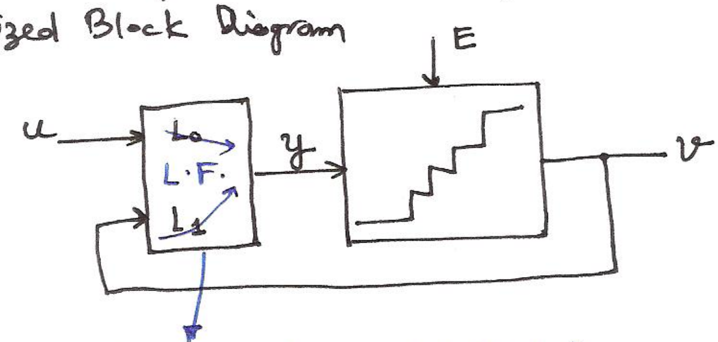
Describing function Analysis

Generalized Block Diagram



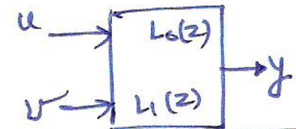
- So far we have assumed Linear Model of noise.

$$Y(z) = L_0(z)\,U(z) + L_1(z)\,V(z)$$

where,

$$L_0(z) = \frac{STF(z)}{NTF(z)}, \qquad L_1(z) = \frac{NTF(z) - 1}{NTF(z)}$$

Loop-filter
↳ 2 inputs & 1 output



* for Second-order DSM

$$L_0(z) = \frac{1}{(1-z^{-1})^2}$$

$$L_1(z) = \frac{-z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{(1-z^{-1})^2}$$

* for special cases:

$L_0(z) = L_1(z) = L(z)$

Ex. first-order DSM

For the whole modulator:

$$\Rightarrow \quad V(z) = Y(z) + E(z)$$

$$= L_0(z)\,U(z) + L_1(z)\,V(z) + E(z)$$

$$\Rightarrow \quad V(z) = \underbrace{\frac{L_0(z)}{1 - L_1(z)}}_{STF} \cdot U(z) + \underbrace{\frac{L_1(z)}{1 - L_1(z)}}_{NTF} \cdot E(z)$$

$$= STF(z) \cdot U(z) + NTF(z) \cdot E(z)$$

So far a linear model. But how to model the quantizer so as to understand the effects of its non-linearity.

- Overload (or saturation) of the quantizer causes instability:
  - ↳ when input exceeds the range of the quantizer, the output of the quantizer doesn't change at all.
  - ↳ feedback breaks down!

- Definition of stability: Bounded input → Bounded output (BIBO)
  - for LTI systems, $\Rightarrow \sum_n |h(n)| < \infty$

Definition for a DSM (non-linear feedback system):

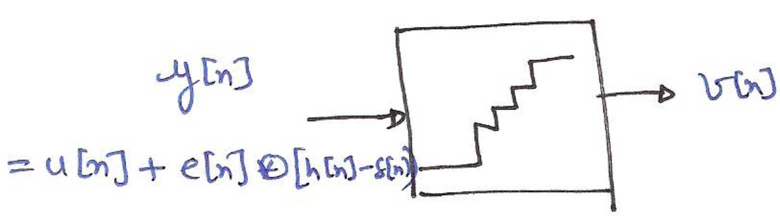
* If the state variables become unbounded for a bounded input, the system is unstable.

- State variables ≜ integrator outputs in the loop filter.

- for higher-order modulators
  - for small input amplitudes → system is stable
  - when the amplitude (u) is increased, the output of the quantizer either gets clipped at the maximum or the minimum output, or wildly oscillates between the two.
  - the output of the loop filter, (y_1) hits ∞ ~~at~~ when unstable.

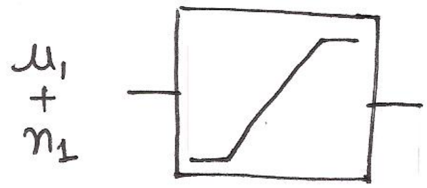
- Non-linear system with feedback:
  - ↳ can't use linear feedback theory directly.
  - ↳ sampled non-linear feedback system ≡ DSM
    - ↳ simulation?
    - ↳ need to develop some intuition.

* Approximate the non-linear (NL) system by a linear one.
  - ↳ Additive white quantization noise was a good working model.
  - ↳ Quantizer overload is the primary reason for instability.



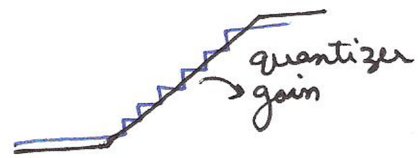
the sum of input and ~~step~~ quantization noise, shaped by the filter ($h[n]-\delta[n]$) is quantized by the quantizer.

Describing function Method (by Ardalan & Paulos)



$$v_1 = f(u_1, n_1) = k_0 u_1 + k_1 n_1$$

Approximating the quantizer by a two input linear system.



$\Rightarrow$ $v_1 = k_0 u_1 + k_1 n_1$

the quantizer output is a linear function of the input (u_1) and quantization noise (n_1).

Find the gains k_0 and k_1

$$v_1[n] = k_0 u_1[n] + k_1 n_1[n].$$ for 'N' samples

$$\underset{N\times 2}{\begin{bmatrix} u_1 & n_1 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} k_0 \\ k_1 \end{bmatrix}} = \underset{N\times 2}{V_1}$$

let $A = [u_1 \;\; n_1]$ be $\mathbb{R}^{N\times 2}$, then

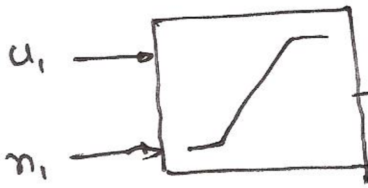
$$A \begin{bmatrix} k_0 \\ k_1 \end{bmatrix} = V_1$$

$$\Rightarrow \quad A^T A \begin{bmatrix} k_0 \\ k_1 \end{bmatrix} = A^T V_1$$

$$\Rightarrow \quad \boxed{\begin{bmatrix} \hat{k}_0 \\ \hat{k}_1 \end{bmatrix} = (A^T A)^{-1} A^T V_1}$$ ← optimal curve fitting

curve fitting using

error $\left\| v - [A] \begin{bmatrix} \hat{k}_0 \\ \hat{k}_1 \end{bmatrix} \right\|_2 = 0$ if the system is linear.



$\rightarrow v_1 = k_0 u_1 + k_1 n_1$ + fitting error

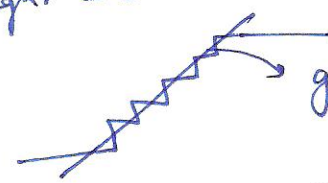
$\Rightarrow$ least square fit between the inputs and the output.

↳ k_0 & k_1 will be different for different inputs.

see Matlab file: "Describing_function_Analysis.m".

for the toolbox $\Delta = LSB = 2$ & step_height $= 2$

* when the fitting error is small



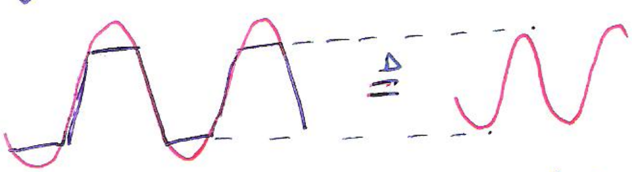
$\rightarrow$ gain k_0 and k_1

(a) Input 'u' is ~~...~~ within the quantizer range (no clipping):

$k_0 \simeq k_1 \simeq 1$.

(b) Quantizer output clips:

gains k_0 and k_1 are very different



$\triangleq$ + harmonics

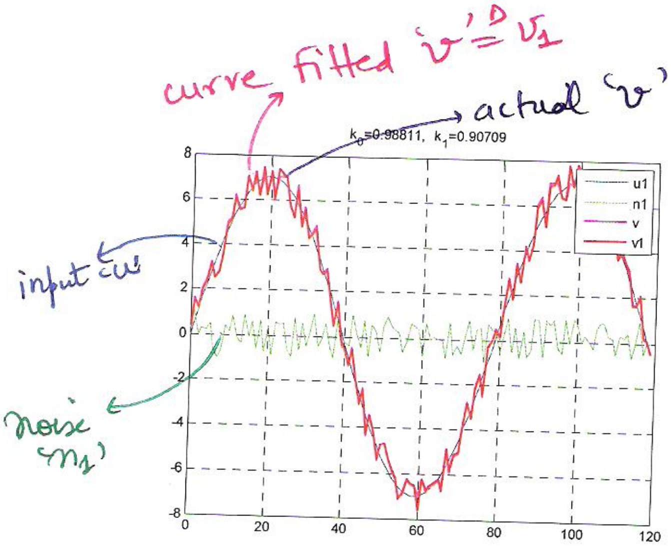
* the gain for the input (u) is the amplitude of the first harmonic of the clipped output. ($k_0 \simeq 0.9$) for $A = 1.2 \cdot fs$

* Since the noise is riding on the input, during the clipping the noise is eliminated in the output $\Rightarrow$ noise gain is zero during clipping

$\Rightarrow$ on average k_1 is much smaller than k_0.

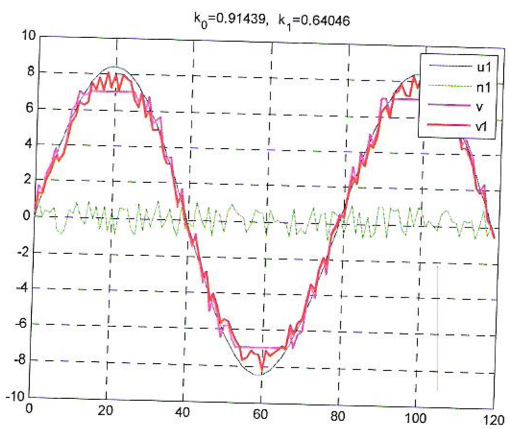
$\Rightarrow$ for $A = 1.2\, fs$

$k_1 \simeq 0.7$



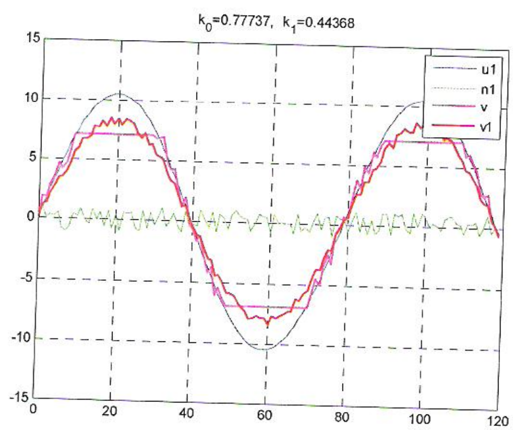
input amplitude → $A = FS$ ← full scale range of the quantizer

$k_0 = 0.99$, $k_1 = 0.91$



$A = 1.2\ FS$

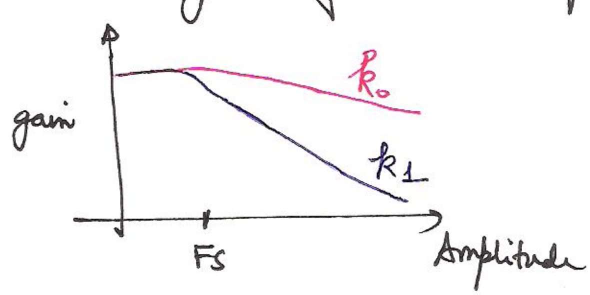
$k_0 = 0.91$, $k_1 = 0.64$



$A = 1.5\ FS$

$k_0 = 0.78$, $k_1 = 0.44$

With overloading,
⇒ gain for noise, k_1, falls much faster than the gain for the input signal u, i.e k_0.



⇒

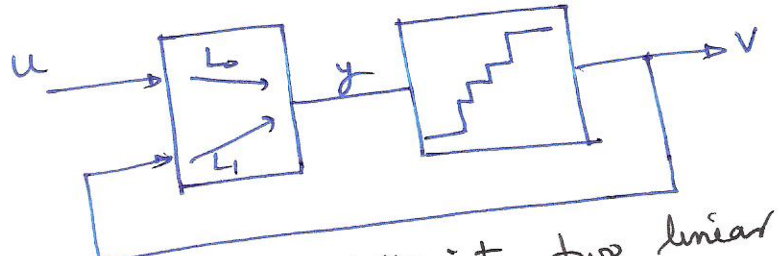
- Signal and noise gains through the 'quantizer' $\simeq 1$, when there is no overload
- With overload,

Signal gain: $k_0 < 1$

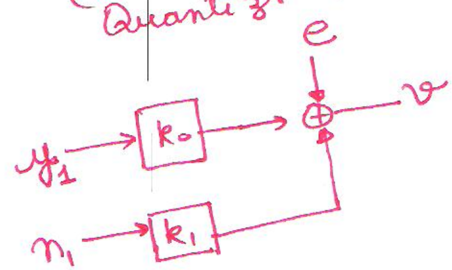
Noise gain: $k_1 \lll k_0 < 1$

⇒ Noise gain (k_1) depends upon the signal 'u'. when signal amplitude is high ⇒ noise gain (k_1) is smaller.

⇒ Introduce gain into the DSM model

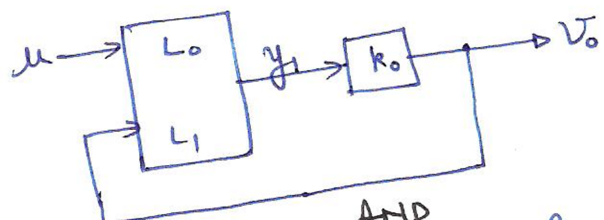


'Quantizer Model'



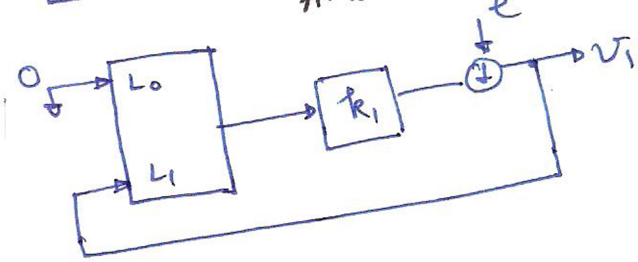
The modulator splits into two linear systems.

⇒



'Signal loop'

AND

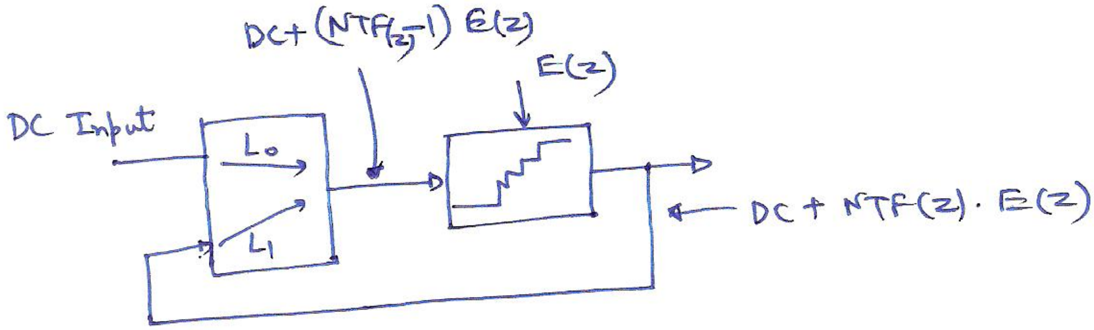


'noise loop'

From these 'linear' loops.

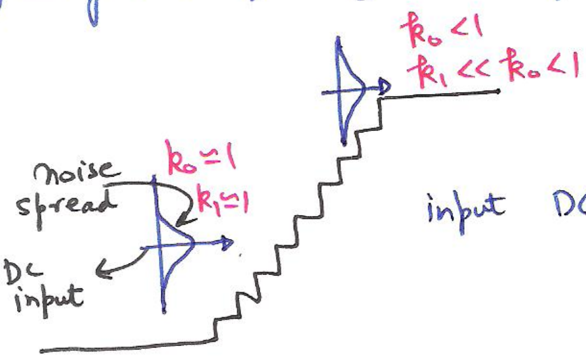
$$NTF'(z) = \frac{V_1(z)}{E(z)} = \frac{L_1(z)}{1 - k_1 L_1(z)} = \frac{NTF(z)}{k_1 + (1-k_1)\,NTF(z)}$$

$$STF'(z) = \frac{V_0(z)}{U(z)} = \frac{L_0(z)}{1 - k_0 L_1(z)}$$



• Gain of the quantizer is dependent upon the input level.

* Here << is of the order of 1/4th



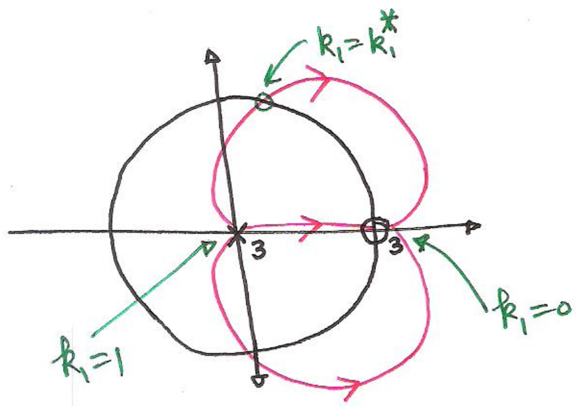
input DC level up ⇒ $k_1 \downarrow\downarrow$ ⇒ noise 'gain' decreases
⇒ poles of the NTF(z) start moving towards the unit-circle.

$$NTF'(z) = \frac{NTF(z)}{k_1 + (1-k_1)NTF(z)} = \frac{L_1(z)}{1 - k_1 L_1(z)}$$

* At what point the poles move out of the unit circle?
↳ Plot root locus of NTF(z) w.r.t. 'k_1'

<u>Example</u>:

$NTF(z) = (1 - z^{-1})^3$ ⇒ { 3 zeros at $z=1$; 3 poles at $z=0$ }



at $k_1 = 0$, poles move to $z=1$
⇒ complete instability.

At $k_1 = k_1^*$, the NTF becomes unstable.

• poles leave the unit circle only for NTF order 3 or higher.

• rlocus(NTF) works ok in MATLAB for plotting root-locus even though the system description (model) is slightly different.

Higher-Order Modulators:

- Signal dependent stability
- Gain falls off as the quantizer overloads (or saturates)
  ↳ when the input levels start to become larger, we hit a point where the modulator becomes unstable.

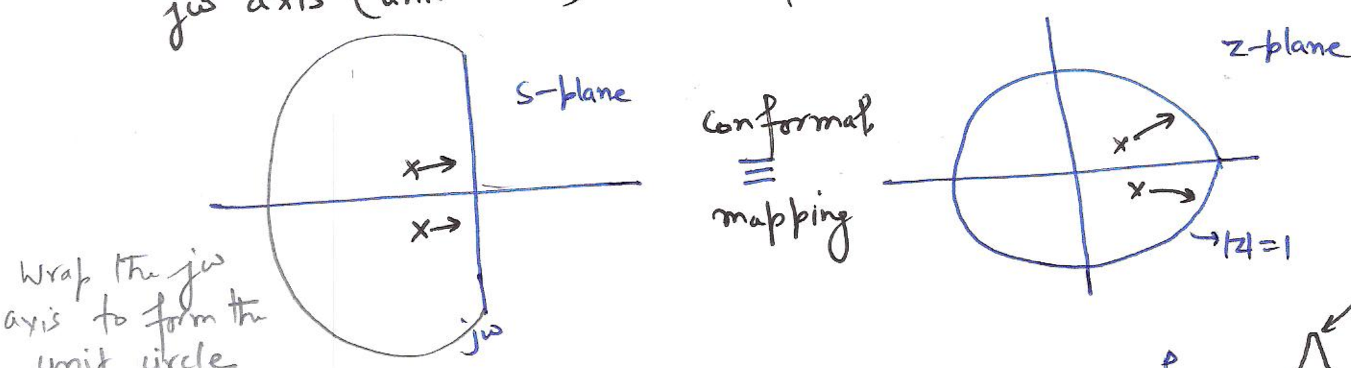
∵ Noise at the quantizer input $= (NTF(z)-1)\,E(z)$

↳ noise mean square value $= \frac{\Delta^2}{12}\left[\|h[n]\|_2 - 1\right]$ → $\sum_n h^2[n]$ By Parceval's Theorem

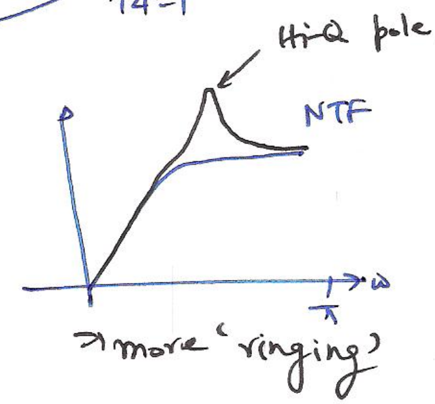
* The modulator 'destabilization' sequence:

① As the input DC level is increased, the gain for the noise falls.

② As, 'k_1' drops, poles of the NTF start moving towards the $j\omega$ axis (unit-circle) ⇒ Q-factor of the poles increases.



③ As the Q factor of the poles increases
↳ length of $h[n]$ increases
↳ $\sigma_q^2 = \frac{\Delta^2}{12}\left(\|h\|_2 - 1\right)$ increases
⇒ quantization noise variance at 'Y' increases



④ Clipping of the output waveform occurs more frequently ⇒ 'k_1' drops further

⇒ poles move faster towards unit circle

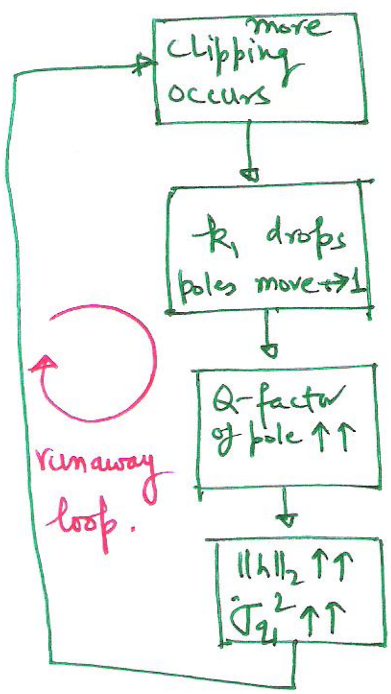
⇒ noise variance increases further

runaway loop

⑤ System becomes unstable

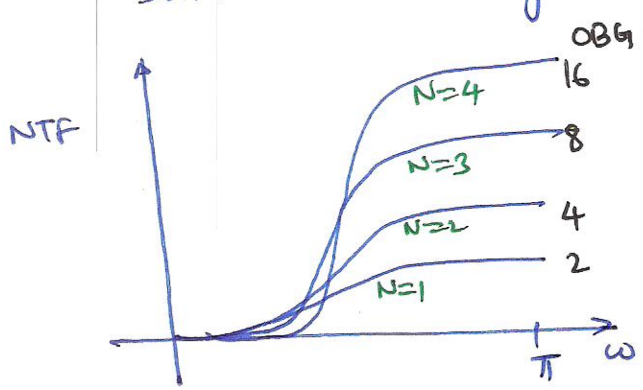
↳ State variable become unbounded.

↳ SNR drops down



- Very rapid runaway to instability.

⇒ Modulator can not be stable for the signal level upto the saturation level of the quantizer. (MSA ⇒ maximum stable amplitude)



As OBG increases, the variance of the noise is higher.

⇒ Quantizer overloads more often

⇒ MSA starts decreasing rapidly.

- OBG = $H(-1) = \|h\|_1$

ECE 697 Delta-Sigma Converters Design

Lecture#10 Slides

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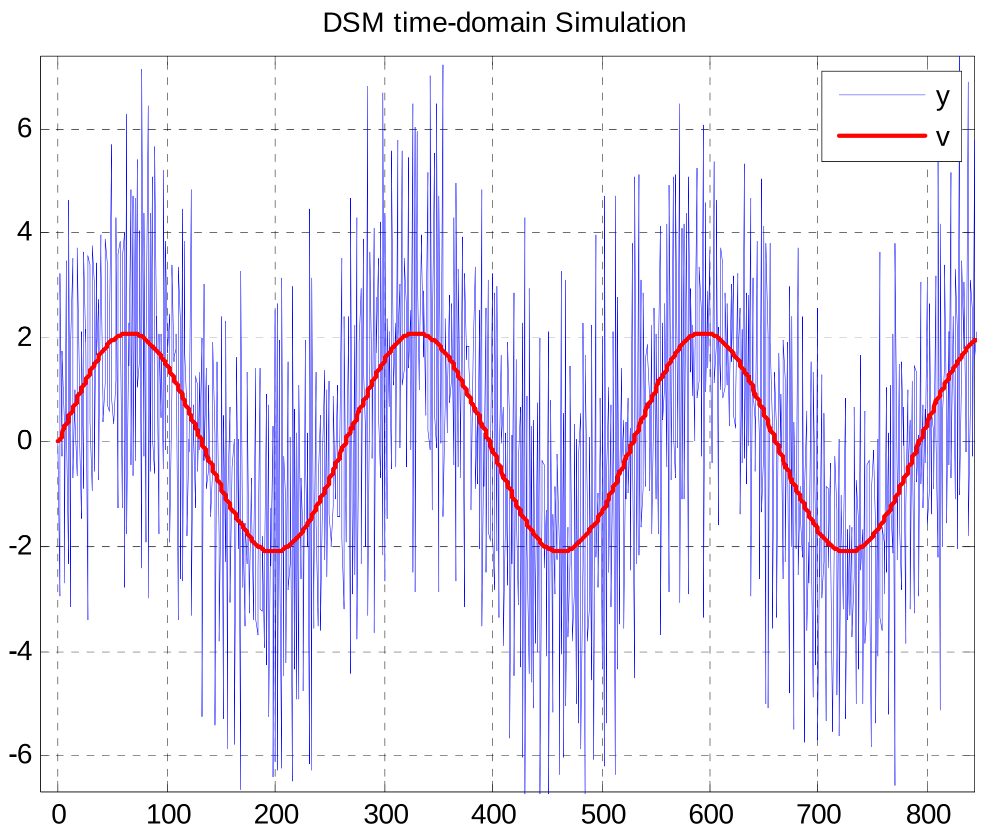
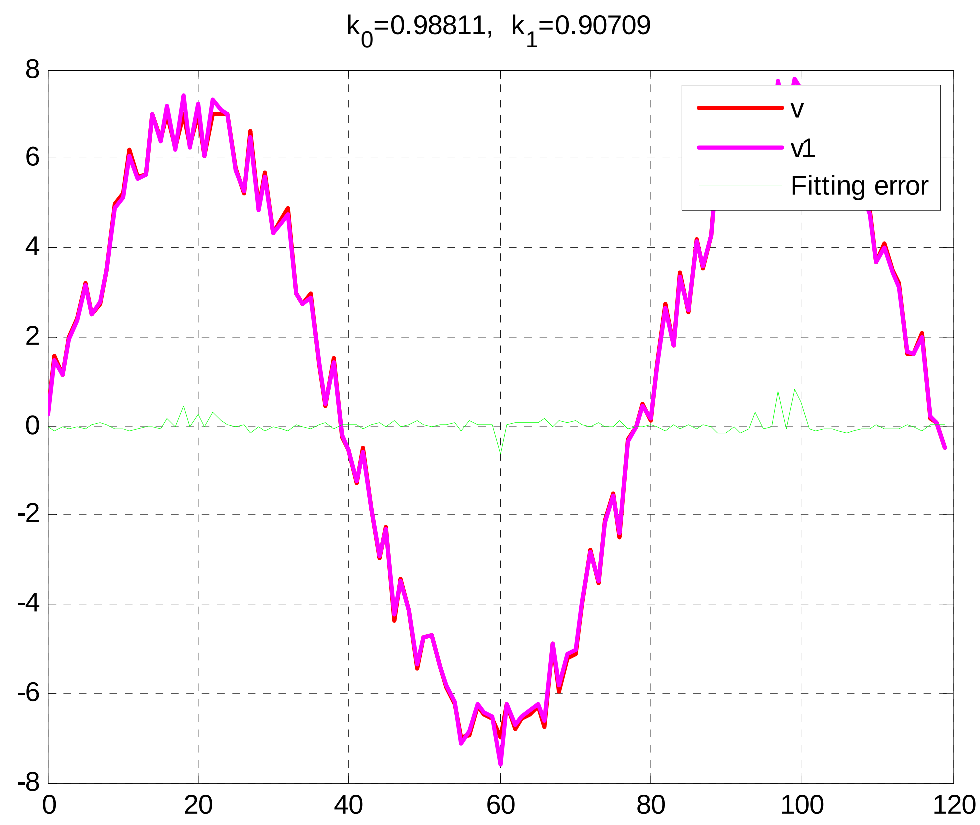
Accumulated Noise at the Quantizer Input



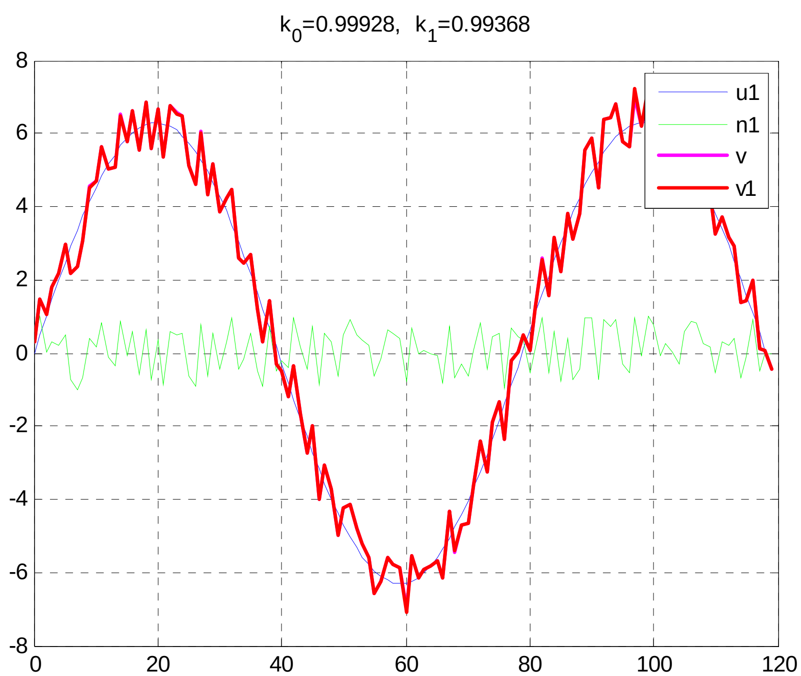
File:Third_Order_DSM_1.m

Describing Function Analysis: Curve Fitting



File: Decribing_Function_Analysis.m

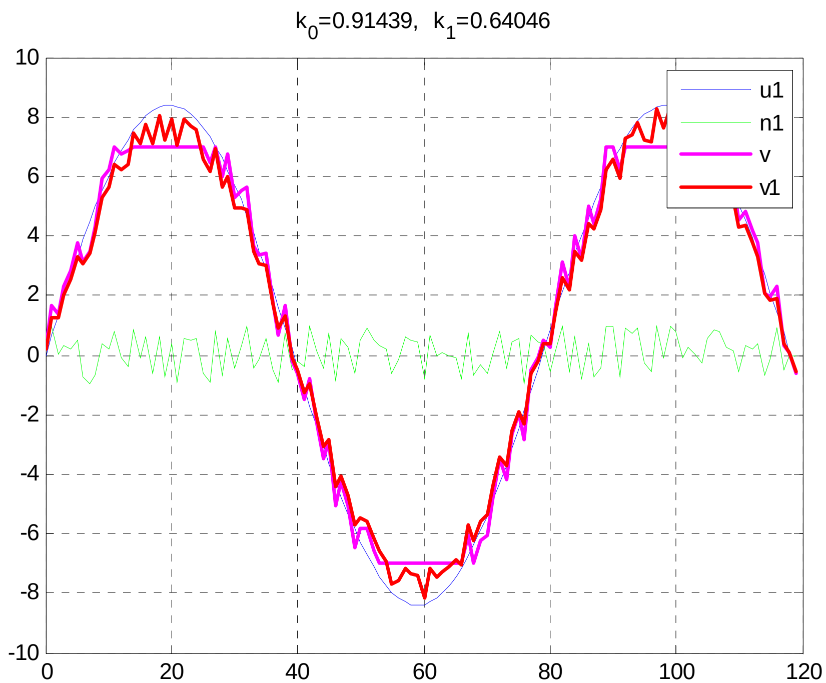
Describing Function Analysis



- Amplitude = 0.9 FS
- $k_0 \approx 1$, $k_1 \approx 1$.

File: Decribing_Function_Analysis.m

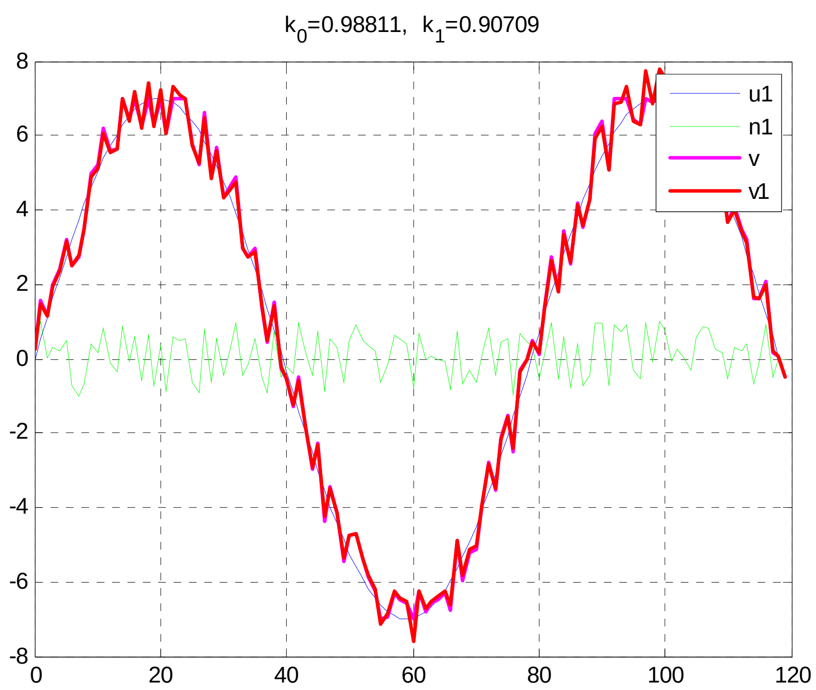
Describing Function Analysis contd.



- Amplitude = 1.2 FS
- k_0 =0.91, k_1 =0.64.

File: Decribing_Function_Analysis.m

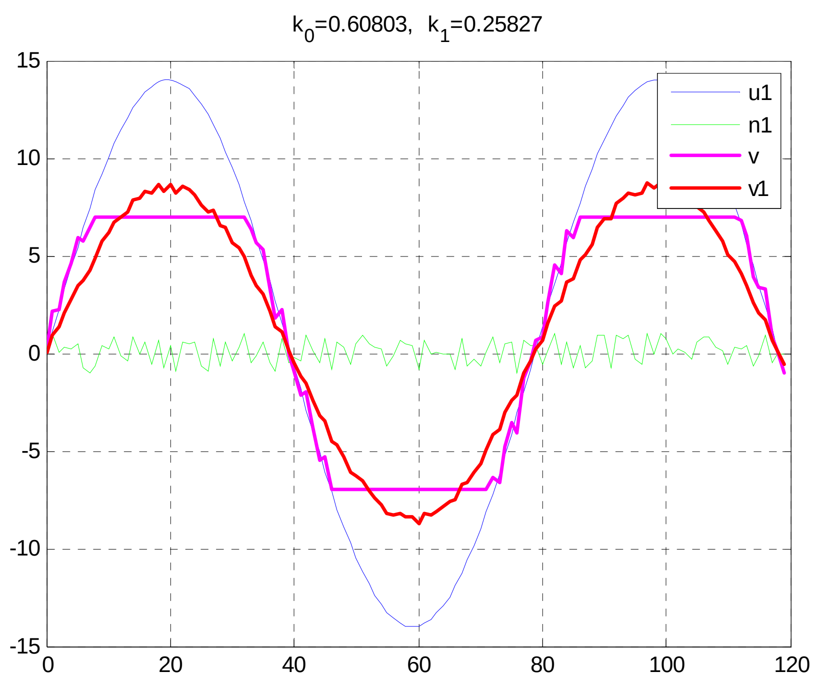
Describing Function Analysis contd.



- Amplitude = 1.5 FS
- k_0 =0.78, k_1 =0.44.

File: Decribing_Function_Analysis.m

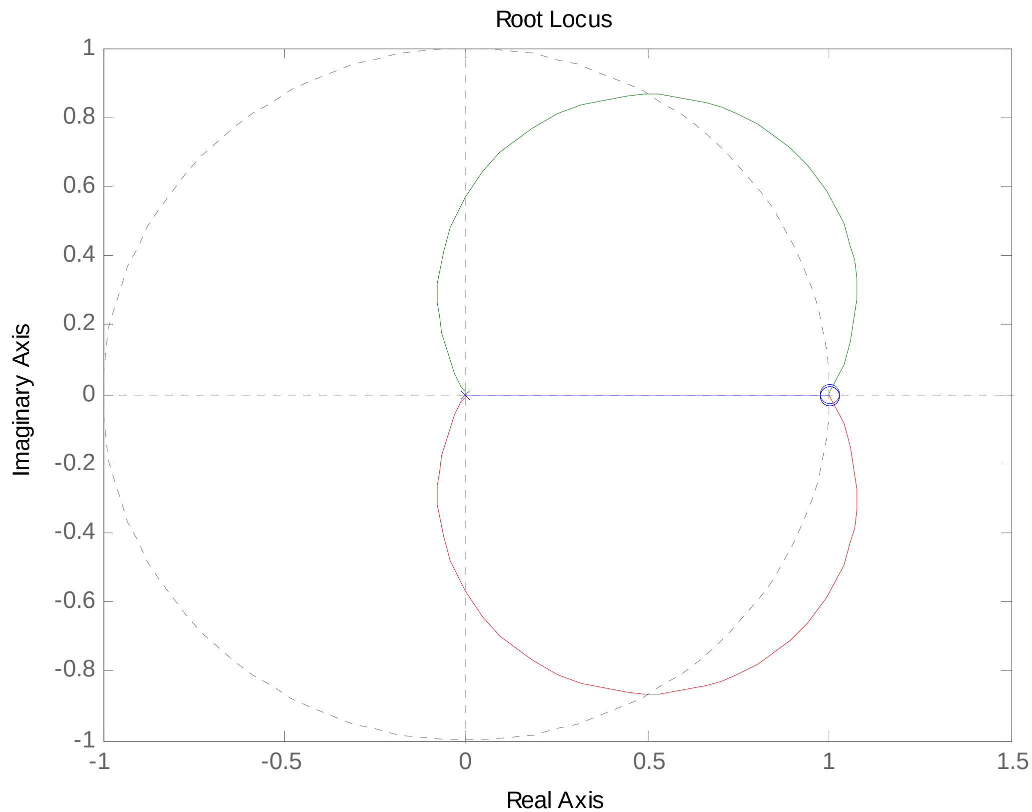
Describing Function Analysis contd.



- Amplitude = 1.5 FS
- k_0 =0.61, k_1 =0.26.

File: Decribing_Function_Analysis.m

3rd-Order DSM: NTF Root Locus



File: Third_Order_DSM_Root_Locus.m