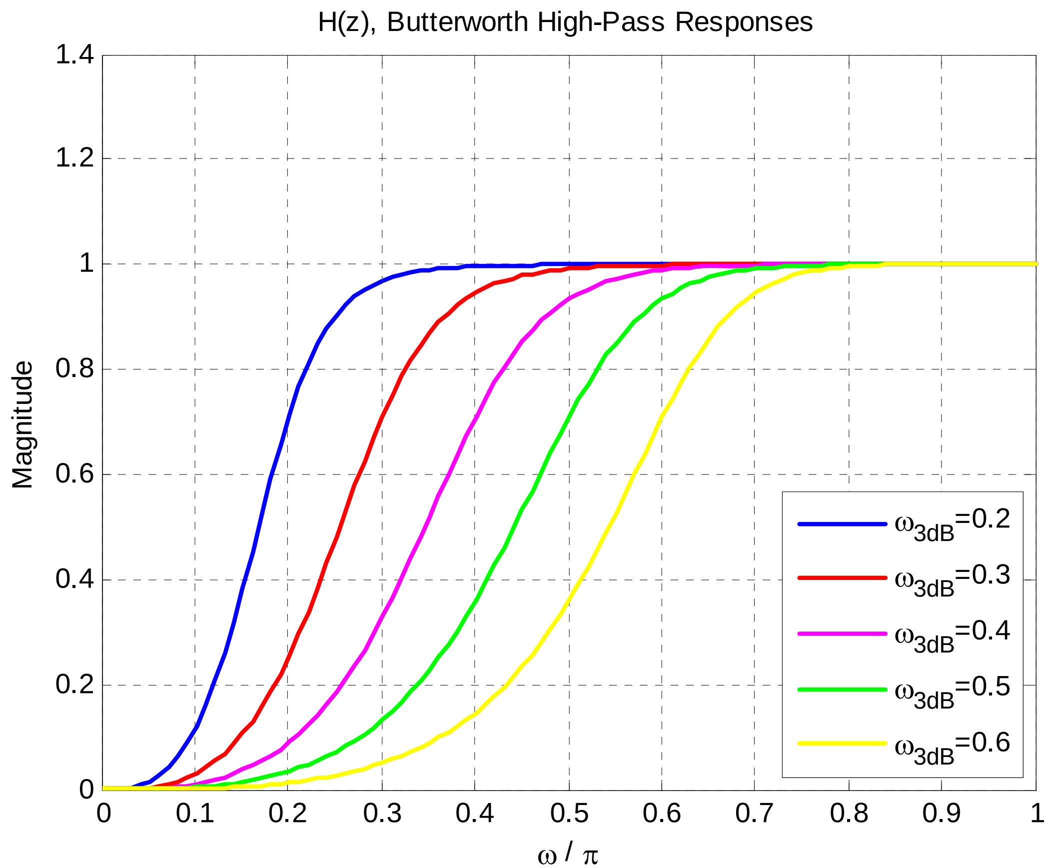
CLANS result.

ECE 697 Delta-Sigma Converters Design

Lecture#12 Slides

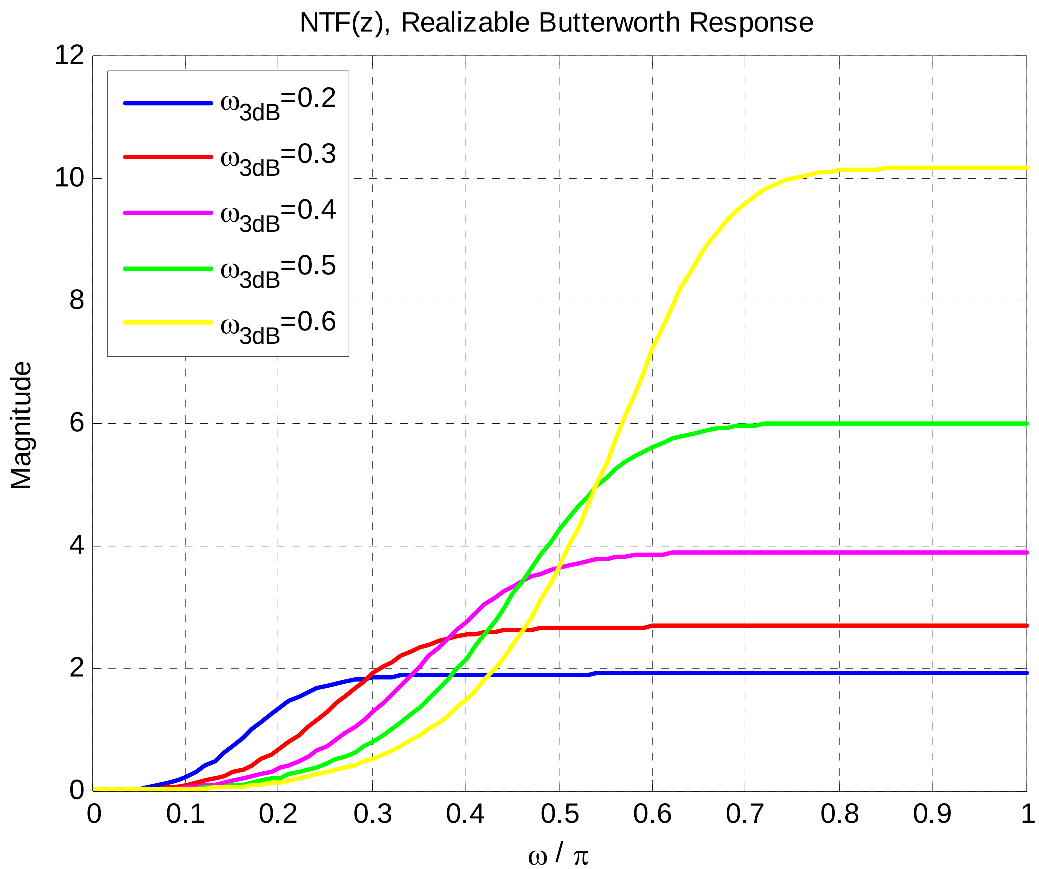
Vishal Saxena
(vishalsaxena@boisetstate.edu)

Butterworth High-Pass Responses



File: ButterworthResponses.m

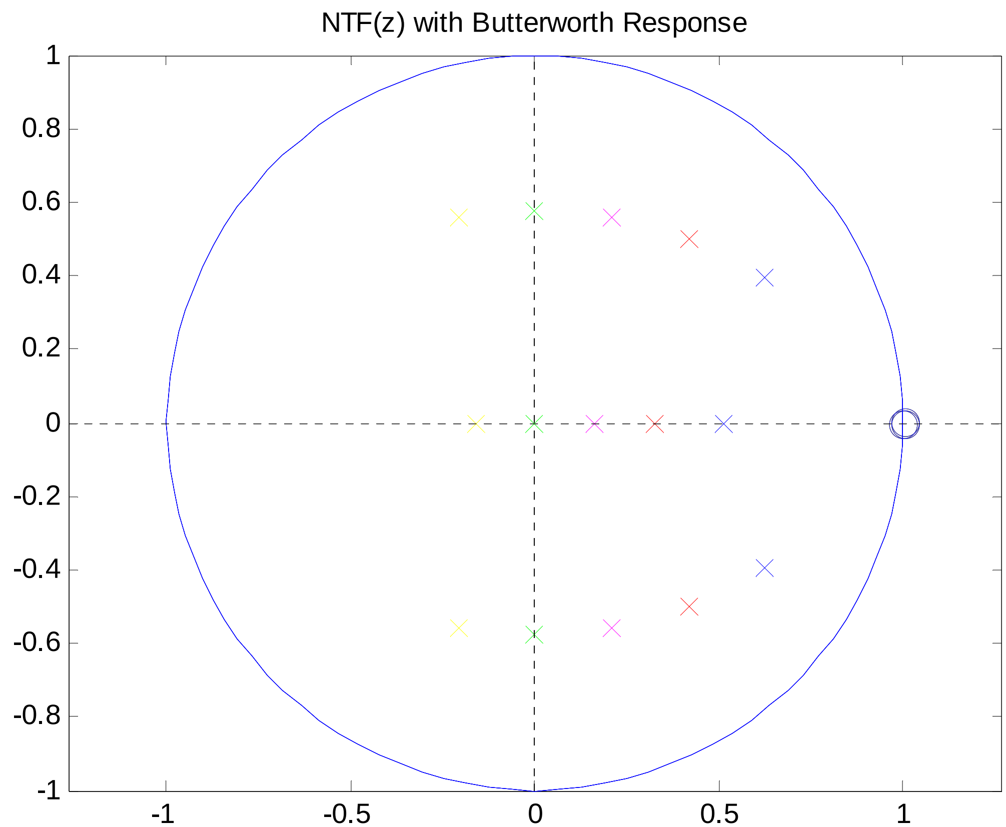
Realizable NTFs with Butterworth Response



$$NTF(z) = \frac{H(z)}{b_0}$$

File: ButterworthResponses.m

NTF Poles for Butterworth Responses

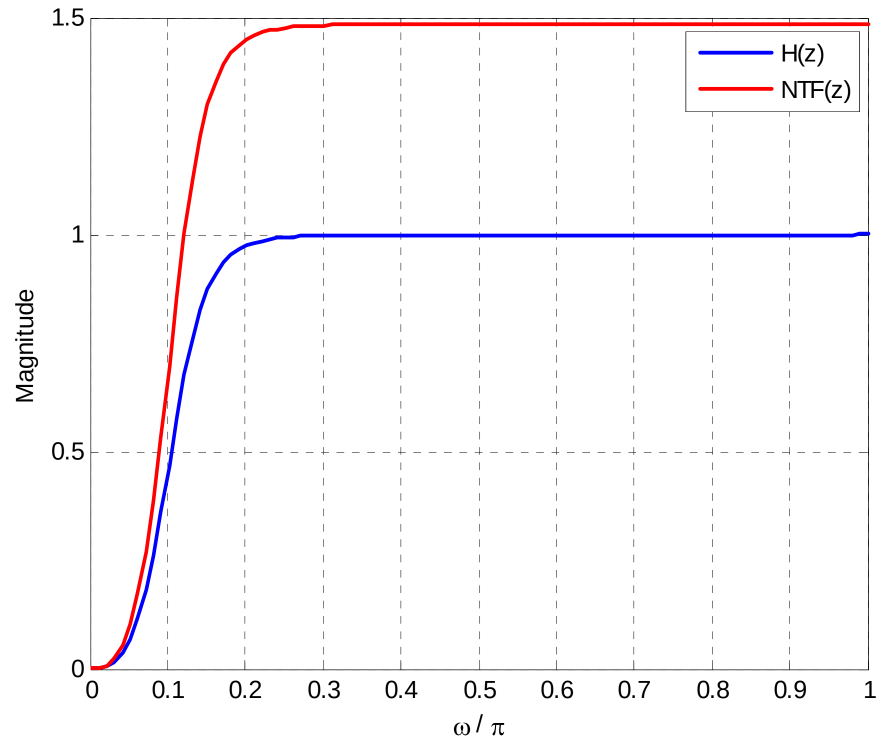


File: ButterworthResponses.m

Systematic NTF Design Example

- Specifications
  - SQNR > 120 dB
  - A signal bandwidth which results in an OSR = 64
    - Study optimal clock rate for the given process and quantizer design.
- Designer's Choice
  - Order = 3
  - Quantizer levels (nLev) = 16
  - Butterworth high-pass response for the NTF.
- Use MATLAB for finding coefficients of the HPF response.
  - `[b,a] = butter(order,` ω_{3dB}`, 'high')`
  - The cutoff frequency ω_{3dB} specifies the transfer function.

Systematic NTF Design contd.



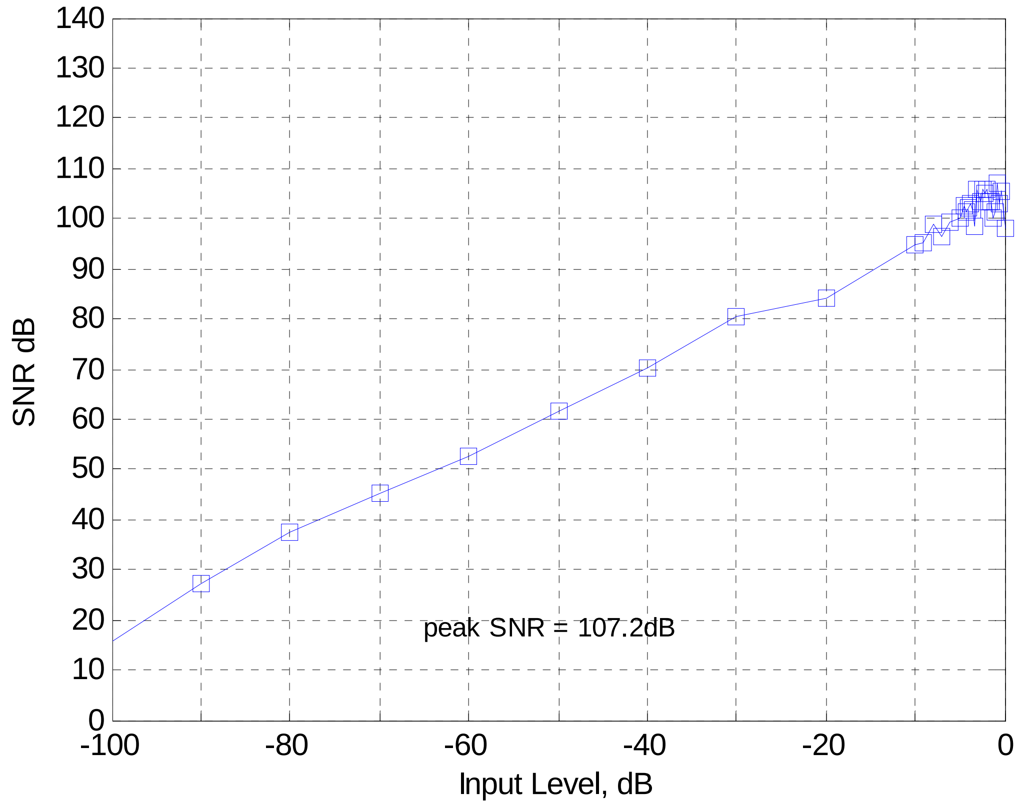
- Start with cutoff frequency $\omega_{3dB}=\pi/8$, for the butterworth HPF H(z).
- Derive a realizable NTF using $NTF(z)=H(z)/b_0$

File: SystematicNTFDesign.m

Systematic NTF Design contd.

- Map the NTF response to a loop-filter architecture (details later).
- Simulate the modulator for all possible amplitudes and input tone frequencies.
- Compute the peak SNR and MSA.
  - May use `simulateDSM` function in the toolbox.

Systematic NTF Design contd.



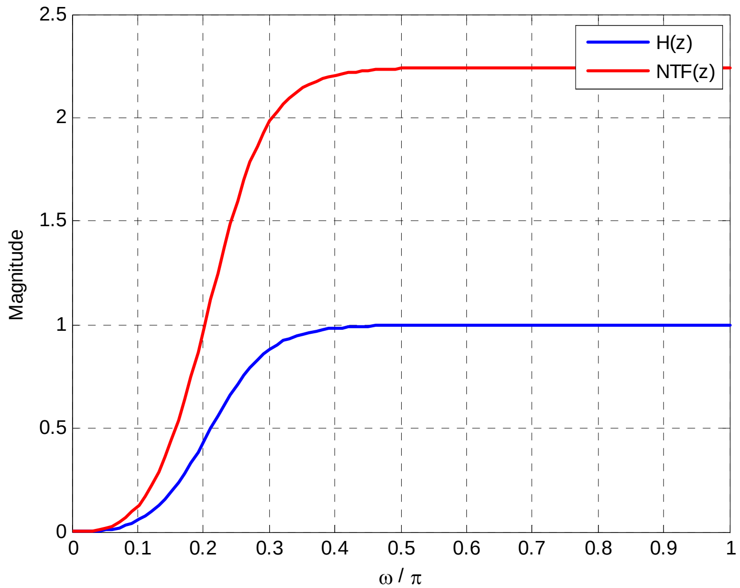
- Peak SNR = 107 dB
- MSA = 0.9

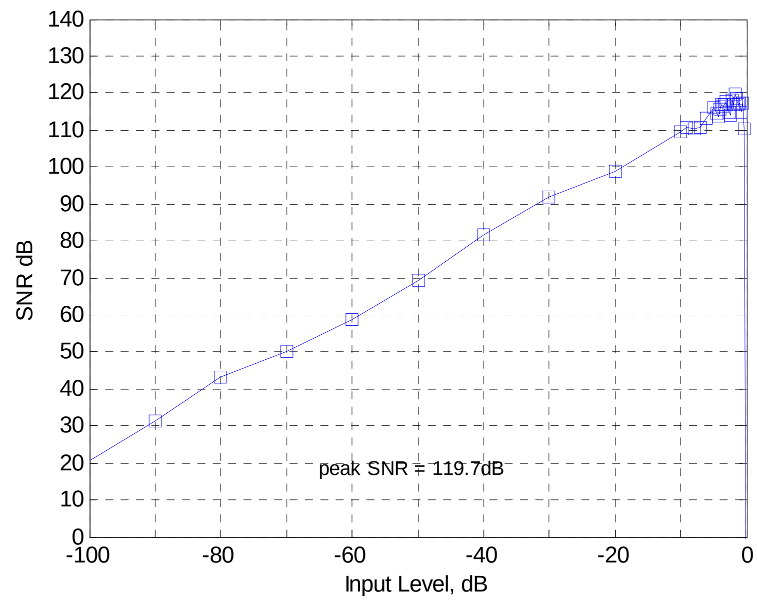
File: SystematicNTFDesign.m

Systematic NTF Design contd.

- If SNR is not enough, repeat the entire procedure with a higher cutoff frequency for the Butterworth HPF
  - IBN ↓, SQNR ↑
  - OBG ↑ and MSA ↓
- If SNR is too high, repeat the entire procedure with a lower cutoff frequency for the Butterworth HPF
  - IBN ↑, SQNR ↓
  - OBG ↓ and MSA ↑

Systematic NTF Design contd.

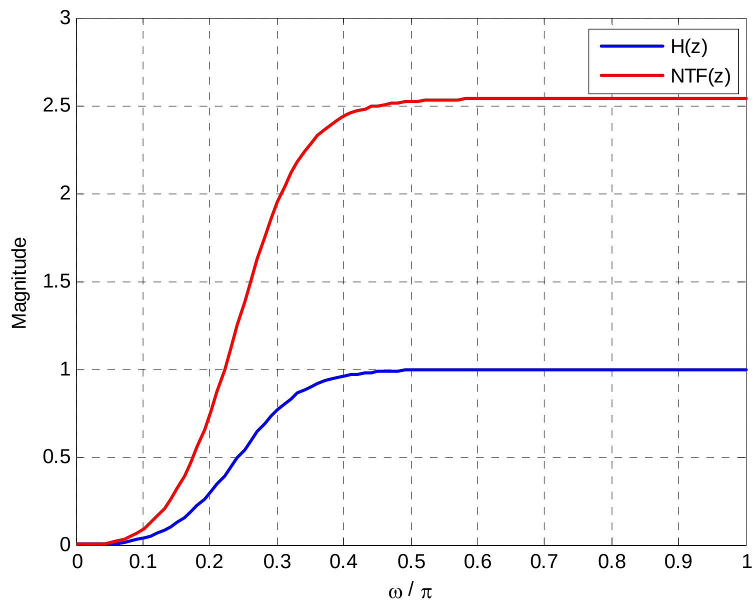


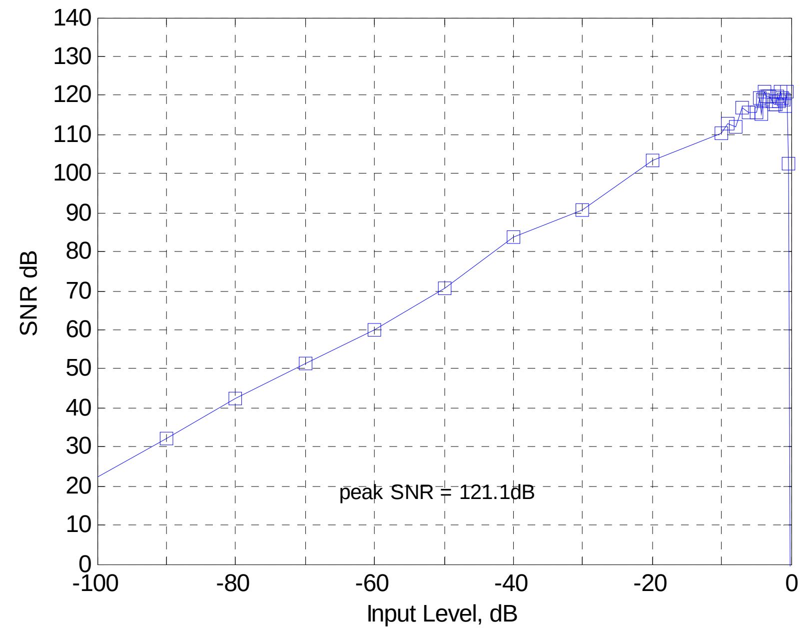


- $\omega_{3dB} = \pi/4$.
- Peak SNR = 119 dB, OBG = 2.25, MSA = 0.8

File: SystematicNTFDesign.m

Systematic NTF Design contd.





- $\omega_{3dB}=2\pi/7$.
- Peak SNR = 121 dB, OBG = 2.54, MSA = 0.8.
  - ✓ Design closed !

File: SystematicNTFDesign.m

Systematic NTF Design contd.

- An advanced version of this iterative process is implemented as the function `synthesizeNTF` in the delta-sigma Toolbox.
  - Several ‘opt’ params for NTF zero (and pole) optimization.
  - Use `synthesizeChebyshevNTF` for low OSR and low OBG designs.
- CLANS algorithm by Kenney and Carley implemented as the `clans` function in the toolbox.
  - Requires Optimization toolbox.
- Exercise: Repeat the design procedure using an Inverse Chebyshev HPF response.
  - `[b,a] = cheby2(n,R,w`$_{st}$`);`

References

[1] S. Pavan, N. Krishnapura, “Tutorial: Oversampling Analog to Digital Converters,” *21st International Conference on VLSI Design*, Jan. 4, 2008. [Online]:http://www.ee.iitm.ac.in/~nagendra/presentations/20080104vlsiconf/20080104vlsiconf.pdf