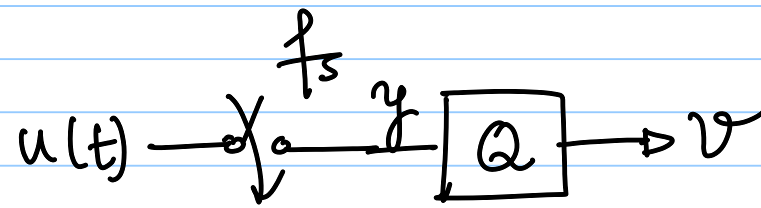
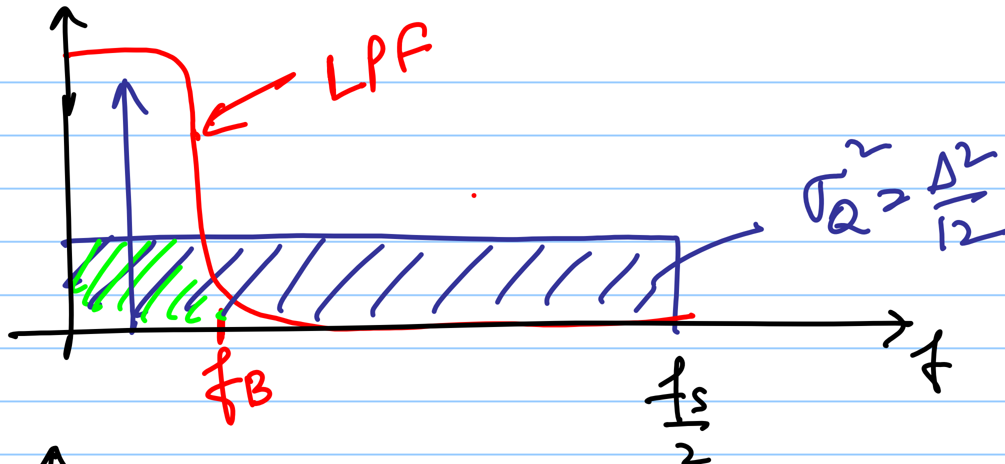
ECE 615 - Lecture 9

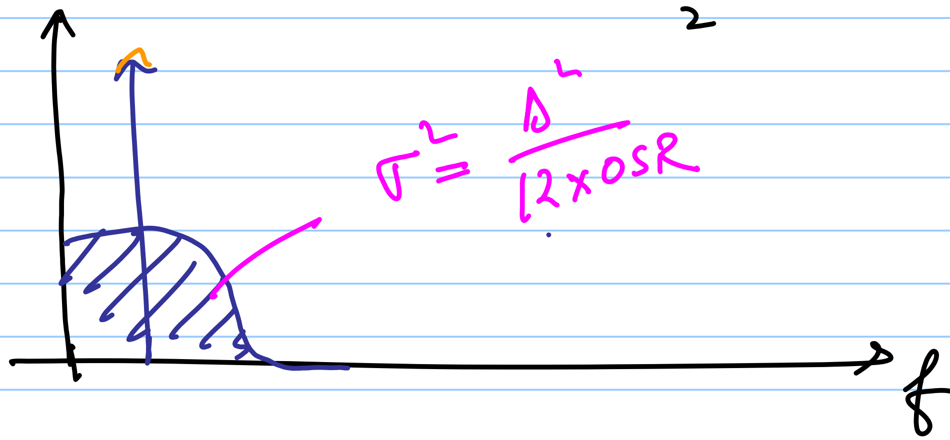
Note Title 2/9/2016

Oversampling:



$$f_s \geq \underbrace{2 f_B}_{\text{Nyquist sampling rate}}$$





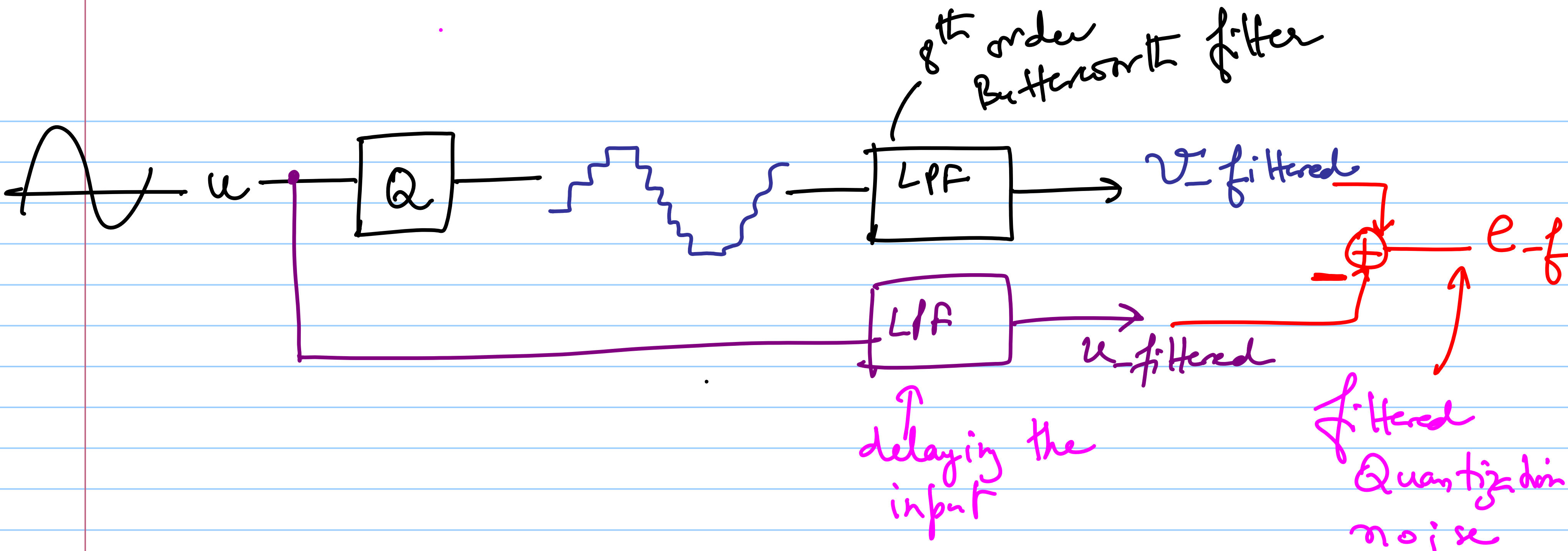
oversampling ratio

$$OSR = \frac{f_s}{2 f_B}$$

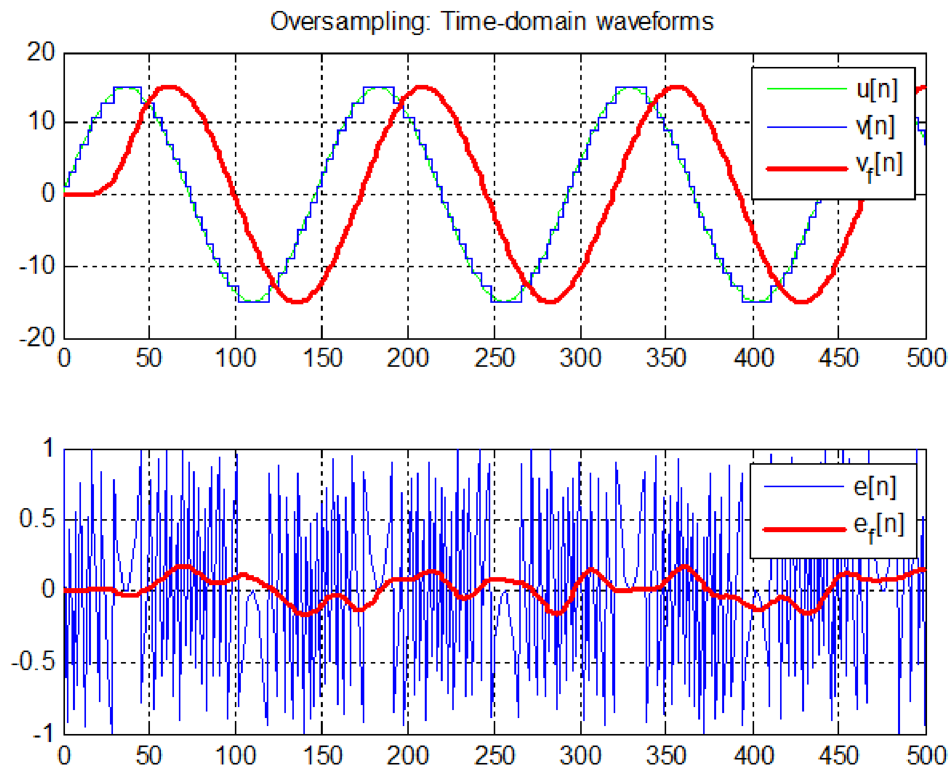
$$f_s = (2 f_B) \times OSR$$

Q-Noise is confined to the bandwidth of the LPF

$$\Rightarrow \sigma^2 = \frac{\Delta^2}{12} \Big/ OSR$$



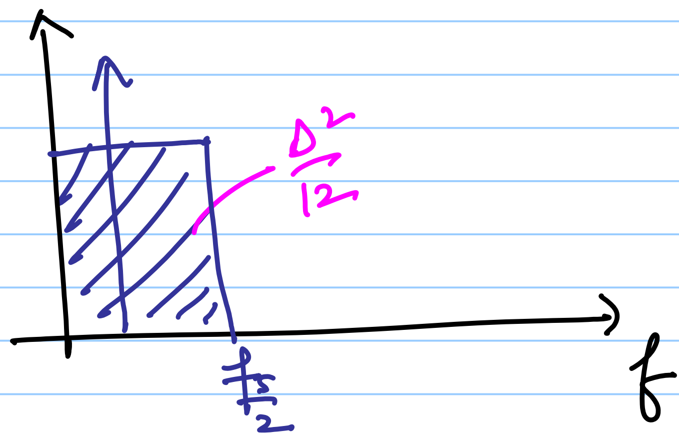
u
Q
LPF
8th order Butterworth filter
v_filtered
LPF
u_filtered
delaying the input
e_f
filtered Quantization noise

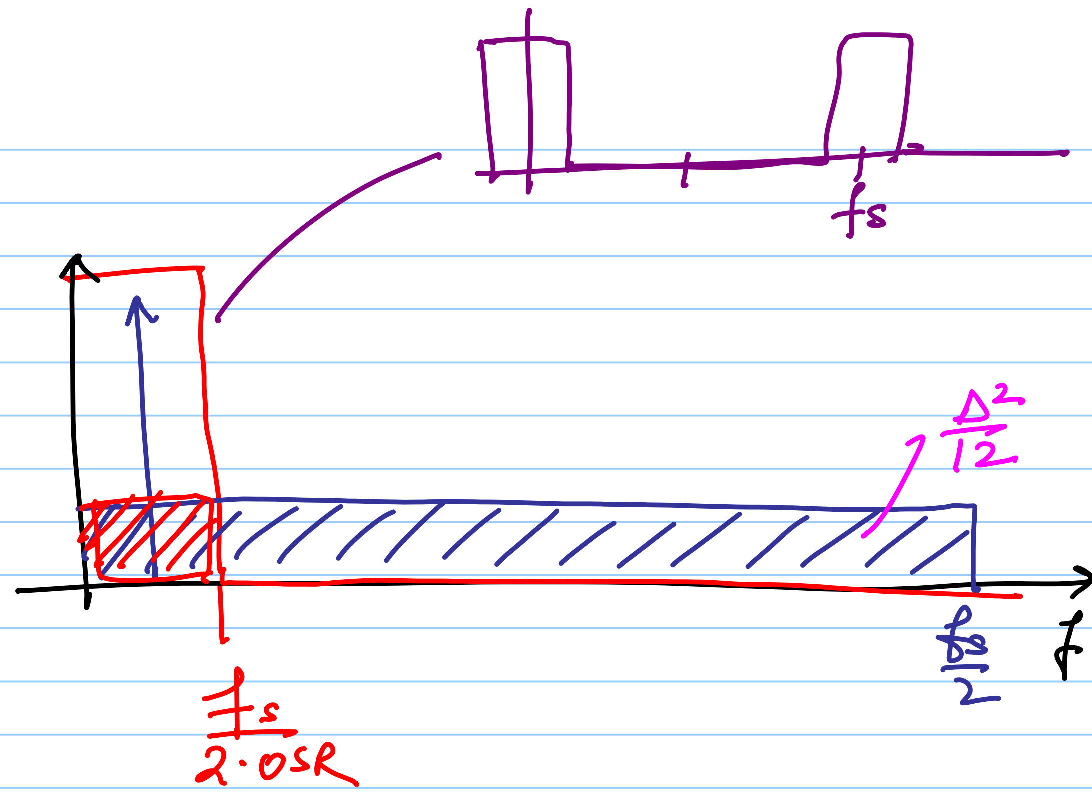


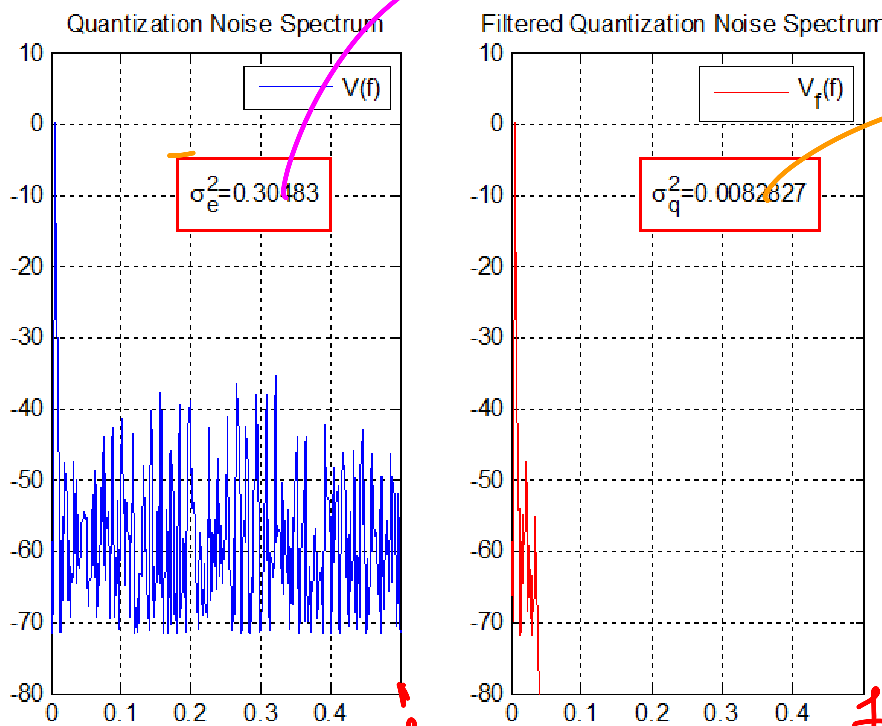
Oversampling: Time-domain waveforms
u[n]
v[n]
$v_f[n]$
e[n]
$e_f[n]$

Nyquist Sampling

$f_s = 2 f_B$







$\Delta^2/12$

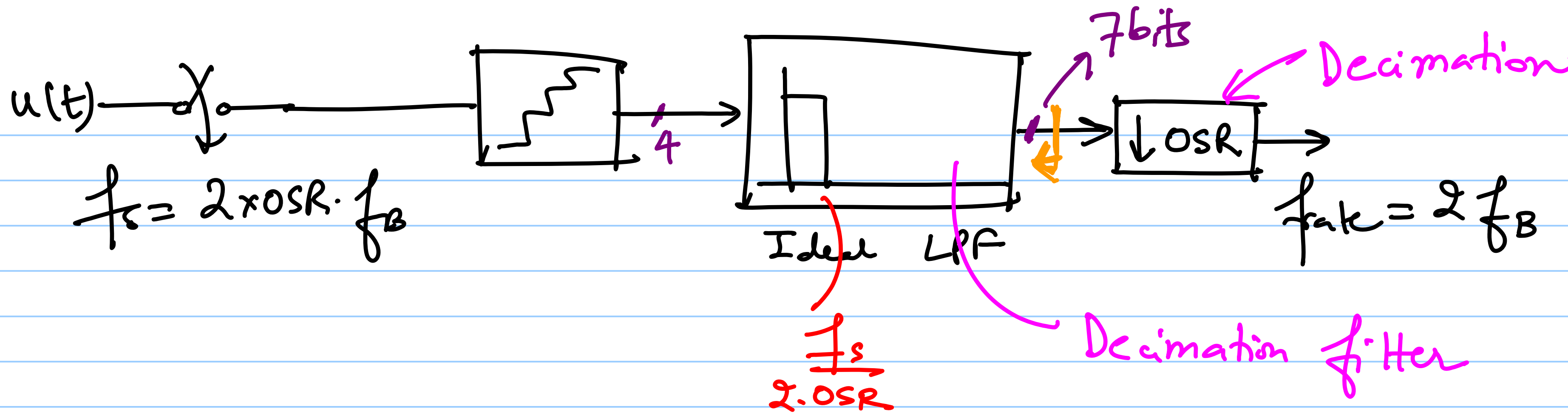
$\simeq \frac{\Delta^2}{12 \cdot OSR}$

$\frac{f_s}{2}$

$\frac{f_s}{2}$

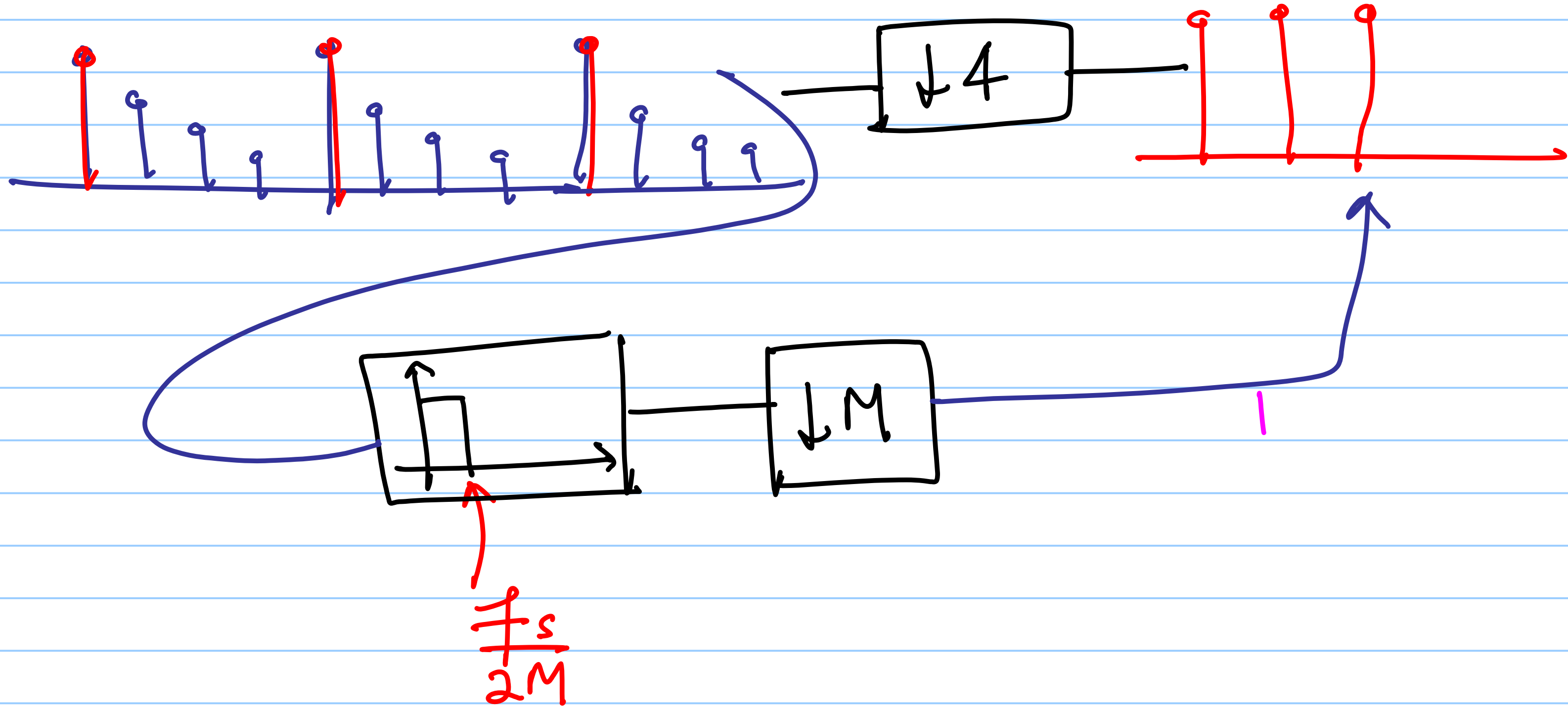
$OSR = 32$

$f_s = 1$

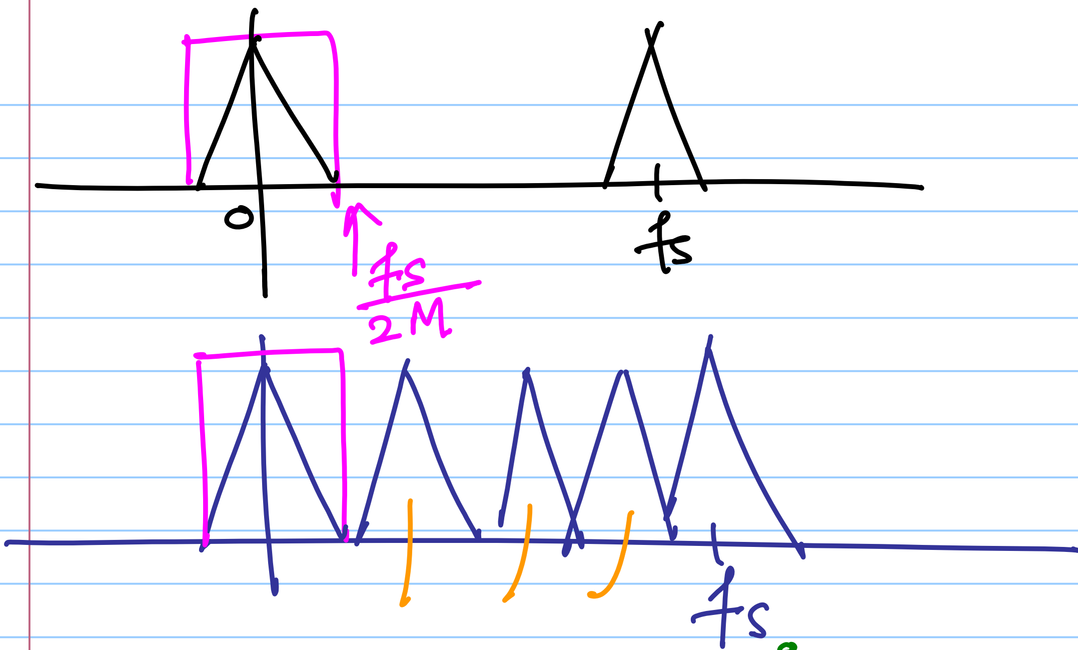


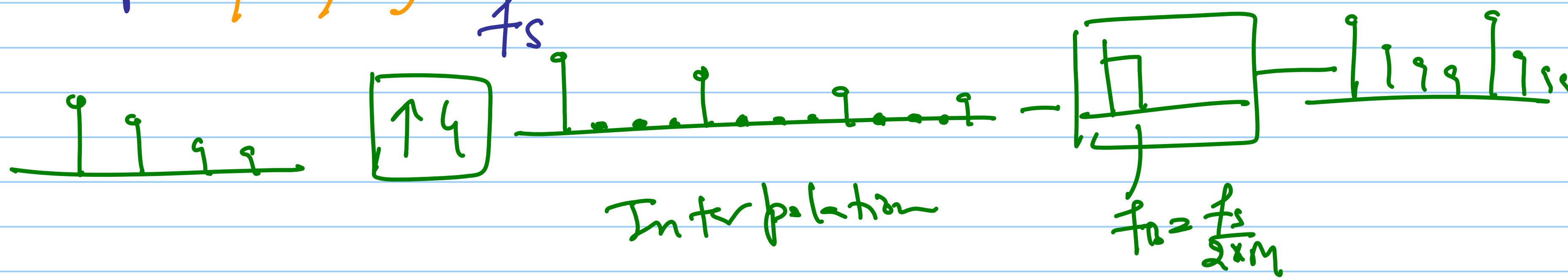
u(t)
$f_s = 2 \times OSR \cdot f_B$
4
Ideal LPF
7 bits
Decimation
$\downarrow OSR$
$f_{dc} = 2 f_B$
$\frac{f_s}{2 \cdot OSR}$
Decimation filter

Decimation



$M = 4$





$A = 2^{N-1}$

$$SQNR = 10\log_{10}\left(\frac{A^2/2}{\frac{\Delta^2}{12\times OSR}}\right) = 10\log_{10}\left(\frac{\frac{(2^{N-1}\Delta)^2}{2}}{\frac{\Delta^2}{12\cdot OSR}}\right)$$

$$= 10\log\left(\frac{3}{2}\cdot 2^{2N}\cdot OSR\right)$$

$$= 6.02N + 1.76 + \boxed{10\log_{10}(OSR)}$$ Extra term

$\hookrightarrow 10 \times P \times \log_{10}(2)$ $\quad OSR = 2^P$

$3 \times P$

$$SQNR = 6.02N + 1.76 + 10\log_{10}(OSR)$$

$$ENOB = \frac{SQNR - 1.76}{6.02}$$

$$= \frac{6.02N + 10\log_{10}(OSR)}{6.02}$$

$$= N + \frac{10\log_{10}(OSR)}{6.02}$$

$$= N + \frac{3\log_2(OSR)}{6}$$

$$= N + 0.5\log_2(OSR)$$

$$\log_{10}(OSR) = \frac{\log_2(OSR)}{\log_2(10)}$$

$$= \log_{10}2 \times \log_2(OSR)$$

for every doubling in OSR $\Rightarrow$ $\frac{1}{2}$ bit increase in resolution

$$N_{eff} = ENOB = N + \frac{1}{2}\log_2 OSR$$

$$= 4 + 0.5 \times 6$$

$$= 7 \text{ bits}$$

$N = 4$ bit

$OSR = 64$

Trading analog complexity with digital complexity

7-bit Flash ADC

$2^7 - 1$ comparators

4-bit flash ADC

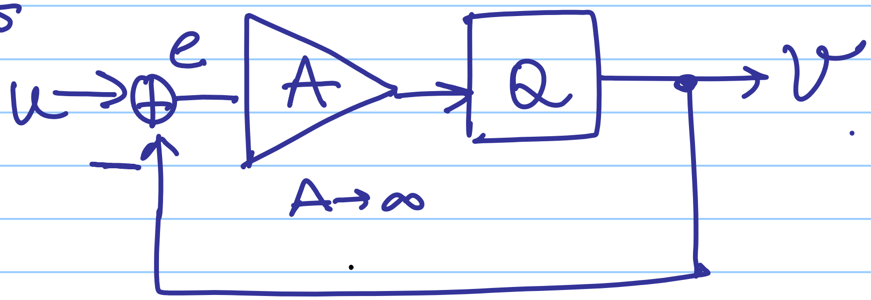
64x clock rate

Digital filter

Can we do better than $\frac{1}{2}$ bit per 2x OSR?

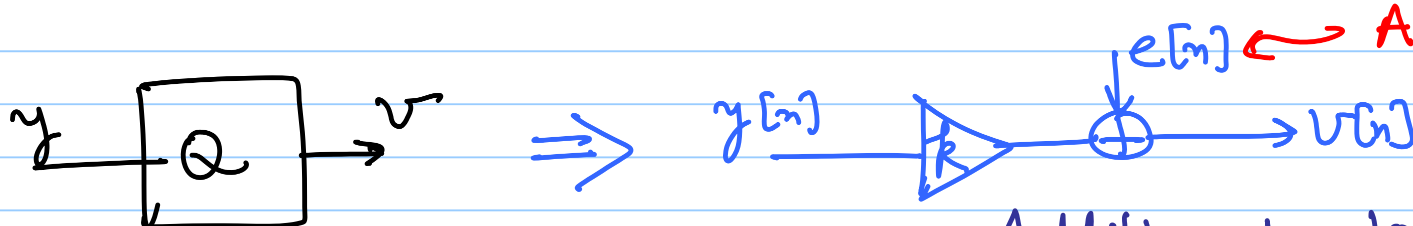
Use feedback to reduce error, e[n].

input



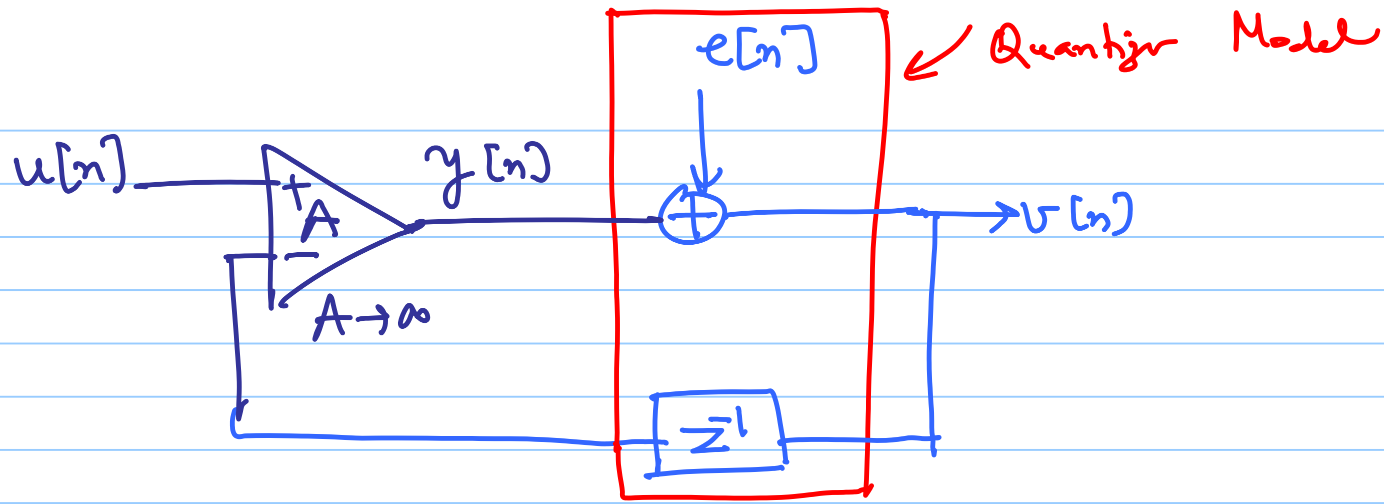
$e \to 0$

reduce $|e| = |u - v|$ by high loop-gain



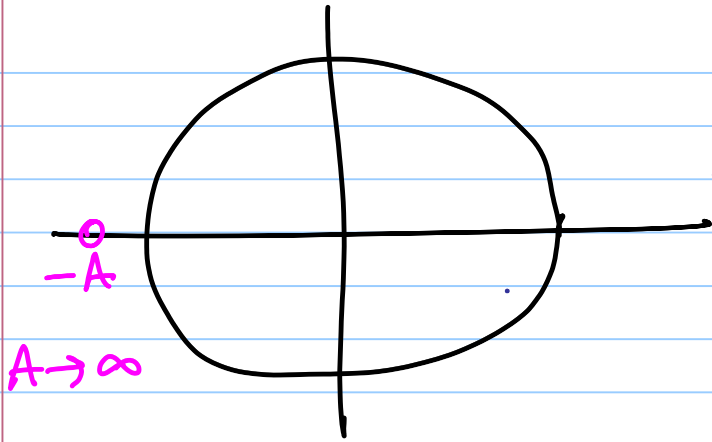
← AWUN

Additive quantization noise model



$$\left(u(z) - z^{-1} v(z)\right)A + E(z) = V(z)$$

$$\Rightarrow \quad V(z) = \left(\frac{A}{1+Az^{-1}}\right) X(z) + \frac{E(z)}{1+Az^{-1}}$$



pole at $1+Az^{-1}=0$

$\Rightarrow z+A=0$

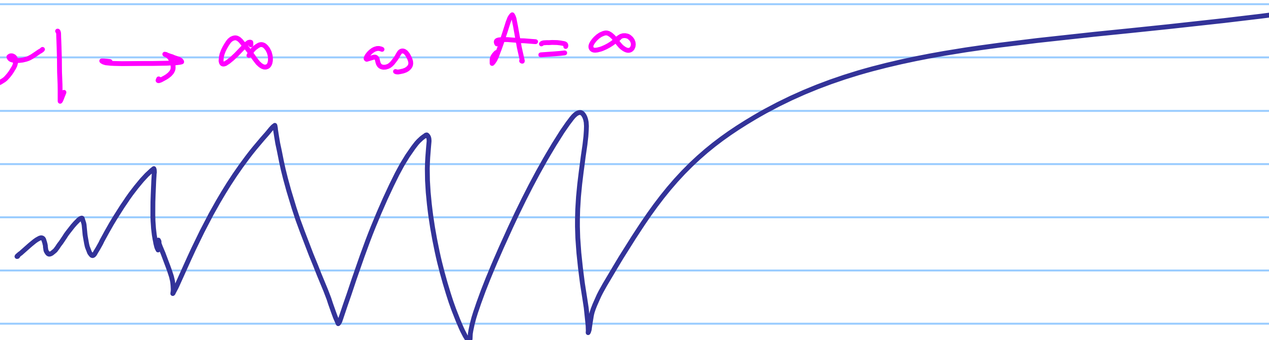
$\Rightarrow z=-A$

pole at $-A \Rightarrow -\infty$

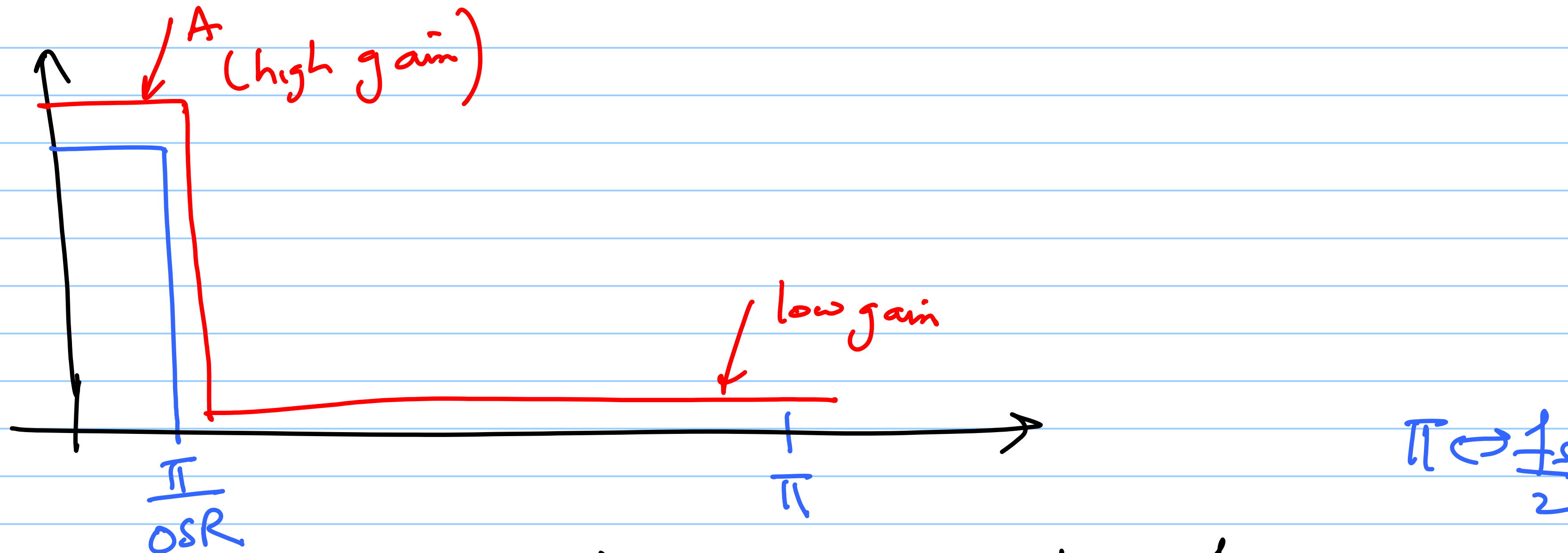
$\Rightarrow$ system is not stable at all!

error $|u-v| \to \infty$ as $A=\infty$

$|u-v|$

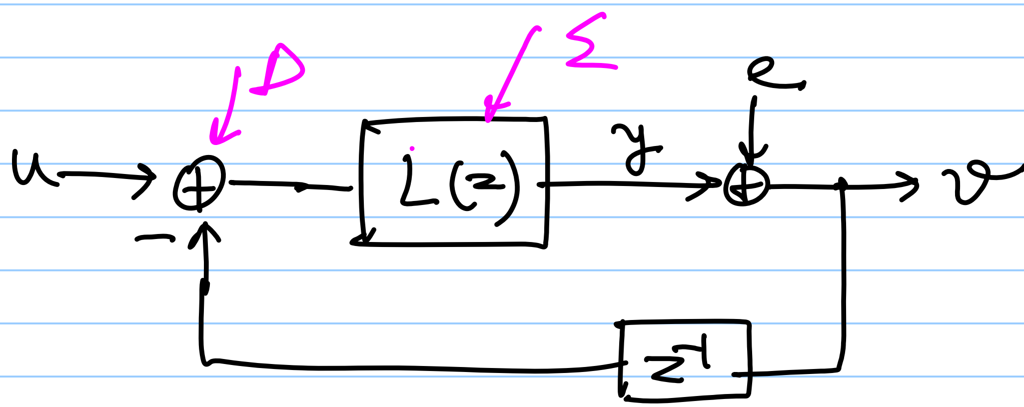


Doesn't work at all!



- Apply high gain at low frequencies to reduce in-band quantization noise
- At high frequencies, keep the gain low to stabilize the loop.

Replace A by $L(z) \Rightarrow$ loop filter



$|L(z)| \to \infty$ at $z=1$

or $\omega = 0$

$L(z) = \dfrac{1}{1-z^{-1}} \Leftarrow$ integrator