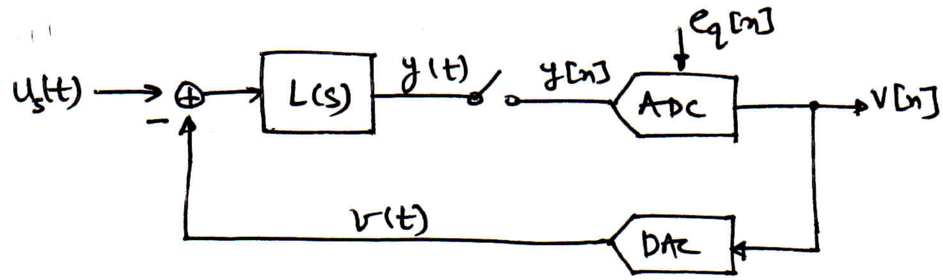
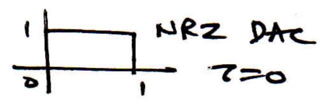
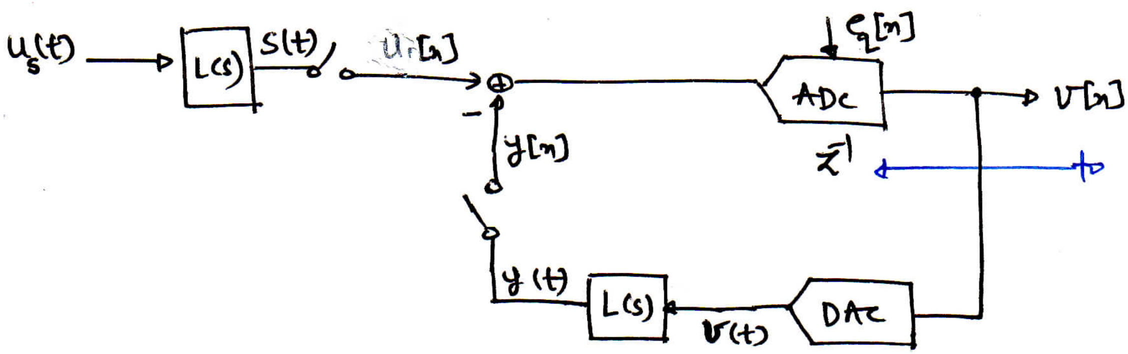
DAC reconstruction jitter derivation in CT ΔΣ Modulators.





Move the CT filter outside the loop:



Here in this figure:

- $s(t) \Rightarrow$ filtered input to the discrete-time loop, $S(s) = U_s(s) \cdot L(s)$
  $s[n]$ sees the $NTF(z)$ of the DT loop.
  Recall that: $STF(e^{j\omega}) = L(s)\Big|_{s=j\Omega} \cdot NTF(e^{j\omega})$ is the overall STF seen by $u(t)$.

Ideal DAC output:

$$v_o(t) = \sum_{k=-\infty}^{\infty} v[k] \cdot \left[u(t-kT) - u(t-kT-T)\right]$$

$$= \sum_{k} \left(v[k] - v[k-1]\right) u(t-kT)$$

Note that here:
$u_s(t) \leftarrow$ input signal
$u(t) \leftarrow$ step signal

with a jitter noise sequence $\Delta t[k]$, we get the jittery DAC output as

$$v(t) = \sum_{k=-\infty}^{\infty} \left(v[k] - v[k-1]\right) \cdot u\left(t - kT + \Delta t[k]\right) \longrightarrow (1)$$

X2

Let the step response of the CT filter, $L(s)$, be "$l_B(t)$"

i.e.,

$$\delta(t) \longrightarrow \boxed{L(s)} \longrightarrow l(t)$$

&

$$u(t) \longrightarrow \boxed{L(s)} \longrightarrow l_B(t).$$

· Then, we have the loop-filter output,

$$y(t) = \sum_{k=-\infty}^{\infty} \left(v[k]-v[k-1]\right) l_B(t-kT+\Delta t[k]) \longrightarrow (2)$$

Taking a Taylor series expansion about $t = t-kT$, we get

$$l_B(t-kT+\Delta t[k]) \simeq l_B(t-kT) + \left.\frac{d}{dt} l_B(t)\right|_{t=t-kT} \cdot \Delta t[k] + \ldots$$

$$\simeq \boxed{l_B(t-kT) + l(t-kT)\cdot \Delta t[k]} \longrightarrow (3)$$

Here we used the fact that $l(t) = \frac{d}{dt} l_B(t)$, by definition of linear T.I. system, step response.

The sampled output of the loop-filter is

$$y(nT) = y[n] = \sum_{k=-\infty}^{\infty} \left[v[k]-v[k-1]\right]\left[l_B(nT-kT) + l(nT-kT)\cdot \Delta t[k]\right].$$

writing $y[n]$ as a convolution we get

$$y[n] = l_B[n] \circledast \left(v[n]-v[n-1]\right) + \left(l[n] \circledast \hat{v}[n]\right)$$

where $\hat{v}[n] = \Delta t[n]\cdot\left(v[n]-v[n-1]\right)$ represents the jitter induced feedback error in the DAC output

Define $l[n] \xleftrightarrow{\mathcal{Z}} L(z)$ and $l_B[n] \xleftrightarrow{\mathcal{Z}} L_B(z)$

Also, $v[n] = y[n-1] + e_q[n-1]$, as per the block diagram

⇒ Taking Z-transform on $v[n]$, we obtain (Linearized model)

$$V(z) = z^{-1} \cdot \frac{E_q(z) + S(z) + L(z)\cdot \hat{V}(z)}{1 - L_B(z)\cdot z^{-1}(1-z^{-1})} \longrightarrow (4)$$

$E_q(z) \rightarrow$ quantization noise model (linearized)

$\hat{V}(z) \rightarrow$ DAC reconstruction jitter term

We can rewrite (4) as,

$$V(z) = NTF(z)\cdot\left(E_q(z) + S(z)\right) + STF(z)\cdot \hat{V}(z) \longrightarrow (5)$$

Here,

$$\left.\begin{aligned} NTF(z) &= \frac{z^{-1}}{1 - z^{-1} L_B(z)(1-z^{-1})}, \text{ and} \\ STF(z) &= \frac{z^{-1} L(z)}{1 - z^{-1} L_B(z)(1-z^{-1})} \end{aligned}\right\} \longrightarrow (6)$$

Eq^n (5) includes the effect of DAC jitter in the output $V(z)$.
In the absence of any jitter, we have $\hat{V}(z) = 0$,

$$\Rightarrow \quad V_0(z) = NTF(z)\cdot\left[E_0(z) + S(z)\right] \longrightarrow (7)$$

⇒ from (5)–(7), the error in the modulator output due to the jitter is

$$E_{vj}(z) = V(z) - V_0(z) = \cancelto{0}{NTF(z)\cdot \Delta E_q(z)} + \boxed{STF(z)\cdot\hat{V}(z)}$$

$$\simeq \boxed{STF(z)\cdot\hat{V}(z)} \longrightarrow (8)$$

⇒ In time-domain the error in the output due to jitter is

$$e_{vj}[n] = stf[n] \circledast \hat{v}[n]$$

$$= stf[n] \circledast \{ (v[n] - v[n-1]) \cdot \Delta t[n] \} \longrightarrow (9)$$

If, for convenience we assume that $STF(z) \simeq 1$ in the region of interest, we have

$$\boxed{e_{vj}[n] \simeq (v[n] - v[n-1]) \cdot \Delta t[n]}$$ for an NRZ DAC.

- usually the notation is $dv[n] = v[n] - v[n-1]$.
- In the papers by Cherry (1999) and Reddy (2007), only the effect of $dv[n]$ is considered for the DAC jitter noise analysis
- I believe that the effect of "STF(z)" must be included to refine our mathematical modeling of jitter noise effects in CT $\Delta\Sigma$ modulators.

- Similarly, jitter error in the output can be derived for RZ DAC. [see TAO [1999]]

In Reddy (2007), the jitter noise term is analyzed and related to NTF(z) using the ~~be~~ observation that

$$e_{vj}[n] \simeq (v[n] - v[n-1]) \cdot \Delta t[n]$$

and

$$dV(z) = V(z)(1 - z^{-1})$$

$$= (U(z) + NTF(z) \cdot E(z))(1 - z^{-1}), \quad \text{assuming } STF(z) = 1$$

$$\simeq (1 - z^{-1}) NTF(z) \cdot E(z), \quad \text{if input varies slowly or for "idle-channel" jitter noise}$$

⇒ Jitter noise $\propto \left| (1 - z^{-1}) NTF(z) \right|_{z = e^{j\omega}}$

⇒ optimize in-band quantization noise and jitter noise.

Read Reddy (2007) for details.