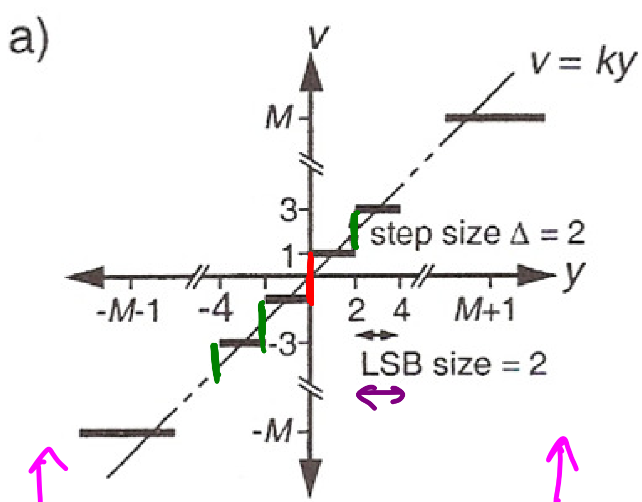
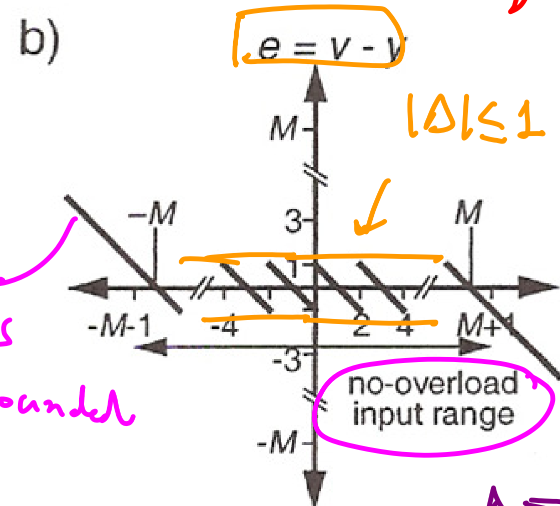
ECE 615 - Lecture 3

Note Title 1/19/2016

Even number of levels (Midrise Quantize)





$|\Delta| \le 1$ LSB ⇒ least significant bit

Δ is unbounded

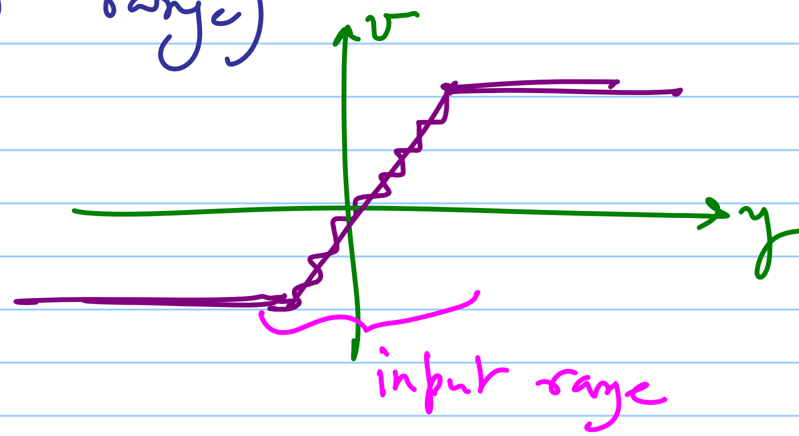
$\Delta\Sigma$ Toolbox

LSB = $\Delta = 2$

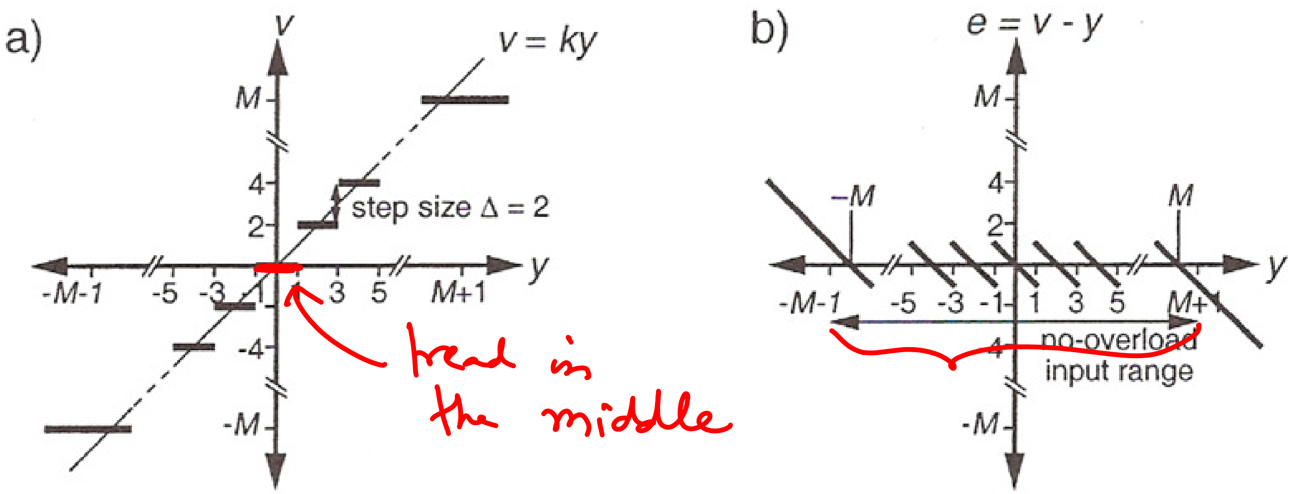
# of levels = $M+1$ ⇒ Even

M ⇒ number of steps ⇒ M = odd

no-overload input range $\Rightarrow$ $|\Delta| < 1$
(input range) bounded

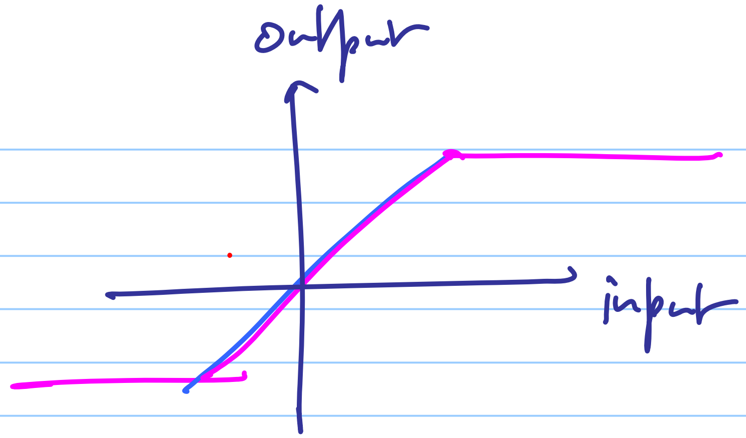


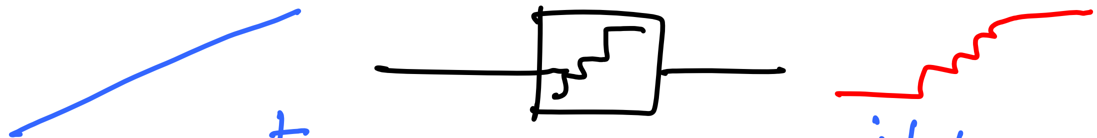
Mid-Tread Quantizer



$M \Rightarrow$ # of steps = Even

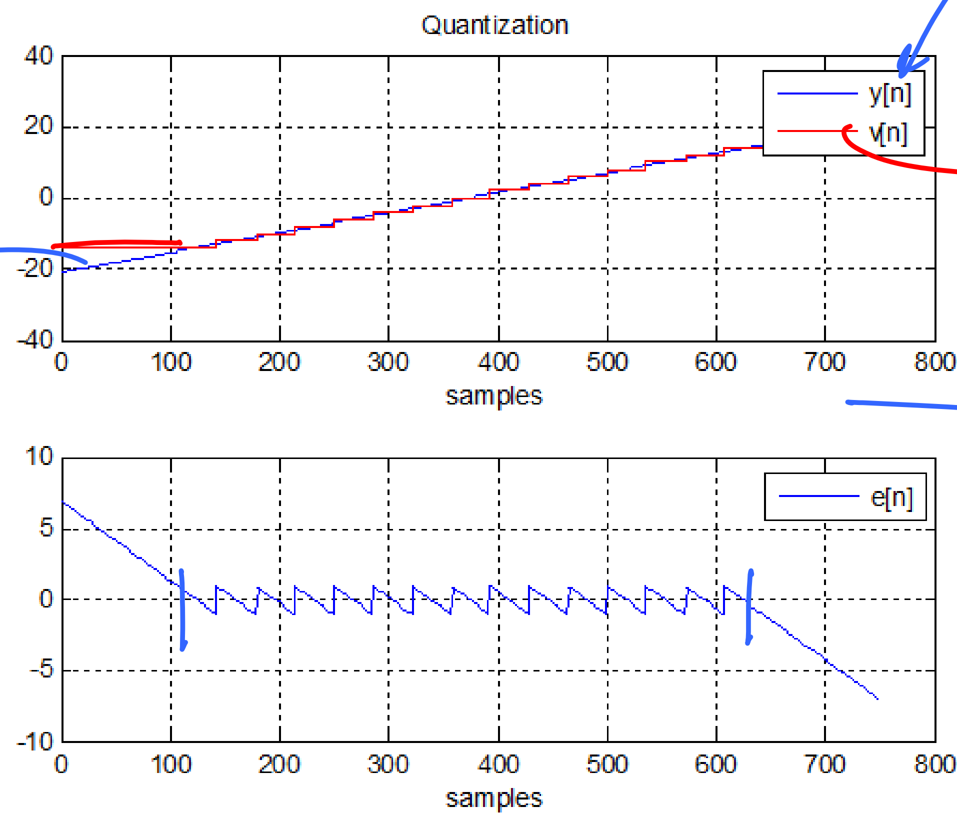
$(M+1) \Rightarrow$ # of levels is Odd


output
input



t

input



output

Slow ramp

$e[n]$

n (time)

614

"DNL"

$|DNL| \leq \frac{1}{2}$ LSB for monotonicity

$\Rightarrow$ no missing codes

Sine Input

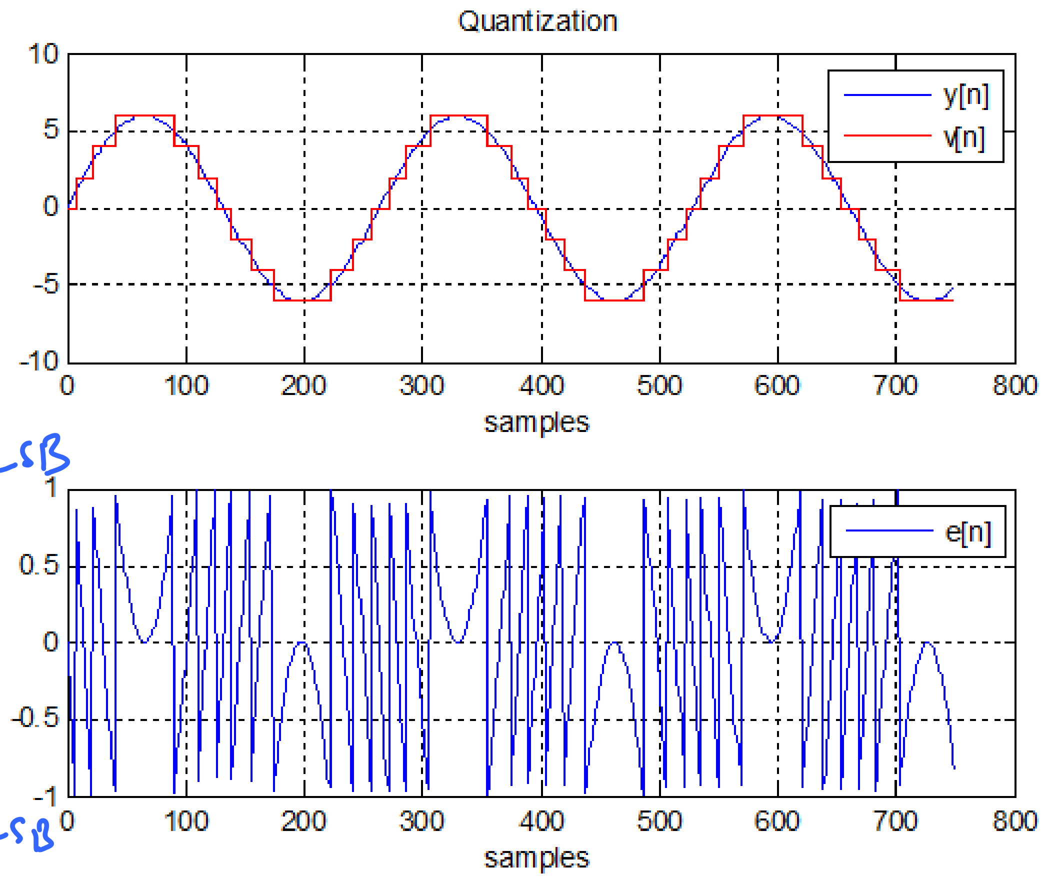
A < input range

no-overload

$+\frac{1}{2}$ LSB

e[n]

$-\frac{1}{2}$ LSB



Spectral Estimation Basics

C.T. $x(t) \xrightarrow{nT_s} x(nT_s) \sim x[n]$ Discrete Time

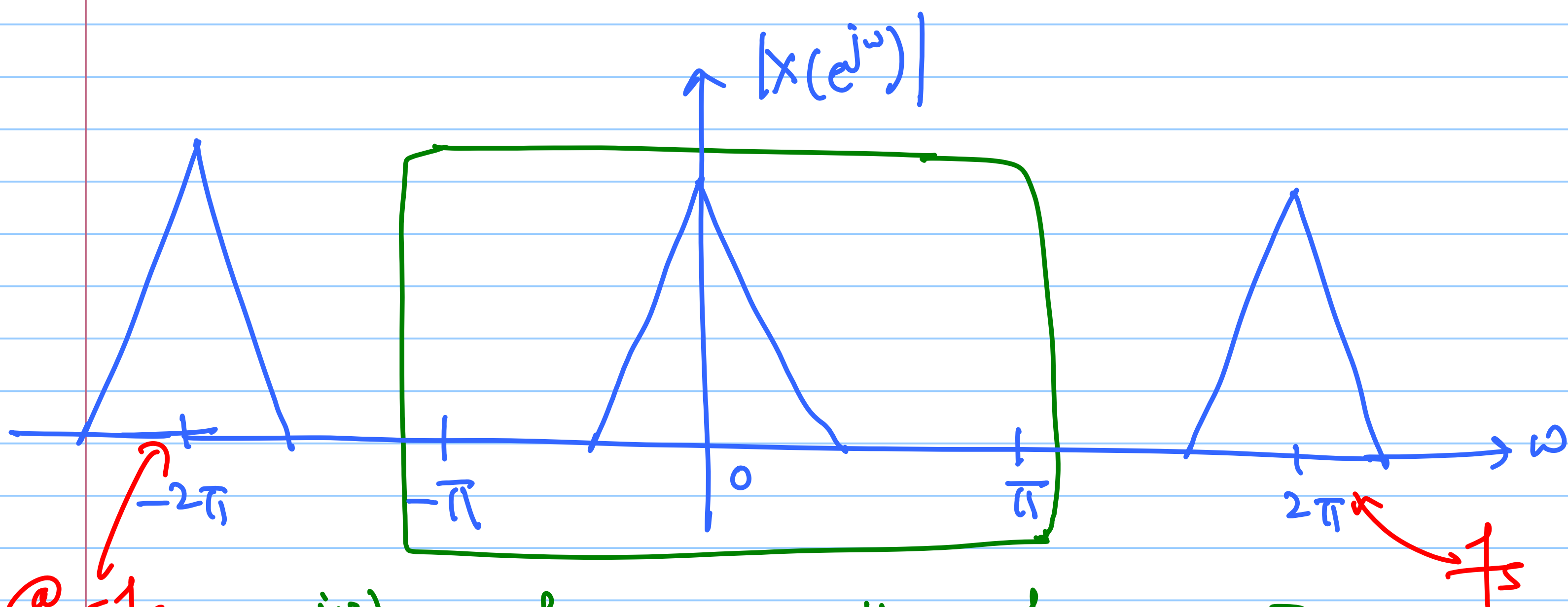
time \ frequency	Continuous	Discrete
Continuous	Fourier Transform $X(f)=\int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$	X
Discrete	DTFT $X(e^{j\omega})=\sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$	DFT → FFT $X(k)=\sum_{n=0}^{N-1} x[n]\, W_N^{nk}$, $W_N = e^{-j\frac{2\pi}{N}}$

DTFT (Discrete Time Fourier Transform)

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n},$$

$x[n]$ is absolutely summable

$$\sum |x(n)| < \infty$$

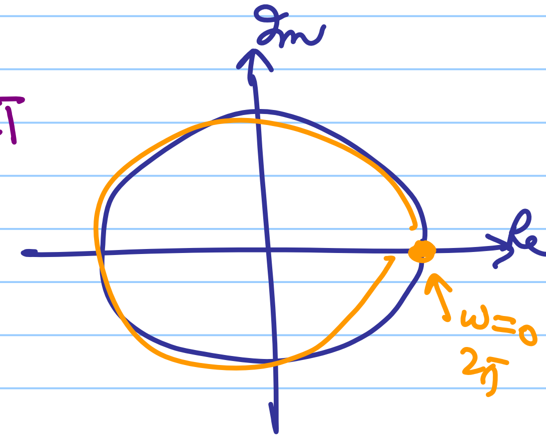


@ $-f_s$ $X(e^{j\omega})$ is periodic with period 2π

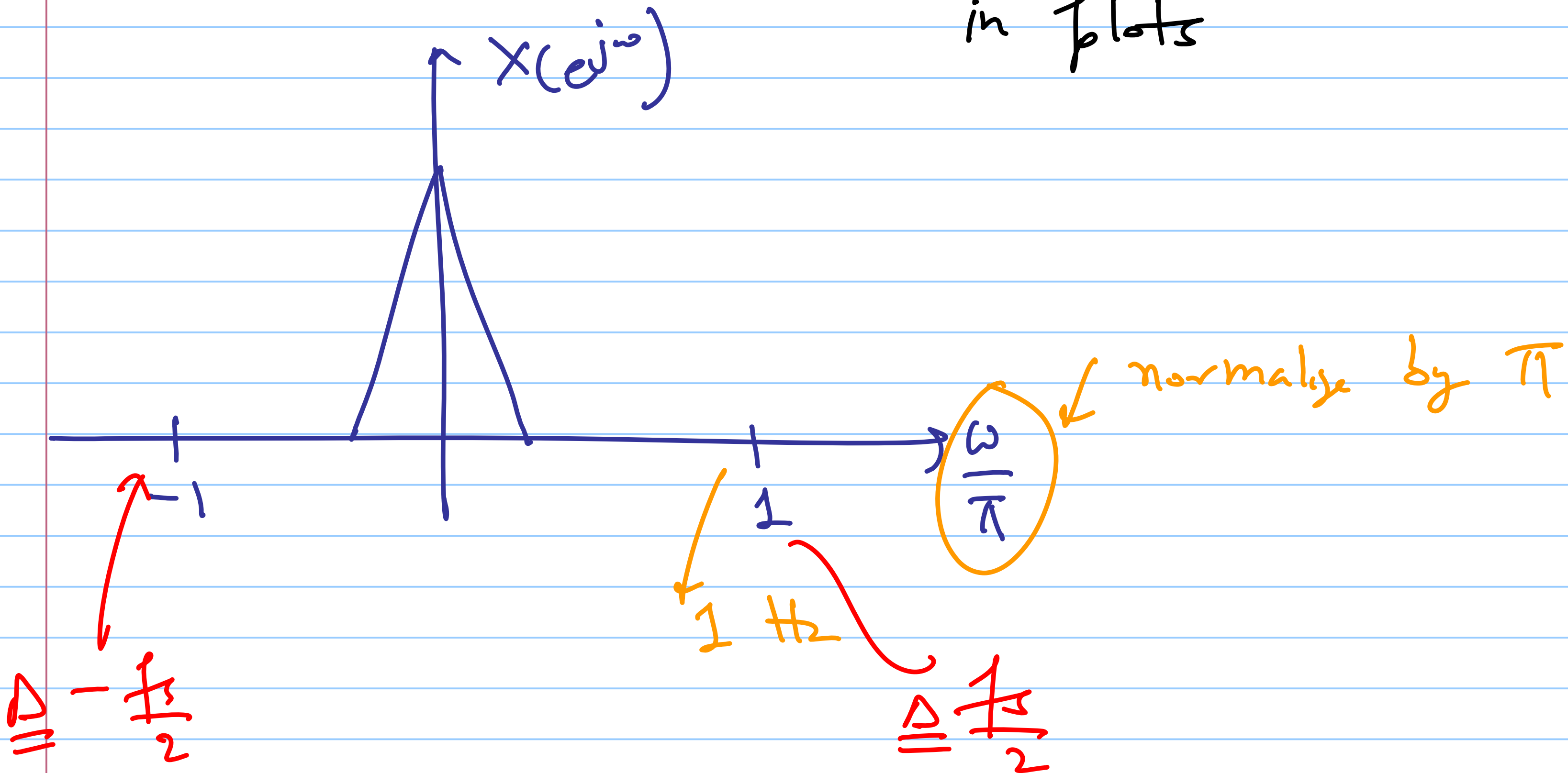
2π corresponds to $\omega_s = 2\pi f_s$

↳ similar to the sampled spectrum

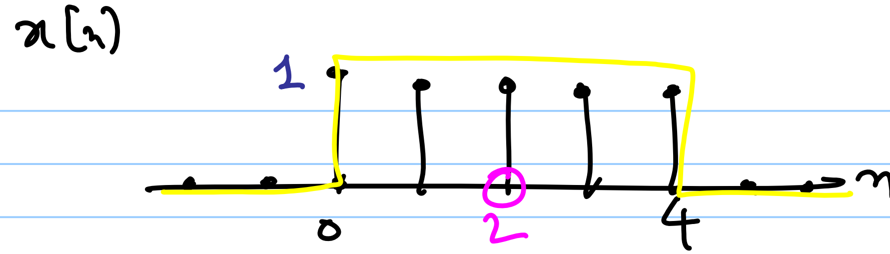
$X(e^{j\omega})$ $e^{j\omega}$ is periodic with period 2π



* Sometimes ω_s is normalized to 1Hz in plots



Example :



$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} = \sum_{n=0}^{4} e^{-j\omega n}$$

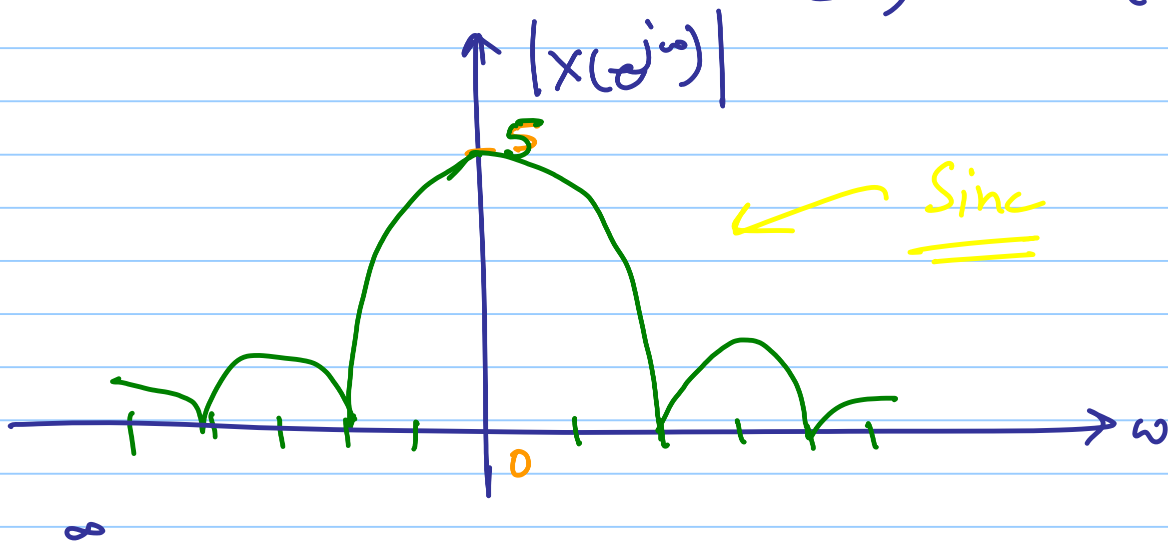
$$\left(\sum_{n=0}^{N} r^{n} = \frac{1-r^{N+1}}{1-r} \right)$$

$$= \frac{e^{-j\frac{5}{2}\omega}\left(e^{j\frac{5}{2}\omega} - e^{-j\frac{5}{2}\omega}\right)}{e^{-j\omega/2}\left(e^{j\omega/2} - e^{-j\omega/2}\right)}$$

$$e^{j\theta} - e^{-j\theta} = 2j\sin\theta$$

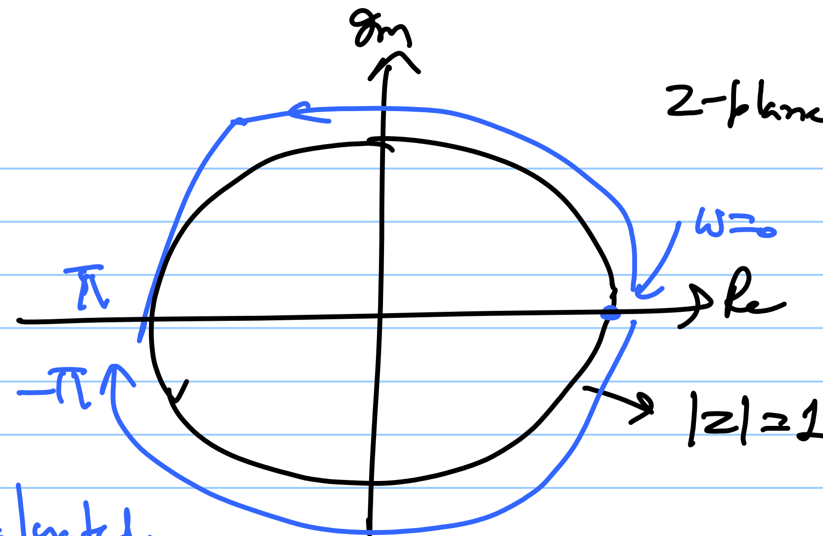
$$= e^{-j2\omega} \times \frac{\cancel{2j}\sin(5/2\,\omega)}{\cancel{2j}\sin(\omega/2)} = e^{-j2\omega} \times \frac{\sin(5\omega/2)}{\sin(\omega/2)}$$

$$= e^{-j2\omega} \times 5 \times \frac{\sin\left(\frac{5\omega}{2}\right)}{\left(\frac{5\omega}{2}\right)} \times \frac{\left(\omega/2\right)}{\sin\left(\omega/2\right)} \quad - - - -$$



Z-Transform

$$X(z) = \sum_{n=-\infty} x(n)\, z^{-n}$$



DTFT is z-transform evaluated along the unit circle $|z|=1$.

$|\cancel{X(z)}|$ $\quad \angle \cancel{X(z)}$

$X(e^{j\omega})$

$|X(s)| \to |X(j\omega)|$

DTFT is continuous in frequency $\qquad X_1(e^{j\omega}) \cdot X_2(e^{j\omega})$

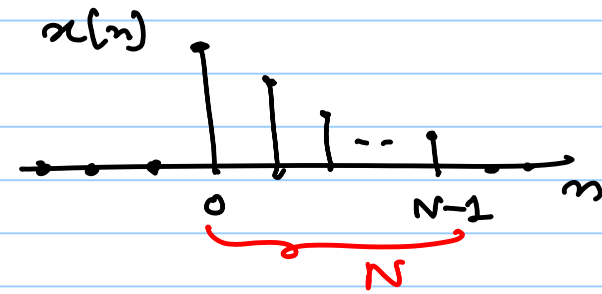
↳ not good for computation using digital computers

↳ Need some transform which is also discretized in the frequency-axis

↳ DT fourier Series ⇒ discrete in frequency

Consider a finite length sequence $x[n]$ of length N.

$\Rightarrow$ $x[n] = 0$ outside $0 \le n \le N-1$

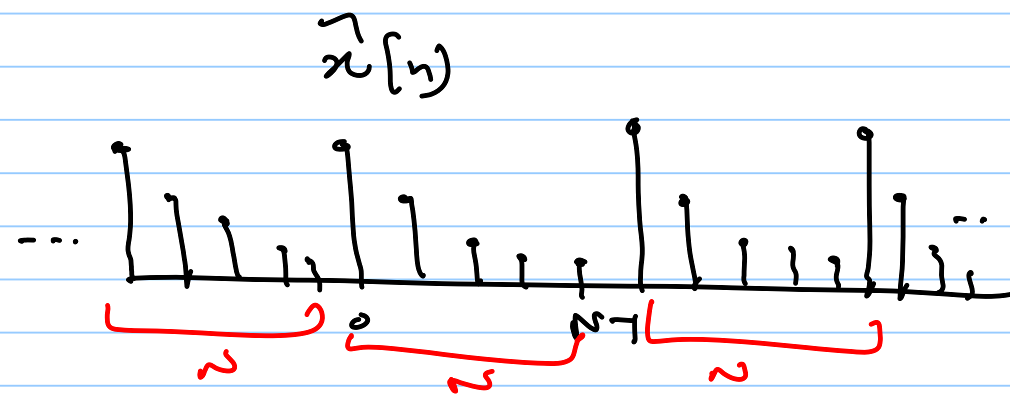


Create a periodic sequence

$$\tilde{x}[n] = \sum_{r=-\infty}^{\infty} x[n - rN]$$

$$= x[n \text{ modulo } N]$$

$$= x[(n)_N]$$



recover $x[n]$

$$x[n] = \begin{cases} \tilde{x}[n], & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$$

$\tilde{x}[n]$ is periodic with period N

↳ can be represented as a summation of complex exponentials with a frequency equal to $\left(\frac{2\pi}{N}\right)$

↳ Discrete-Time Signal Processing → Oppenheim & Schafer

* periodic complex exponentials

$$e_k[n] = e^{j\left(\frac{2\pi}{N}\right)kn}$$

$$= e_k[n + rN]$$

$\frac{2\pi}{N}$ is the fundamental frequency

$$\triangleq e^{j\frac{2\pi}{T}kt}$$

Fourier series

$f_0 = 1/T$

$\omega = \frac{2\pi}{T}$

$$\tilde{x}[n] = \frac{1}{N}\sum_{k} \underbrace{\tilde{X}[k]}_{a_k} \cdot \underbrace{e^{j\left(\frac{2\pi}{N}\right)kn}}$$

$\Leftarrow$ Discrete Fourier Series Representation (DFS)

a_k coefficients of DFS

tones or complex exponentials

Test: $e_{k+lN}[n] = e^{j\frac{2\pi}{N}(k+lN)}$

$= e^{j\frac{2\pi k}{N}} \times e^{j2\pi l}$

$= e^{j\frac{2\pi k}{N}} \times \left(e^{j2\pi}\right)^{l}$

$= e_k[n] \Leftarrow$ periodic with N

Thus, we need only $e_0[n]$ to $e_{N-1}[n]$ to represent $\tilde{x}[n] \Rightarrow k = 0$ to $N-1$

$$\Rightarrow \quad \tilde{x}[n] = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}(k)\, e^{j\frac{2\pi}{N}kn} \longrightarrow \text{DFS}$$

$\uparrow$ DFS coefficient

where $$\vec{X}(k) = \sum_{n=0}^{N-1} \vec{x}[n]\, e^{-j\left(\frac{2\pi}{N}\right)kn}$$

$k = 0$ to $N-1$

only N frequency components

$\vec{X}(k)$ is periodic with period N, because $x[n]$ & $e^{-j\frac{2\pi}{N}kn}$ are periodic with period N.