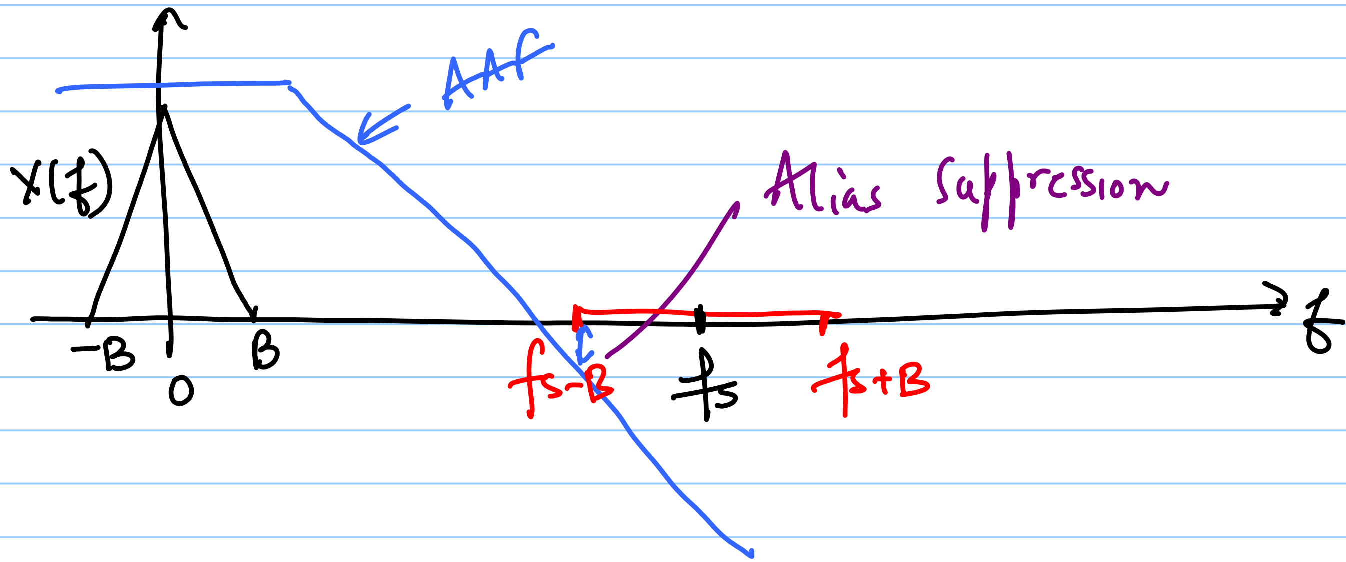
ECE 615 - Lecture 2

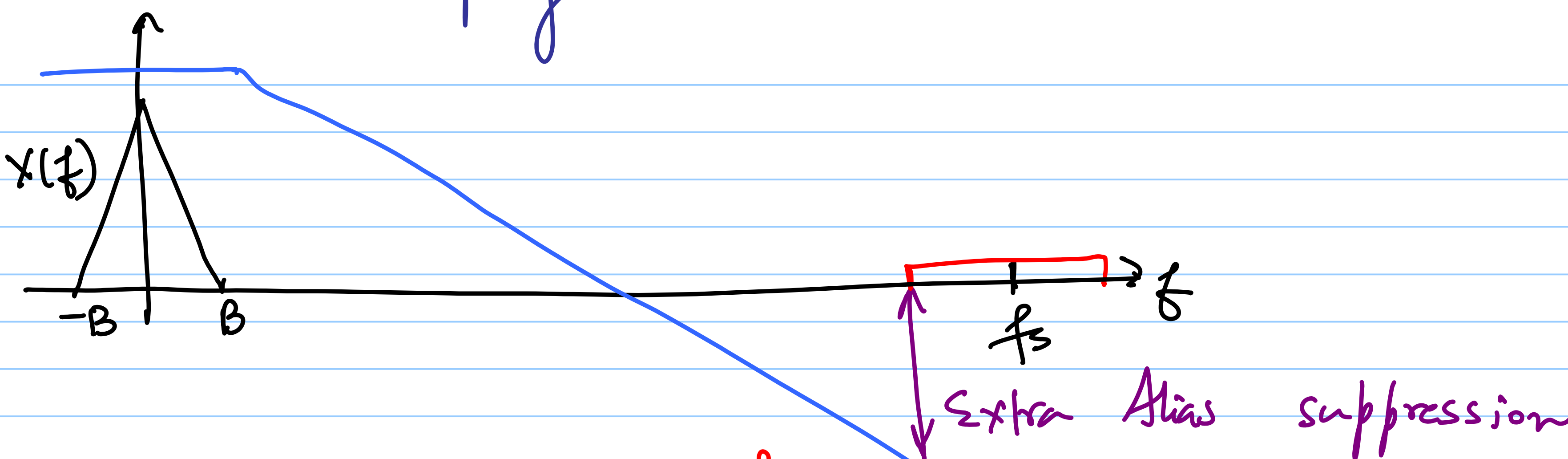
Note Title 1/14/2016



20dB × n

Class time
6:00 - 7:15 PM

Oversampling



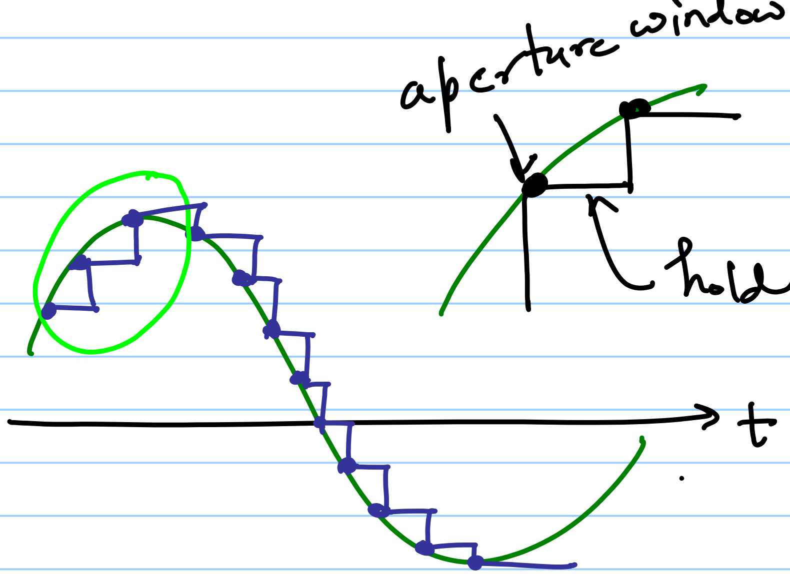
$$\text{Oversampling Ratio (OSR)} = \frac{f_s}{f_{s,\,Nyquist}} = \boxed{\frac{f_s}{2B} \triangleq OSR}$$

Oversampling results in

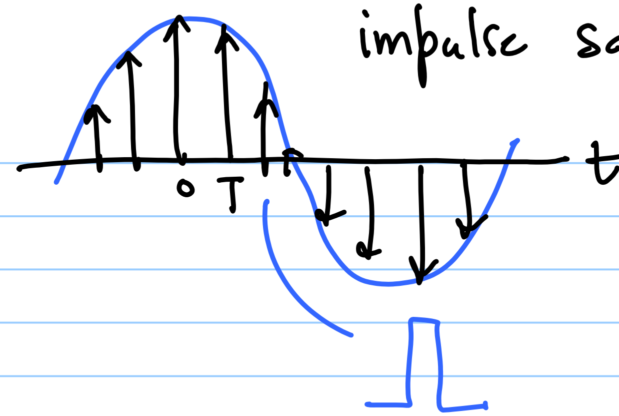
↳ better alias rejection for the same AAF

↳ lower order AAF for the same amount of alias suppression

Sample and Hold:

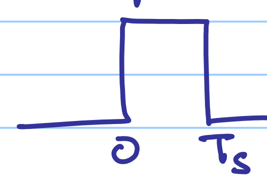


impulse sampling



Also called zero-order hold (ZOH)

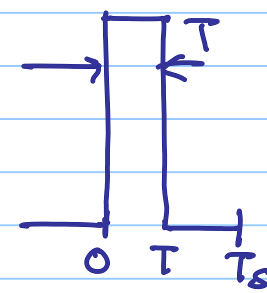
pulse-shape ⇒ NRZ (non-return to zero)



In an ideal S/H → aperture window is sufficiently small w.r.t T_s.

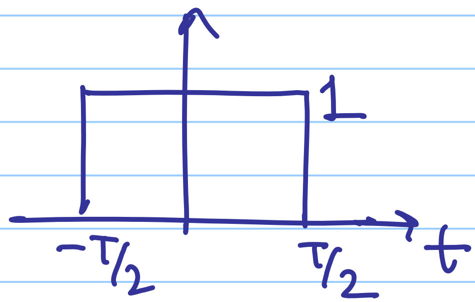
Generalized S/H

return-to-zero (RZ) pulse

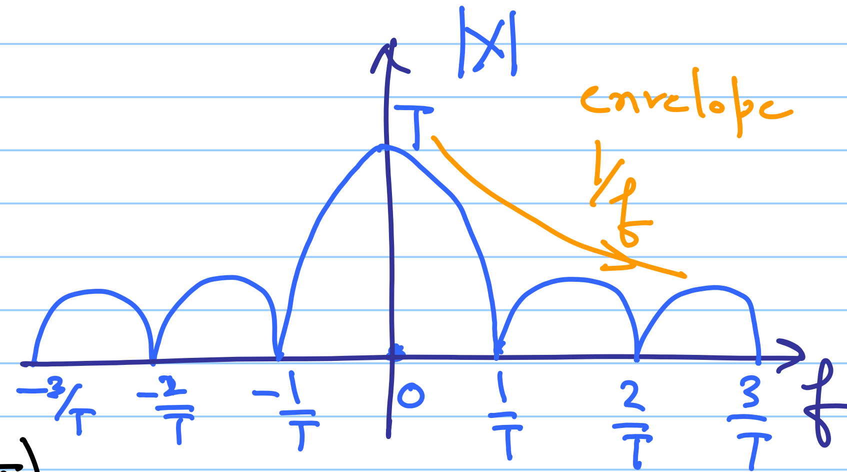


$0 \leq T \leq T_s$

* Recap on Signals



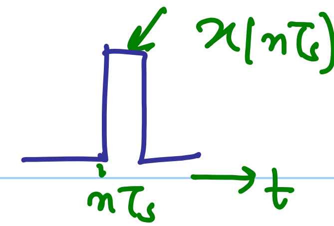
$$\xleftrightarrow{\mathcal{F}}$$



$$\mathrm{rect}\left(\frac{t}{T}\right) \xleftrightarrow{\mathcal{F}} T\,\mathrm{sinc}(fT)$$

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

For a generic S/H, the output



$$y(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \mathrm{rect}\left(\frac{t - T/2 - nT_s}{T}\right)$$

$$= \sum_{n=-\infty}^{\infty} \left[x(t) \cdot \delta(t - nT_s)\right] \circledast \mathrm{rect}\left(\frac{t - T/2}{T}\right)$$

$$= \Big[x(t) \cdot \underbrace{\sum_{n=-\infty}^{\infty} \delta(t - nT_s)}_{p(t),\ \text{impulse train}}\Big] \circledast \underbrace{\mathrm{rect}\left(\frac{t - T/2}{T}\right)}_{h(t)}$$

$h(t) \leftarrow$ RZ pulse shape

$$= \left[x(t) \cdot p(t)\right] \circledast h(t)$$

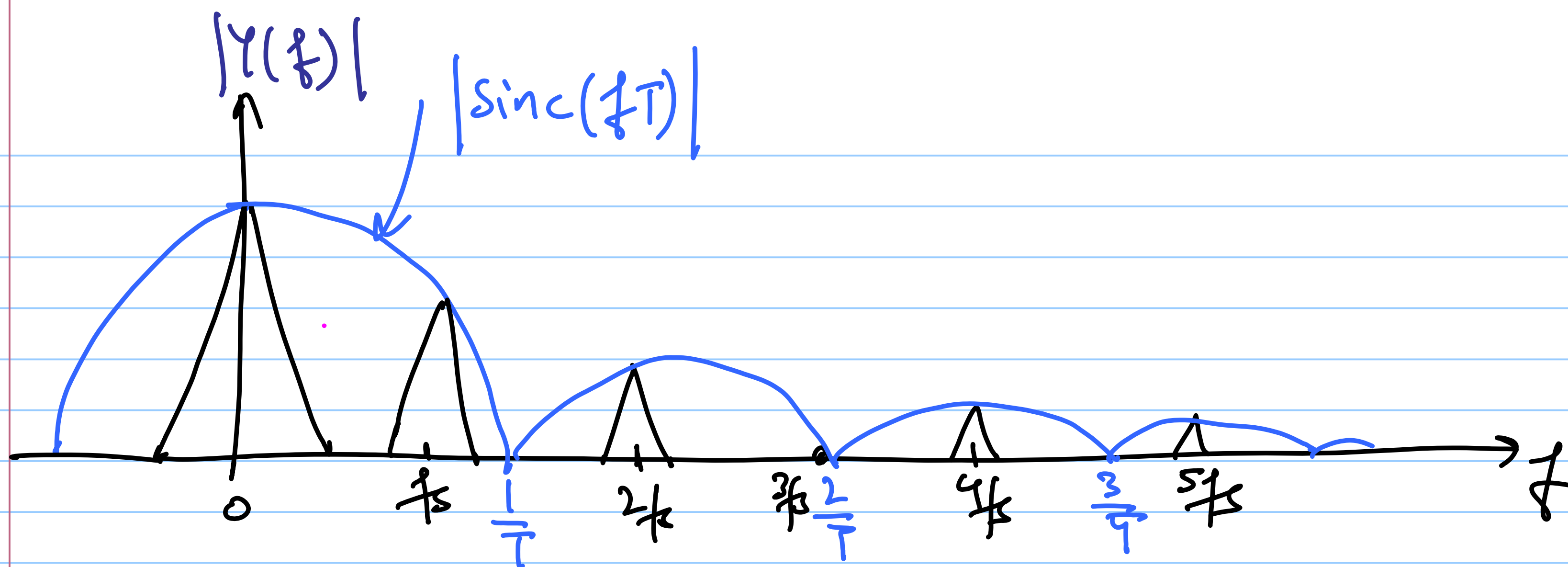
$$Y(f) = \underbrace{\left[X(f) \circledast P(f)\right]} \cdot H(f)$$

$$h(t) = \text{rect}\left(\frac{t - T/2}{T}\right)$$

$$H(f) = T\,\text{sinc}(fT) \cdot e^{-j\pi f T}$$

$$|H(f)| = |T\,\text{sinc}(fT)|$$

$$Y(f) = \underbrace{\left(\frac{T}{T_s} \sum_{k=-\infty}^{\infty} X(f - k f_s)\right)}_{\text{replicas with ideal sampling}} \cdot \underbrace{\text{sinc}(fT)}_{\text{Sinc distortion!}} \cdot e^{-j\pi f T}$$



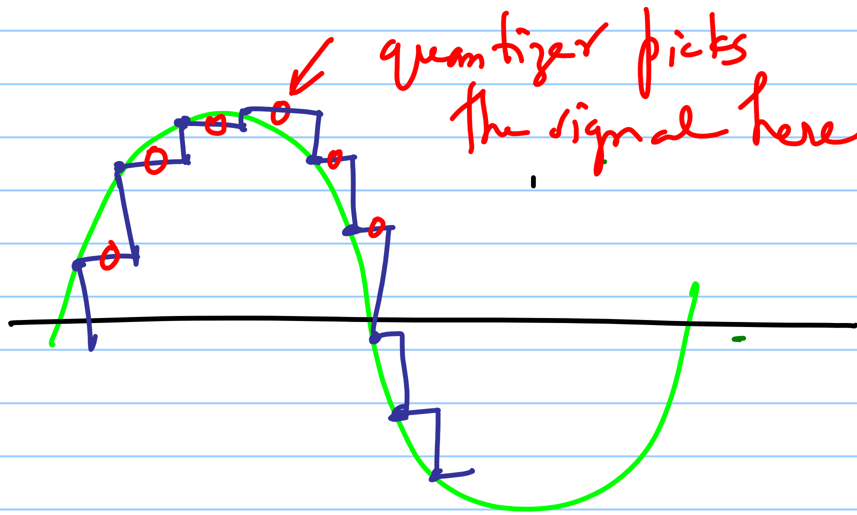
The replicas are weighted by the sinc response.

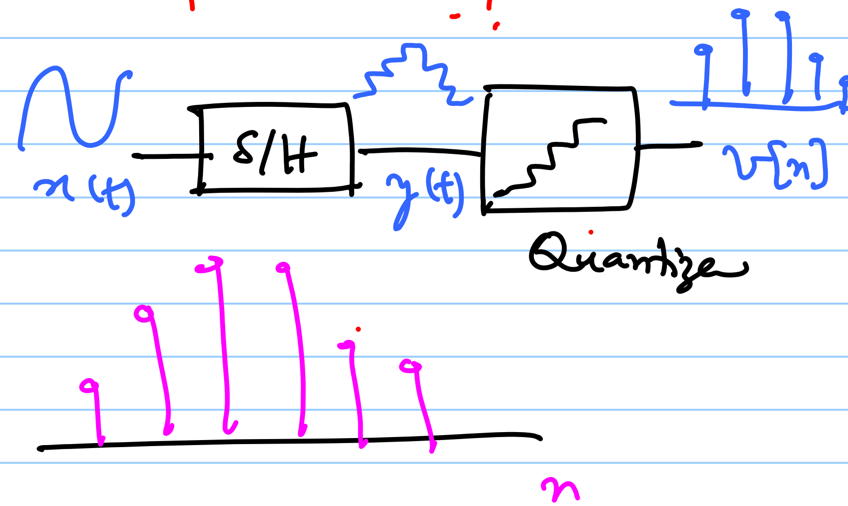
$T = T_s \Rightarrow$ ZoH $\rightarrow$ worst sinc distortion

for $\frac{T}{T_s} \rightarrow 0 \Rightarrow$ sinc distortion vanishes

↳ signal power of the S/H output diminishes

* Is the S/H sinc distortion a problem in an ADC with a S/H in the front-end??



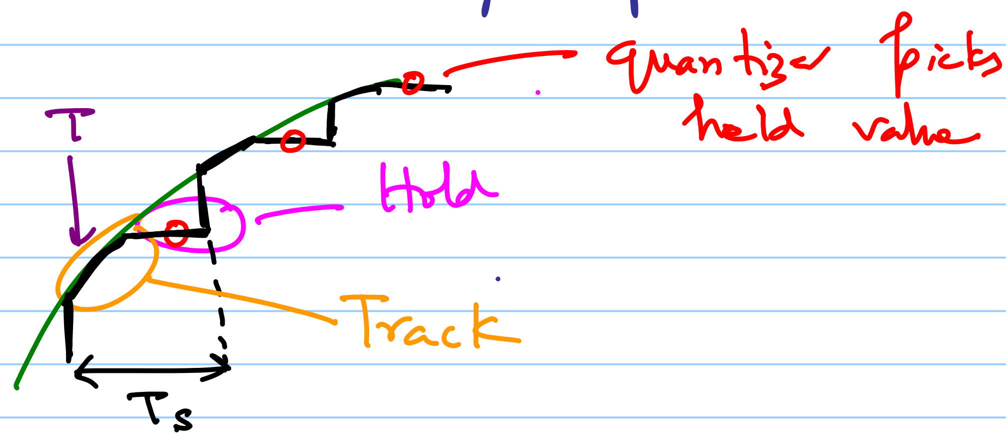


* The quantized value only corresponds to the sampled points on the input

S/H distortion is not an issue in an ADC.

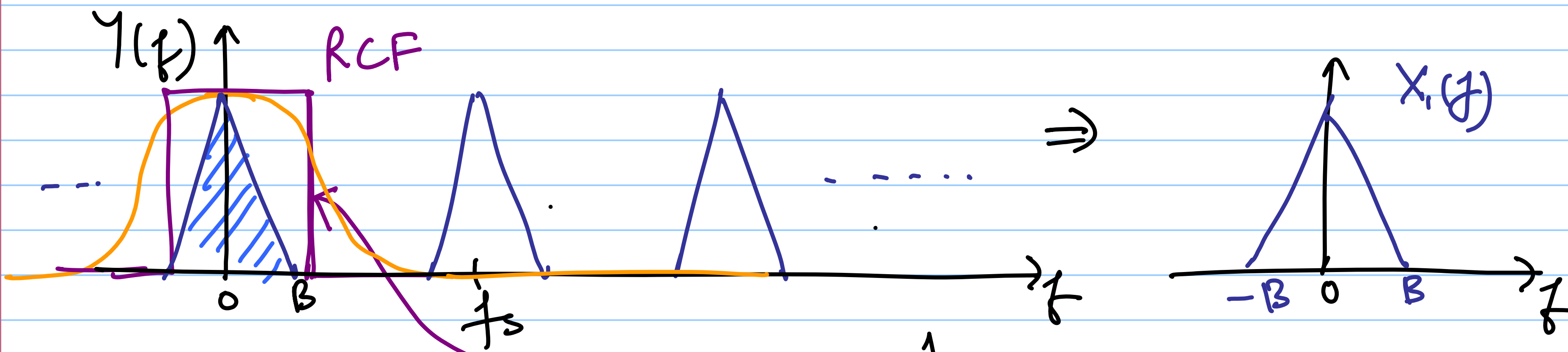
Track and Hold (T/H)

* At high speeds (100MHz → 10's GHz) the aperture time increases wrt the sample period



Reconstruction

Sampled signal

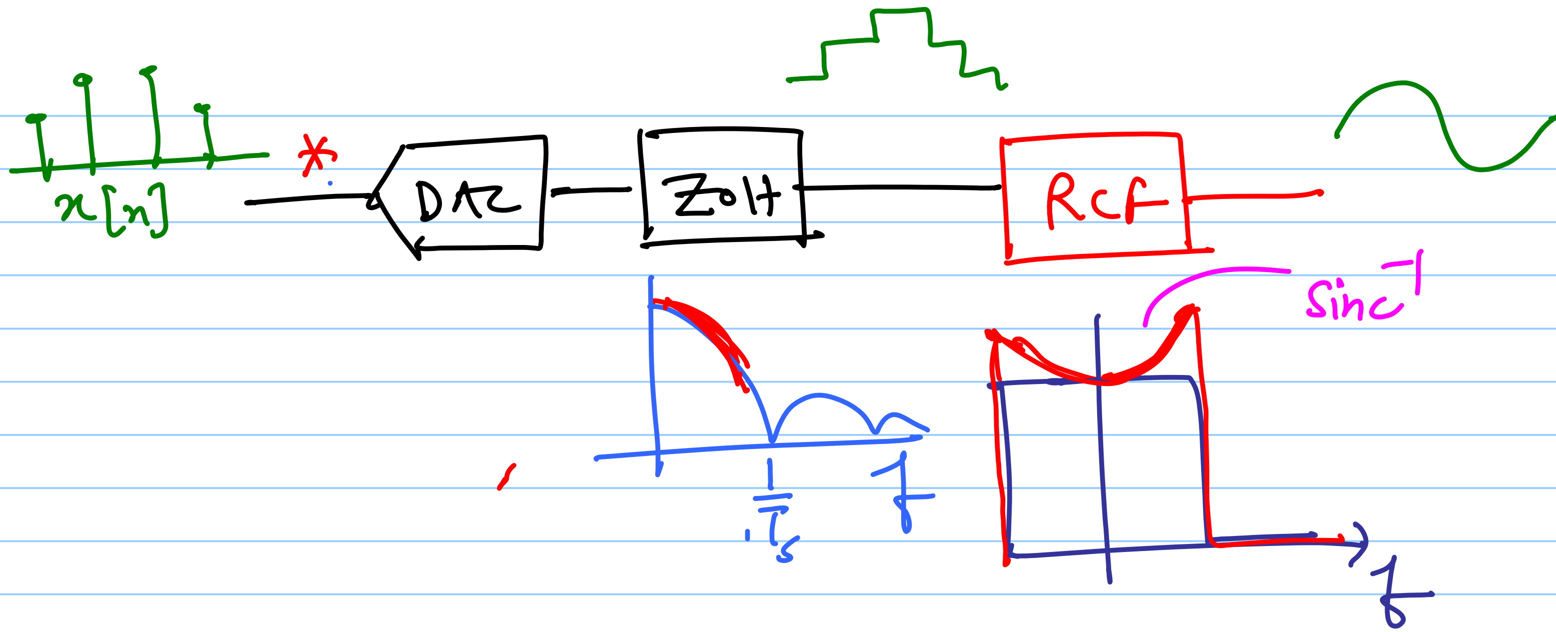


$$Y(f) \cdot \operatorname{rect}\left(\frac{f}{2B}\right) \xleftrightarrow{\mathcal{F}^{-1}} \left(y(t) \circledast \sin(t\pi 2B) \cdot 2B\right)$$

Reconstruction

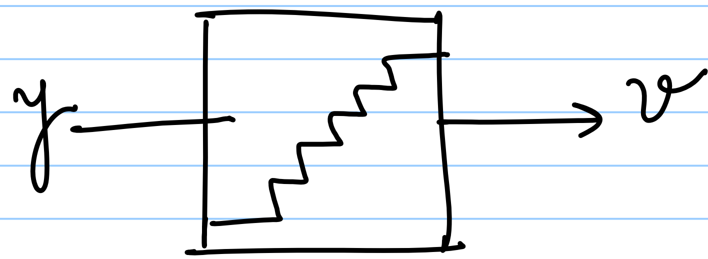
$\Rightarrow$ time-domain look like a sinc interpolation

Again, the requirements on the RCF are relaxed by oversampling.



sinc⁻¹ response shape is used in the RCF to compensate for the sinc distortion in the ZOH.

Quantizer



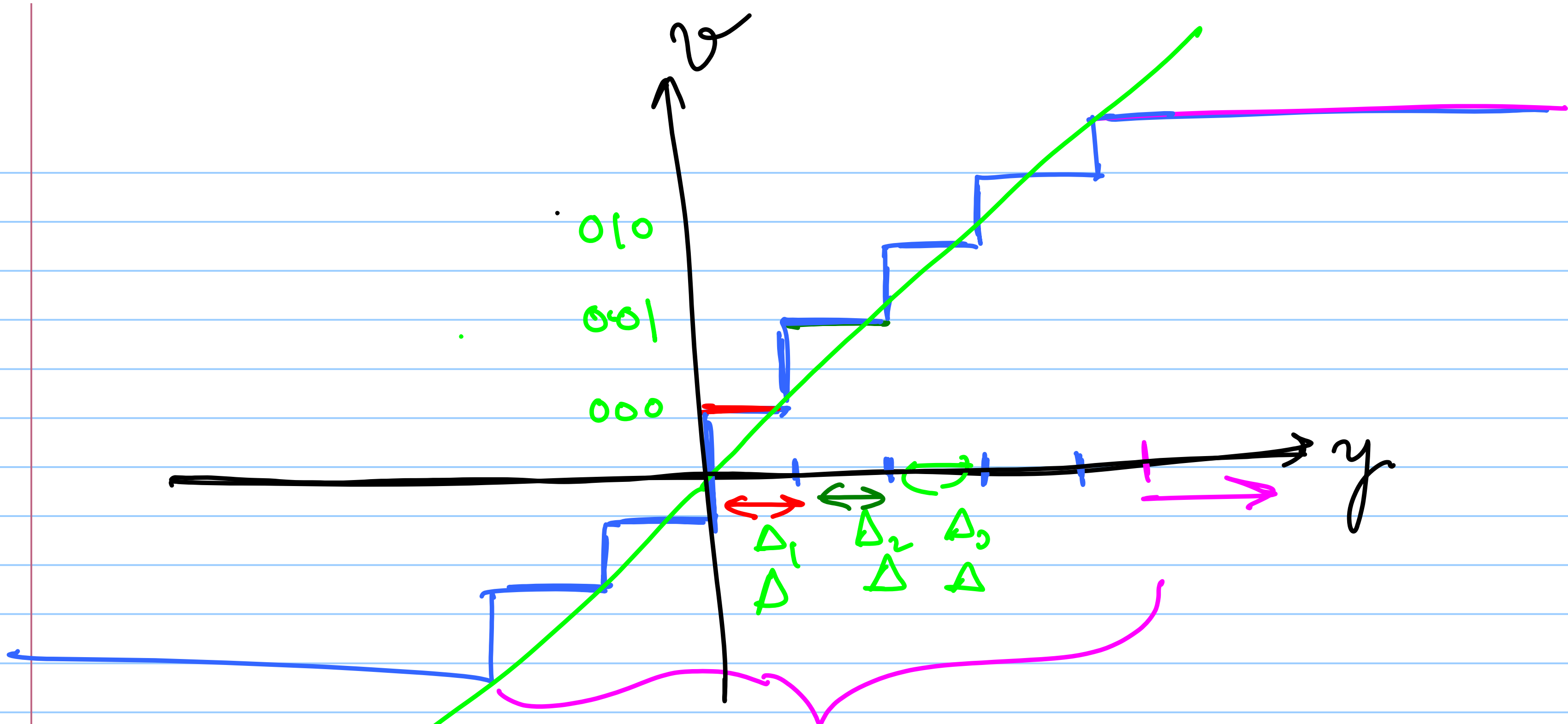
$$v = Q(y)$$

Symbology

$$Y \xrightarrow{Q} V$$

The amplitude of the DT signal 'y' is quantized so that it assumes one of the finite number of allowable values (or levels)

↳ These levels are represented by binary code.



Uniform quantizer : fixed-level spacing (Δ)

The quantizer is a memoryless, non-linear device defined by its input-output characteristics

Unipolar Quantize → only +ve values

bipolar " → +ve as well as −ve input values

Quantize error: $\boxed{e = v - y}$

DSP: Discrete-time Signal Processing

→ Oppenheim & Schafer