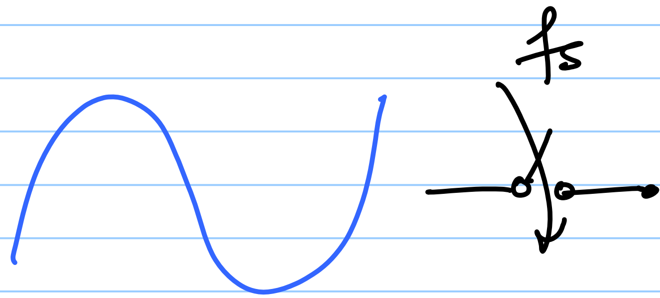
ECE 615 - Lecture 7

Note Title 2/2/2016



$x[n] \xrightarrow{\text{FFT}} X(k)$

f_{in}

Coherent sampling

$x(t)$ $A \sin(2\pi f_{in} t)$

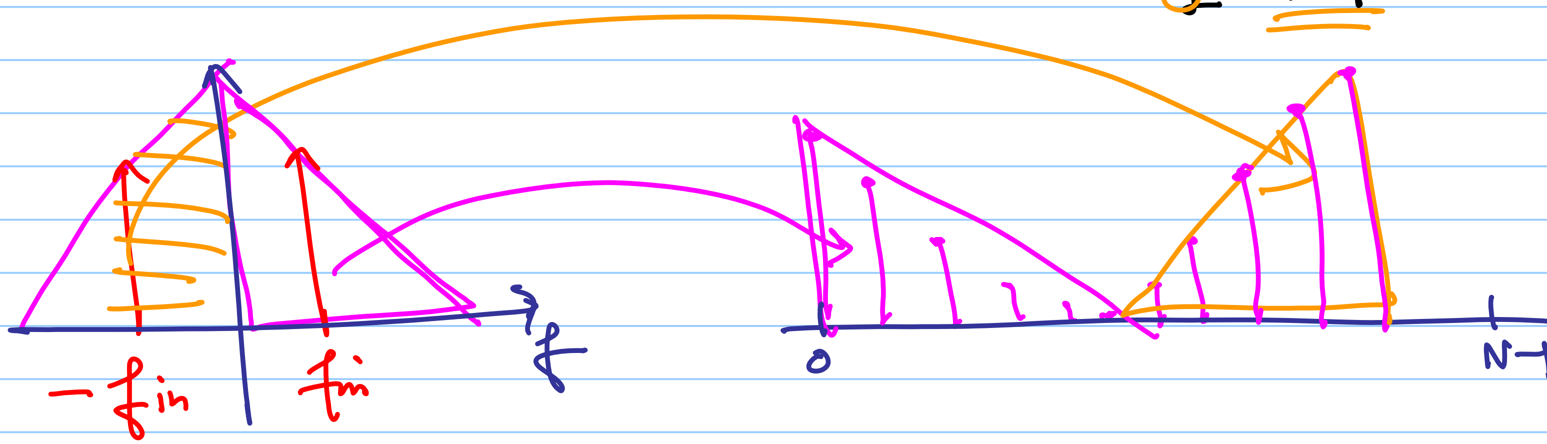
$$\frac{f_{in}}{f_s} = \frac{m}{N}$$

'm' cycles of f_{in} are fitted in 'N' cycles of f_s

$N = 2^p$

$f_{in} = \frac{m}{2^p} f_s$

lets say $f_{in} = \frac{129}{1024} f_s$



$$f_s = 100 \text{ MHz}$$

$$f_{in} = 2.5 \text{ MHz}$$

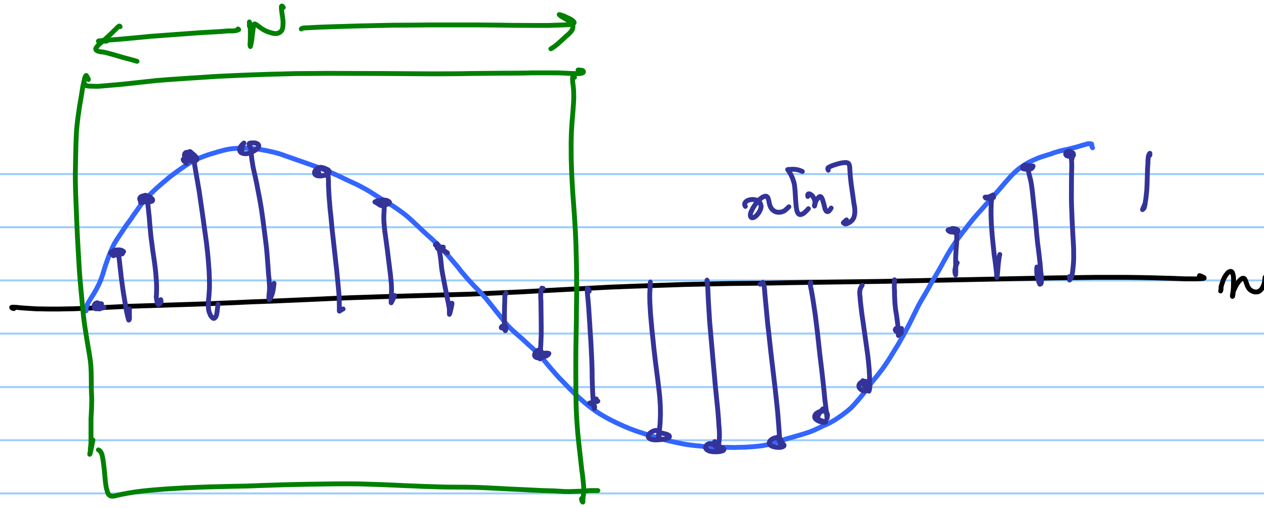
$$\frac{f_{in}}{f_s} = \frac{2.5}{100} = \frac{25}{1000} = \frac{1}{40} = \frac{m}{2^{10}}$$

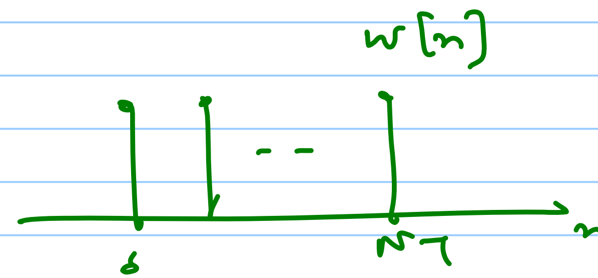
$$N = 2^{10}$$

$$\frac{f_{in}}{f_s} = \frac{m}{2^{10}}$$

$$m = \left\lfloor \frac{f_{in} \times 2^{10}}{f_s} \right\rfloor$$

$$f_{in\_new} = \frac{m f_s}{2^{10}}$$



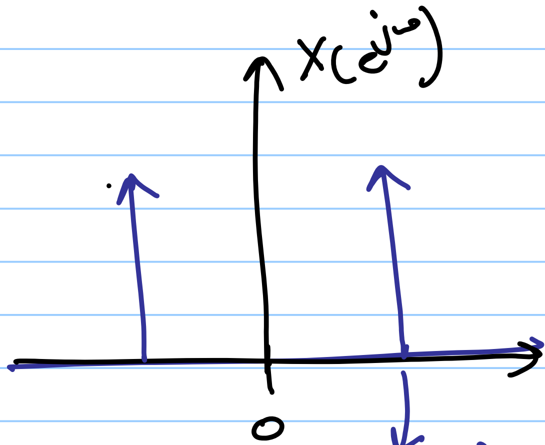


$x[n] \cdot w[n]$

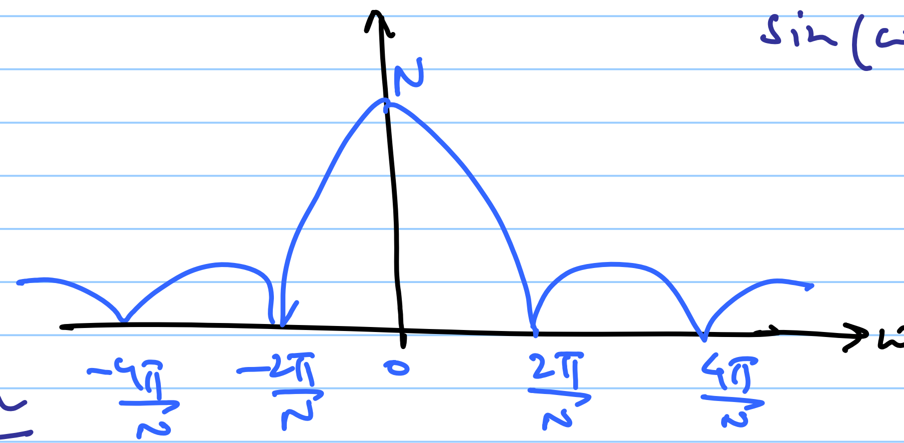
rectangular window

$$p[n] = x[n]\cdot w[n] \overset{DTFT}{\longleftrightarrow} X(e^{j\omega}) \circledast W(e^{j\omega})$$

$$W(e^{j\omega}) = e^{-j\omega\frac{(N-1)}{2}} \frac{\sin(\omega N/2)}{\sin(\omega/2)}$$



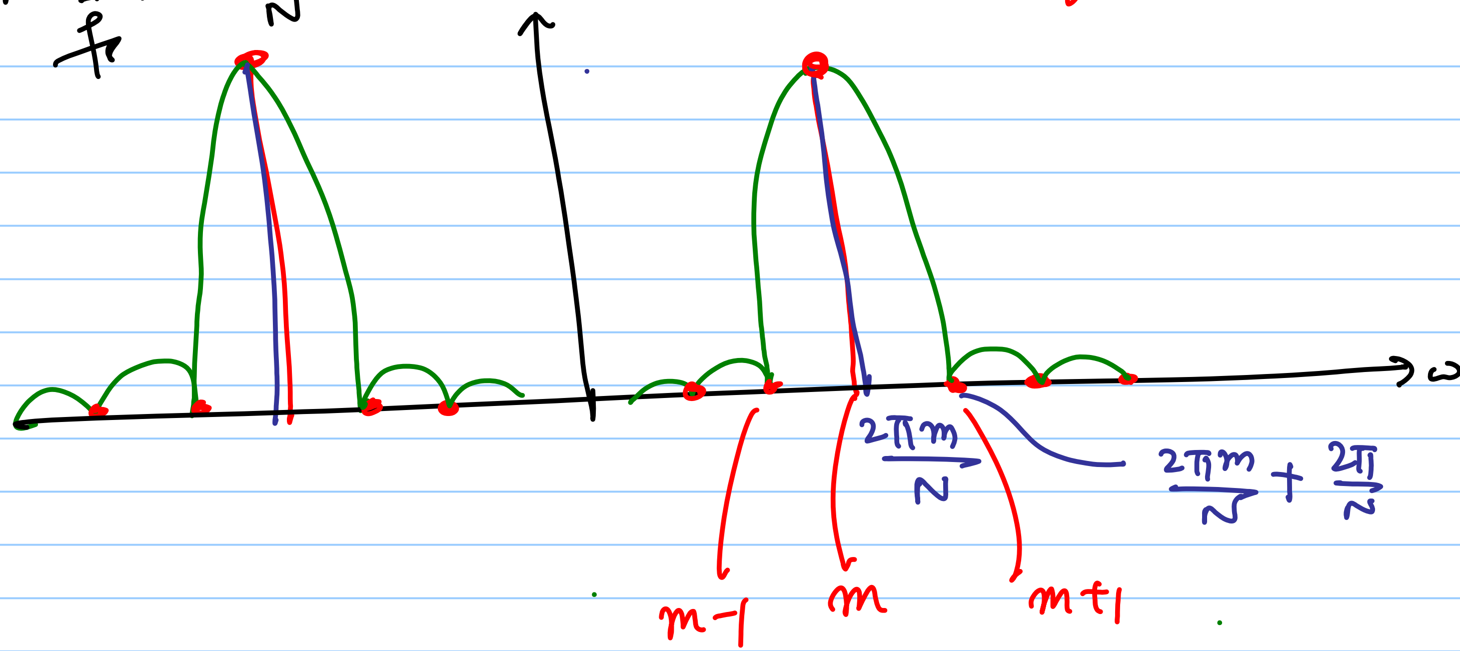
$\circledast$



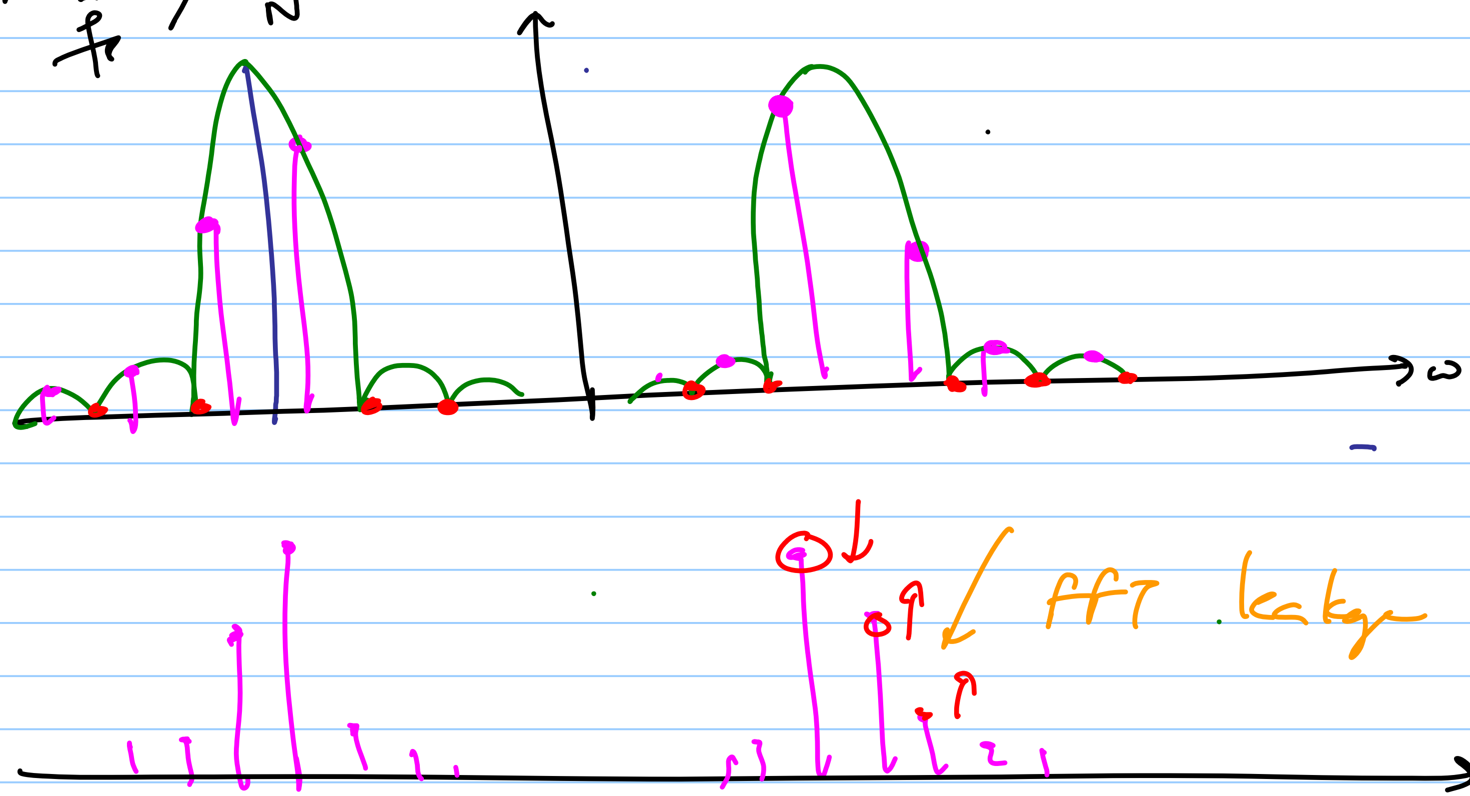
$$\left(\frac{f_m}{f_s/2}\right)\pi = \frac{2\pi m}{N}$$

for $\frac{f_m}{f_s} = \frac{m}{N}$

$P(e^{j\omega}) \Rightarrow P(k)$



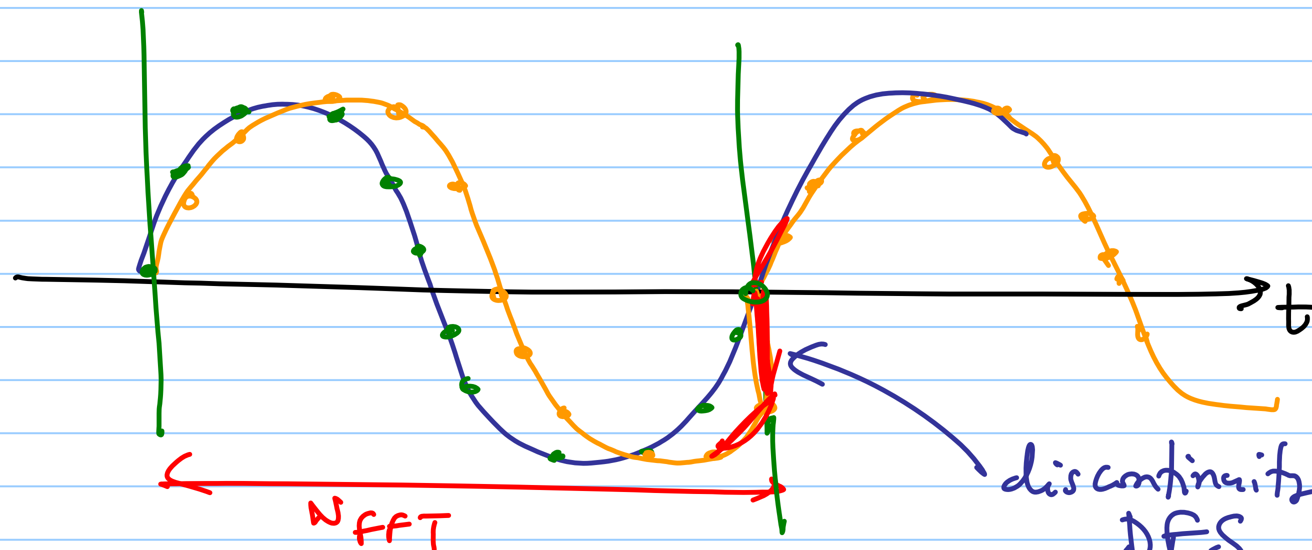
for $\frac{f_{in}}{f_s} \neq \frac{m}{N}$



Coherent sampling

or

Synchronous sampling



discontinuity when evaluating DFS $\tilde{x}[n]$

* Temporal discontinuity in $\tilde{x}[n]$ while taking DFS causes FFT leakage

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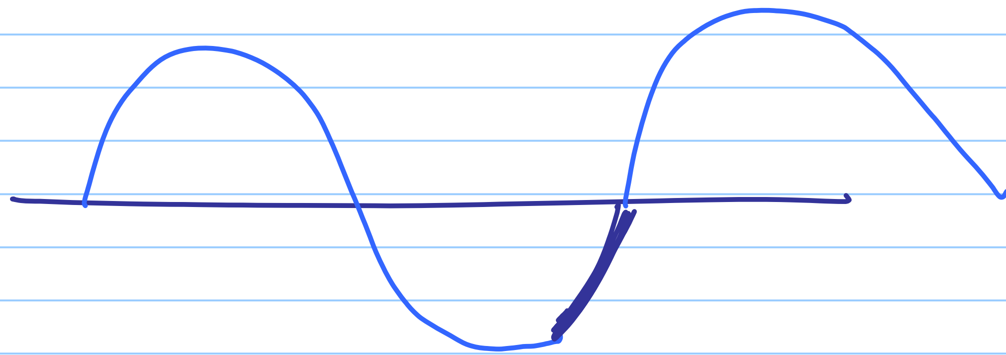
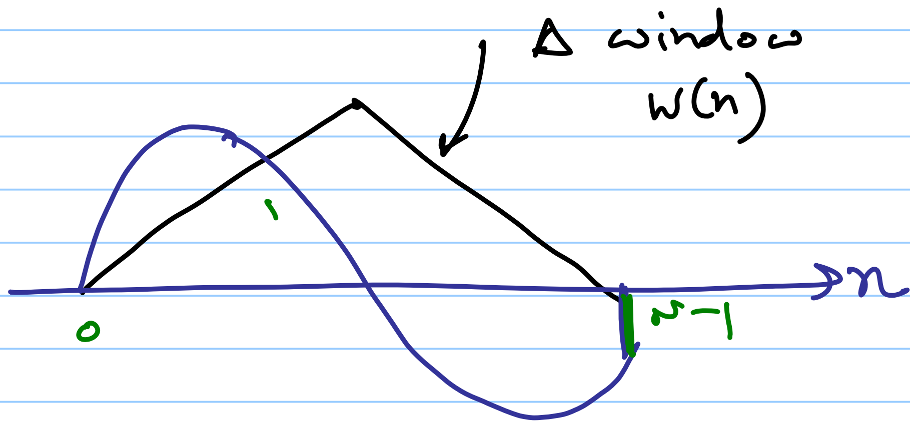
discontinuity $\Rightarrow$ step function $\Rightarrow$ high frequency components

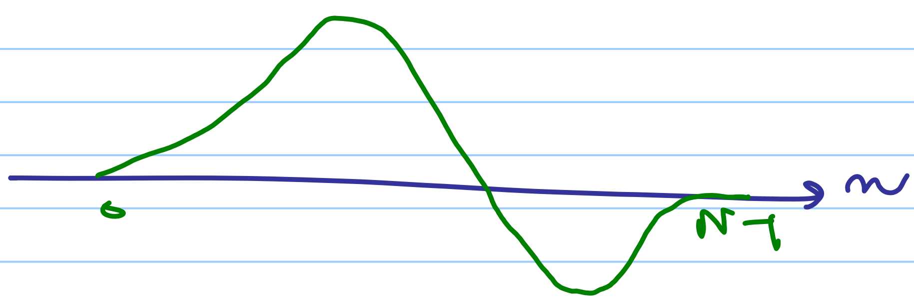
① Signal reconstruction

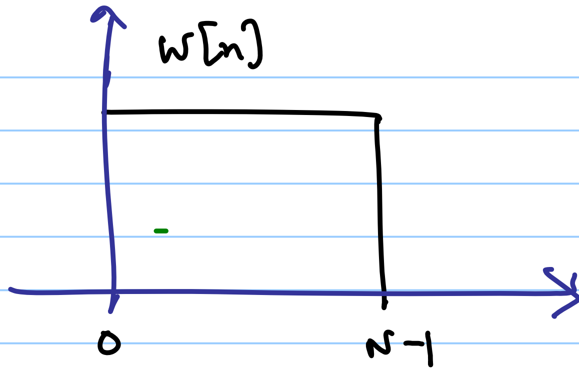


Adjust 'N' such that it has full cycles of
x[n]

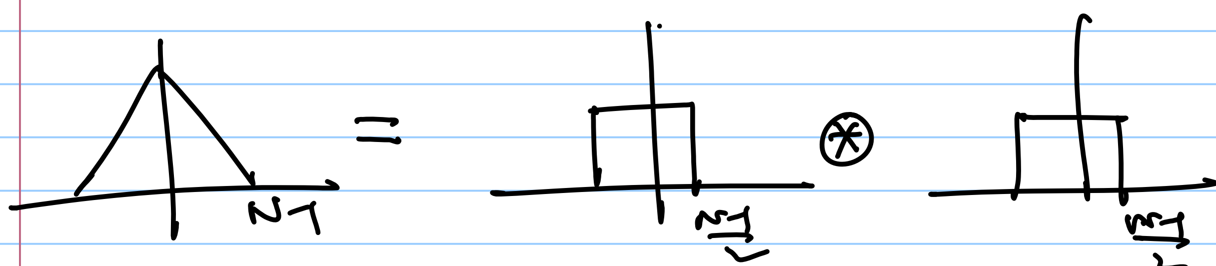
② Attenuate the discontinuity.





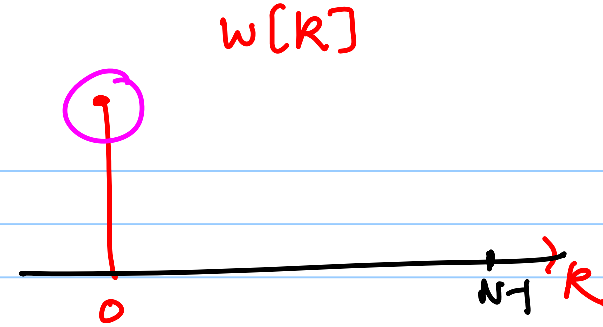


$$\propto \frac{\sin\left(\frac{N\omega}{2}\right)}{\sin\left(\omega/2\right)}$$

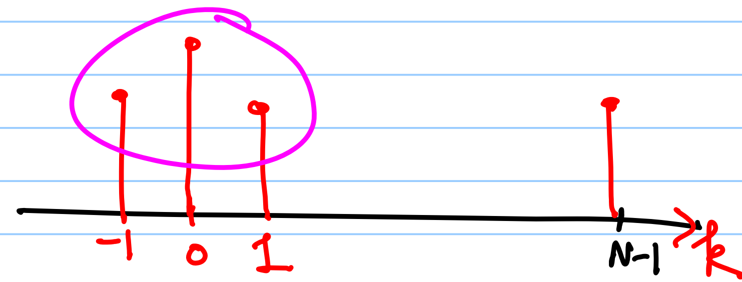


$$\propto \frac{\sin^2\left(\frac{N\omega}{2}\right)}{\sin^2\left(\omega/2\right)}$$

— rect window



triangular window



windows

(+) sidelobe suppression → less FFT leakage

(−) main lobe gets wider → signal bins > 1

Raised Cosine window

(Hann, Hanning)

$$w[n] = \frac{1}{2}\left[1 - \cos\left(\frac{2\pi n}{N}\right)\right], \quad 0 \le n \le N-1$$

$$x[n] = A \sin\left(\frac{2\pi}{1024}\cdot 129 n\right)$$

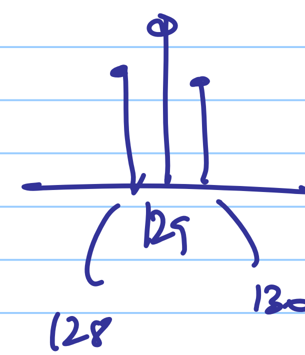
$$p[n] = x[n]\cdot w[n] = A\sin\left(\frac{2\pi}{1024}\cdot 129n\right)\cdot\frac{1}{2}\left[1-\cos\left(\frac{2\pi n}{1024}\right)\right]$$

$$= \frac{A}{2}\sin\left(\frac{2\pi}{1024}\times 129n\right) - \frac{A}{2}\sin\left(\frac{2\pi}{1024}\cdot 129n\right)\cdot\cos\left(\frac{2\pi n}{1024}\right)$$

$$= \frac{A}{2} \sin\left(\frac{2\pi}{1024} \cdot 129n\right) - \frac{A}{4} \sin\left(\frac{2\pi}{1024} \times 128n\right) - \frac{A}{4} \sin\left(\frac{2\pi}{1024} \cdot 130n\right)$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

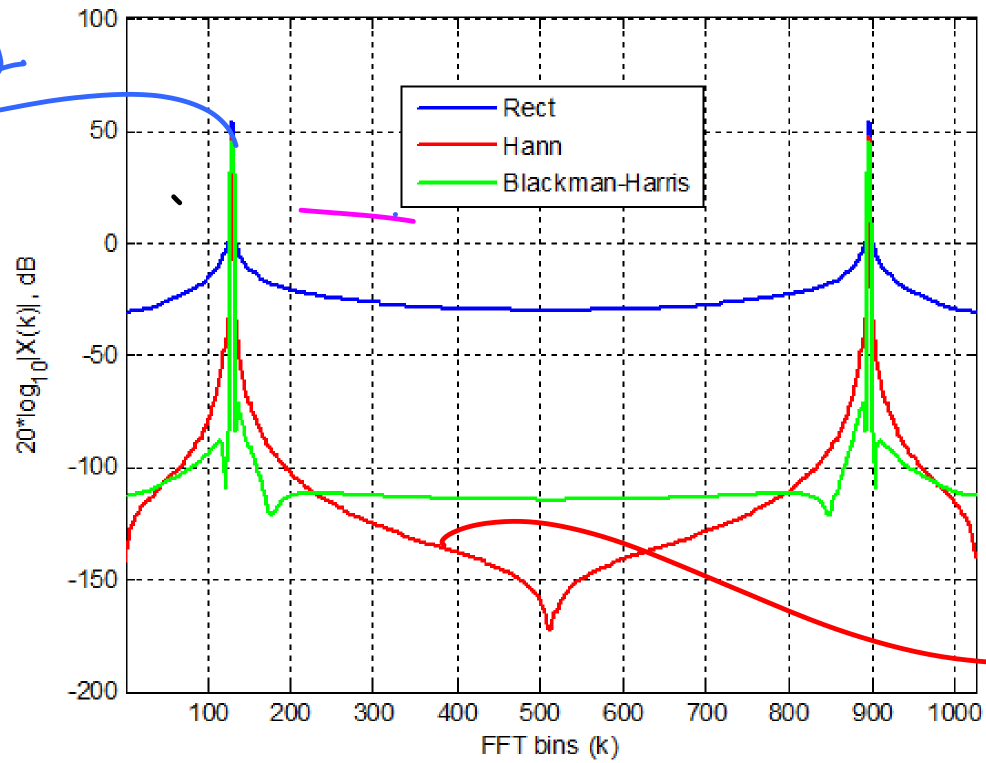
129^{th} bin $\Rightarrow$ $\underbrace{128,\ 129,\ 130}_{\text{3 bins}}$



# of bins $= n_b = 3$

$nb = 1$
@ m

$$f_{in} = \frac{129.01}{1024} f_s$$



$nb = 3$
@ $\{m-1, m, m+1\}$

for circuit simulation (Cadence Spectre)

Hann window is preferred

$\Rightarrow \quad N \leq 2^{10} \sim 2^{13}$

Experiments $\Rightarrow$ Can collect $> 1M$ samples

but hard to control $\frac{f_{in}}{f_s}$

$\hookrightarrow$ Blackmann-Harris window

$$w[n] = a_0 + a_1 \cos\left(\frac{2\pi n}{N}\right) + a_2 \cos\left(\frac{2\pi}{N} 2n\right) + a_3 \cos\left(\frac{2\pi}{N} \cdot 3n\right)$$

1 2 2 2

$$-\frac{N}{2} \le n \le \frac{N}{2}$$

$a_0 = 0.35875$

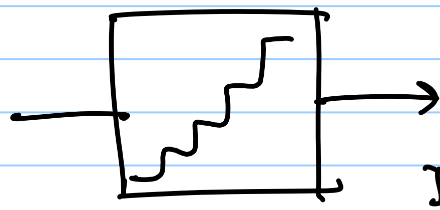
$a_1 = 0.48829$

$a_2 = 0.14128$

$a_3 = 0.01168$

$m_b = 7$ bins

Quantizer Noise Modeling

y  v

DT

Digital $\Rightarrow$ DT + quantized

Quantization Error : $e = v - y$

$\Delta =$ LSB size
↳ least significant bit