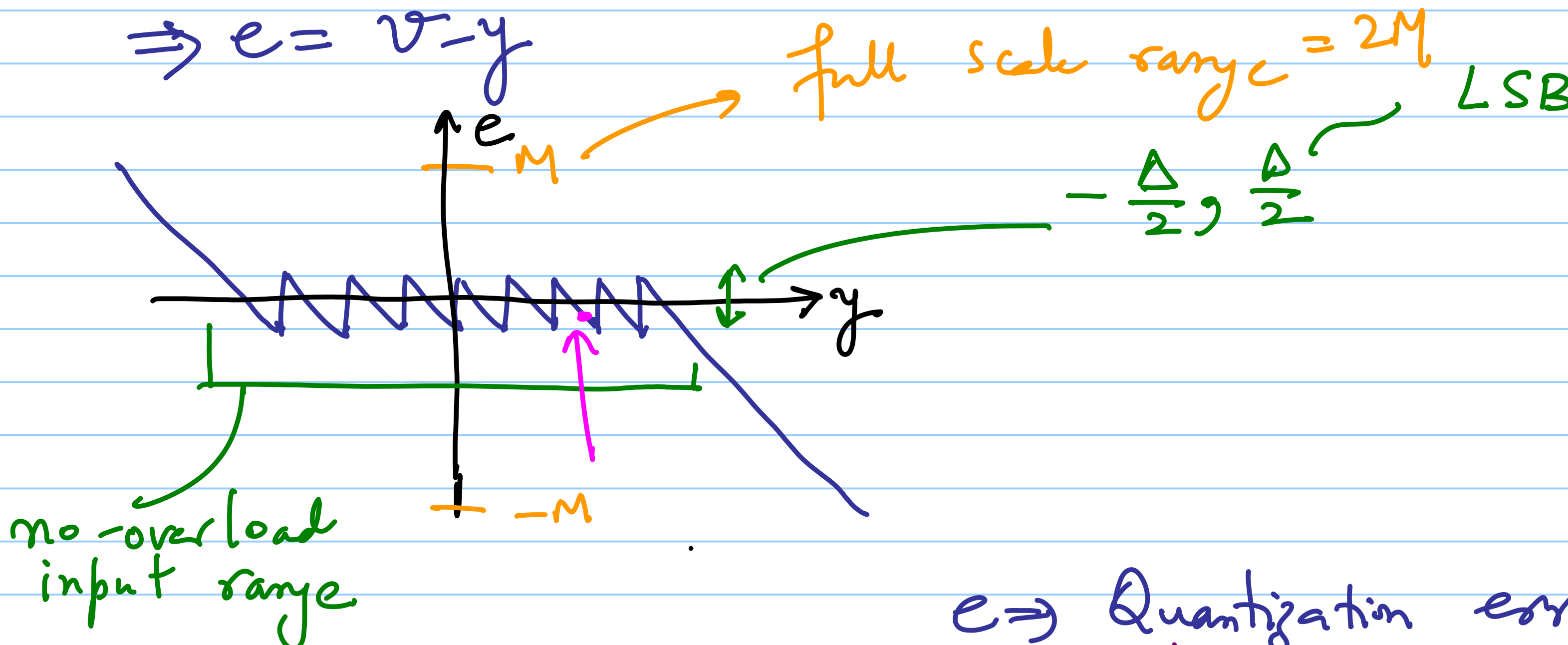
ECE 615 - Lecture 8

Note Title 2/4/2016

$\Rightarrow e = v - y$



$e \Rightarrow$ Quantization error

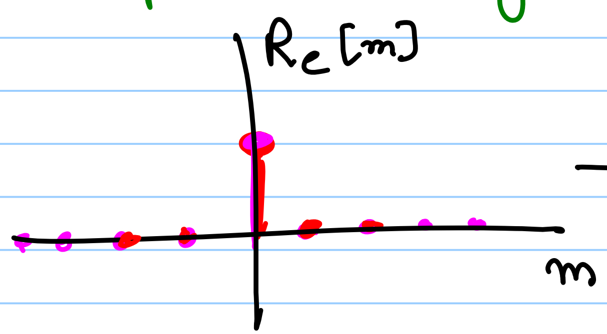
↳ noise

Conditions to model $e[n]$ as noise

① y (input) stays within the no-overload input range

② $e[n]$ is uncorrelated with the input $y[n]$
"input must be busy"

③ Spectrum of $e[n]$ is "white"



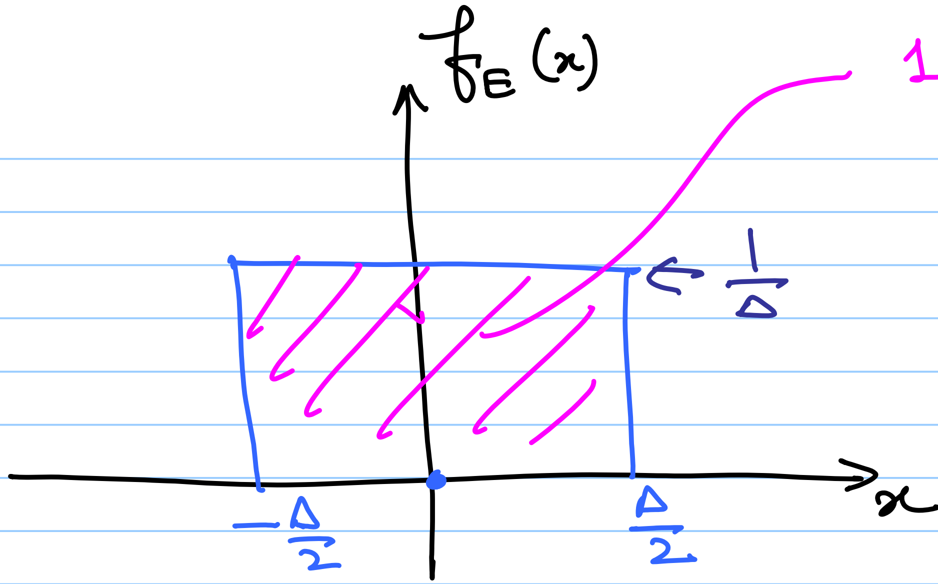
PSD of "e"

$$S_e(f) = \mathcal{F}\Big[\underbrace{E\big(e[n]\, e[n-m]\big)}_{\text{autocorrelation}}\Big] = \delta[m]$$

PSD

④ Quantization error is uniformly distributed

$$e \sim U\left[-\frac{\Delta}{2}, \frac{\Delta}{2}\right]$$



$$1 = \int_{-\infty}^{\infty} f_E(x)\,dx$$

$\mu = 0$

σ_e^2

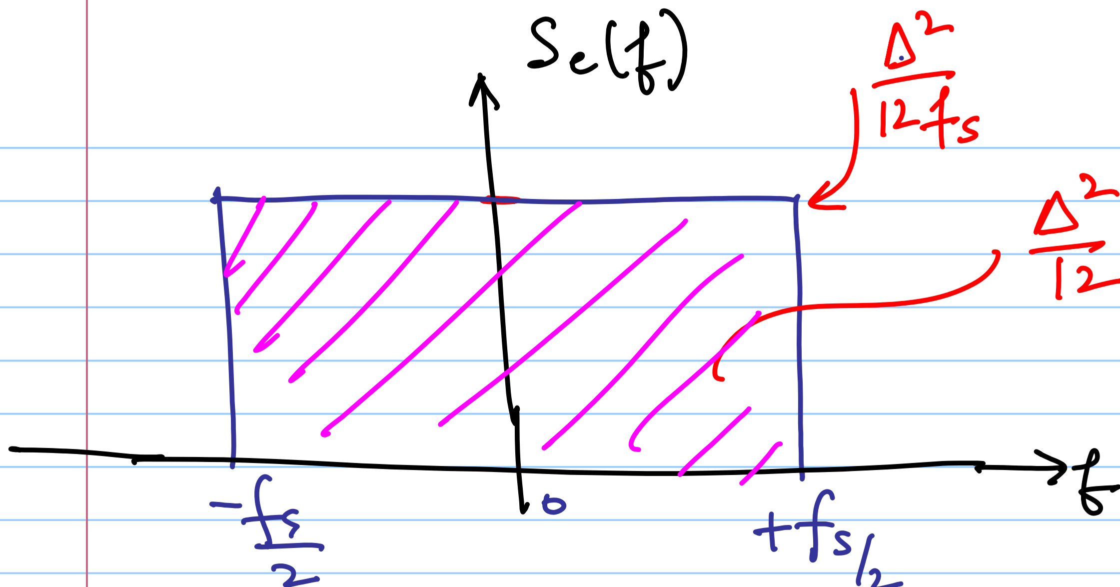
$$\sigma_e^2 = E\left[(e-\mu)^2\right] = E\left[e^2\right] = \int_{-\Delta/2}^{+\Delta/2} x^2 f_E(x)\,dx$$

$$= \frac{1}{\Delta}\left[\frac{x^3}{3}\right]\Bigg|_{-\Delta/2}^{\Delta/2} = \frac{2}{\Delta} \times \frac{x^3}{3}\Bigg|_{0}^{\Delta/2} = \frac{2}{\Delta}\left(\frac{\Delta}{2}\right)^3 \frac{1}{3}$$

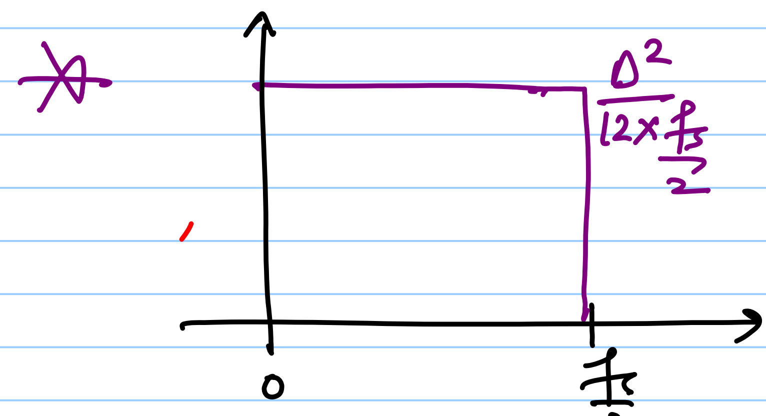
$$= \frac{\Delta^2}{12}$$

Noise power = Variance $= \sigma_e^2$

Q-noise power $= \frac{\Delta^2}{12}$ $= \frac{V_{LSB}^2}{12}$



Sampled white noise distributed from $-\frac{f_s}{2}$ to $\frac{f_s}{2}$



One-sided spectrum

$y = A \sin(\omega_{in} t)$ is quantized with a quantizer which has $LSB = \Delta$

Signal-to-quantization-noise ratio

$$SQNR = \frac{\text{Signal power}}{\text{Q-noise power}} = \frac{P_s}{\Delta^2/12}$$

P_s ← signal power

$P_s = \frac{A^2}{2}$ for sine input

* We have an N-bit ADC

full scale range (FS) $= 2^N \Delta$

maximum amplitude $(A_{max}) = \frac{FS}{2} = 2^{N-1}\Delta$

max. signal power $= \frac{(2^{N-1}\Delta)^2}{2} = \frac{A_{max}^2}{2}$

$$\text{Peak SQNR} = \frac{(2^{N-1}\Delta)^2}{2 \times \frac{\Delta^2}{12}} = \frac{2^{2N-2}}{2} \times 12 = \frac{3}{2} \times 2^{2N}$$

← full scale sine input

$$SQNR_{dB} = 10 \log_{10}\left(\frac{3}{2} \cdot 2^{2N}\right)$$

$$\boxed{SQNR_{dB} = 6.02N + 1.76 \text{ dB}}$$

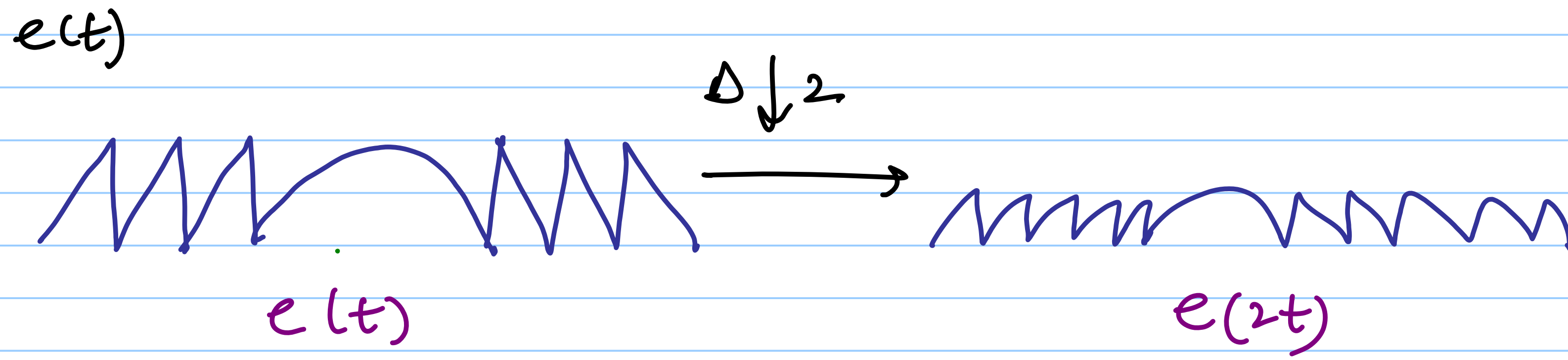
for $\Delta = 0.1 \Rightarrow$ # of periods $= 44$

for $\Delta = 0.2 \Rightarrow$ # of periods $= 20$

When Δ is halved

Δ^2

tone power $= \left(\frac{1}{2}\right)^2 = \frac{1}{4} = -6\,dB$

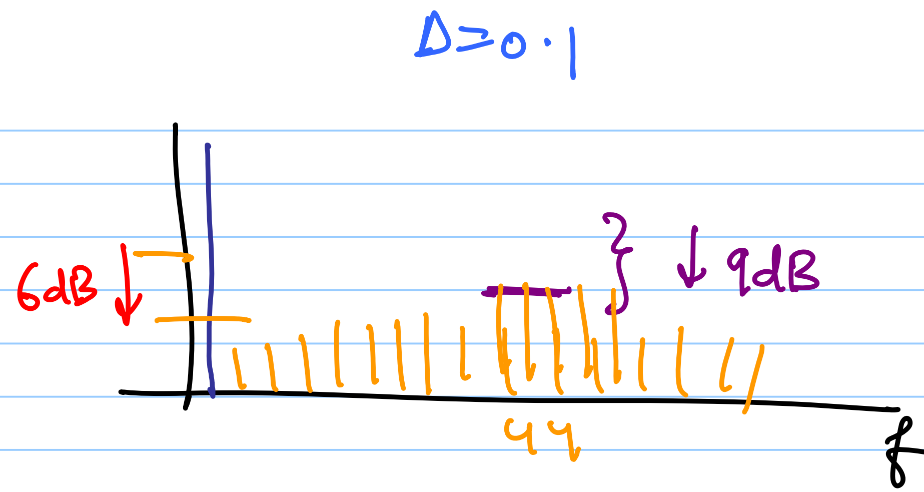


$E(f)$ $\qquad\qquad$ $\frac{1}{2} E(f/2)$

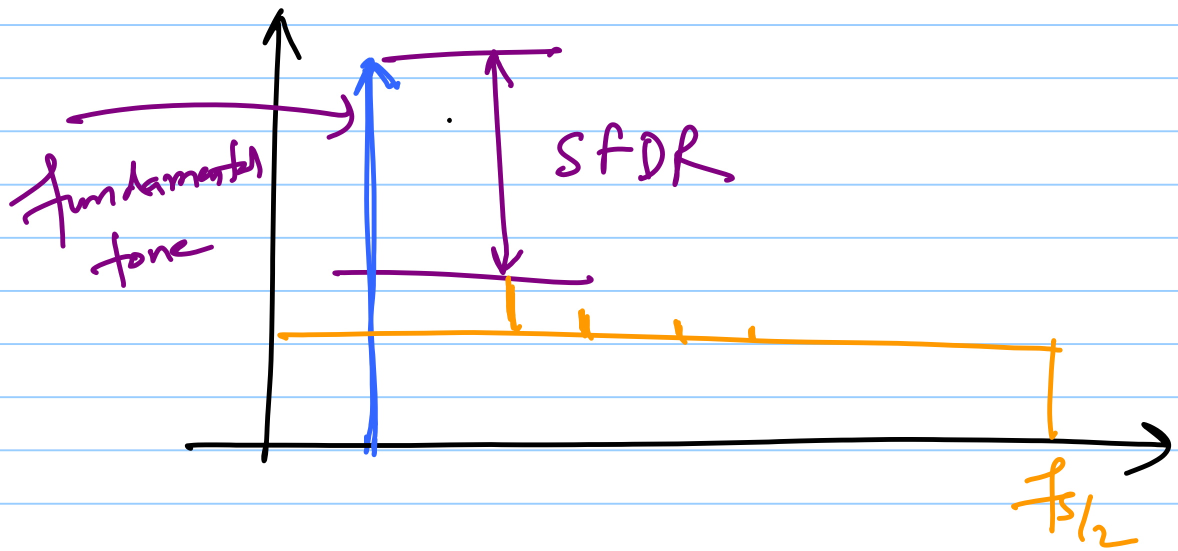
$\Rightarrow$ 3dB reduction in tone power

total tone power reduction = 9dB by halving Δ


Δ= 0·2
20
f

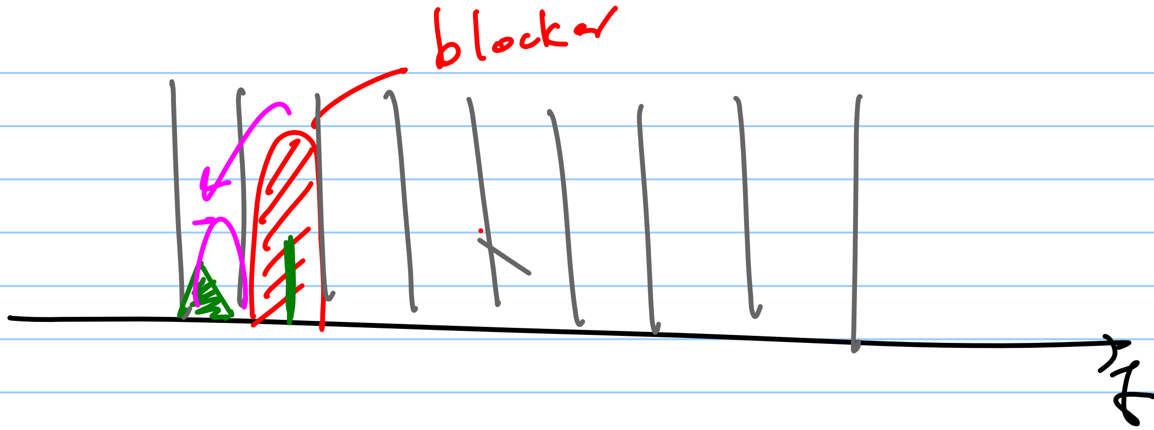

Δ=0·1
6dB
9dB
44
f

SFDR (spurious-free dynamic range)



Spurs ⇒ spurious tones

SFDR = difference (in dB) b/w the signal tone power & the largest spurious tone "in the bandwidth of interest"



halving Δ by 2 ⇒ 9dB better SFDR

& 6dB higher SQNR

Frequency domain measurements

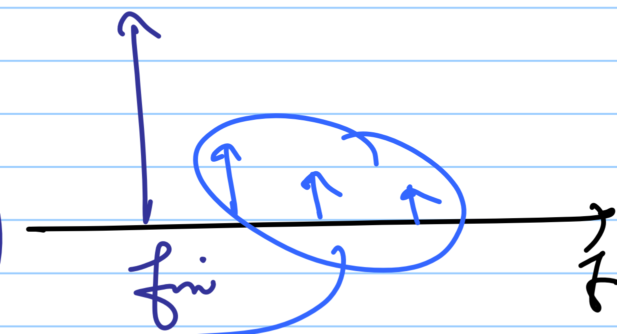
$$SQNR = 10 \log_{10}\left(\frac{P_s}{P_Q}\right)$$

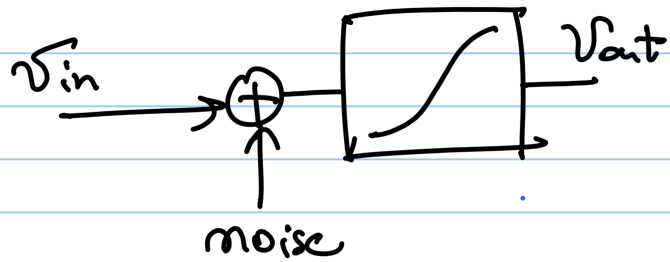
$P_Q \Rightarrow$ quantization noise power

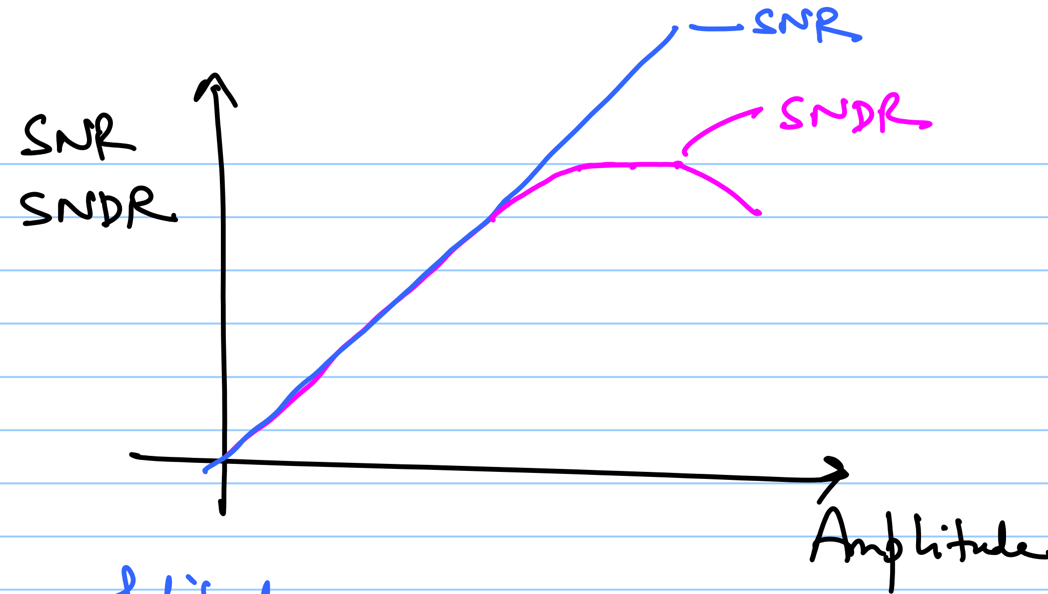
$$SNR = 10 \log_{10}\left(\frac{P_s}{P_N}\right)$$

$$SNDR = 10 \log_{10}\left(\frac{P_s}{P_N + P_{distortion}}\right)$$

signal to noise and distortion ratio (SINAD)







@ higher amplitude

⇒ higher distortion

⇒ lower SNDR

$$\text{ENOB} = \frac{\text{SNDR} - 1.76}{6.02}$$

effective number of bits.

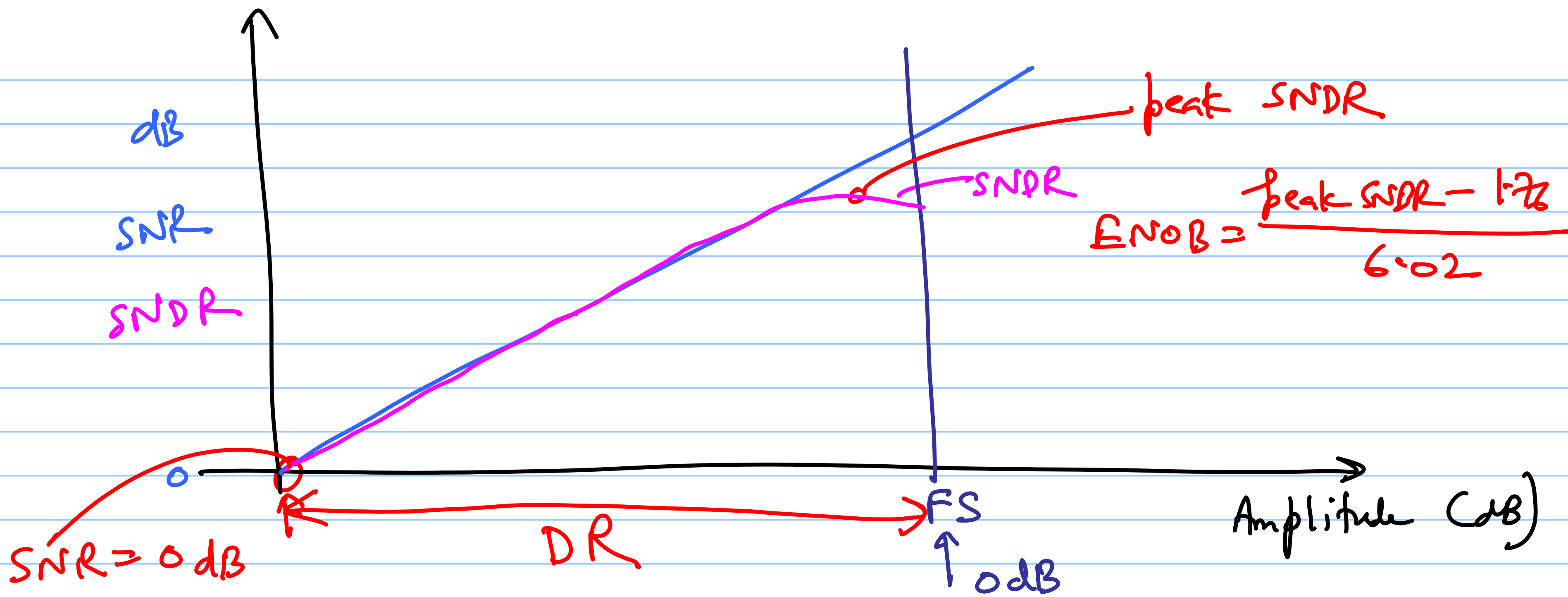
$$\text{SQNR} = 6.02N + 1.76$$

$$N = \frac{\text{SQNR} - 1.76}{6.02}$$

$$\text{SFDR} = 10 \log_{10} \left(\frac{\text{signal power}}{\text{largest spurious power}} \right)$$

$$\text{Dynamic Range (DR)} = 10 \log \left(\frac{\text{Maximum signal power detect}}{\text{Smallest signal power detected}} \right)$$

$$= 10 \log \left(\frac{A_{max}}{A_{min}} \right)$$


dB
SNR
SNDR
peak SNDR
SNDR
ENOB = (peak SNDR − 1.76) / 6.02
0
SNR = 0 dB
DR
FS
0dB
Amplitude (dB)