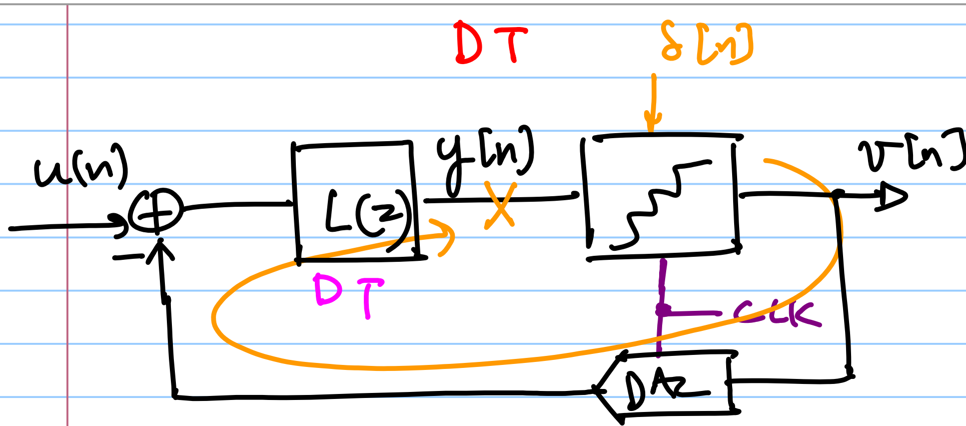
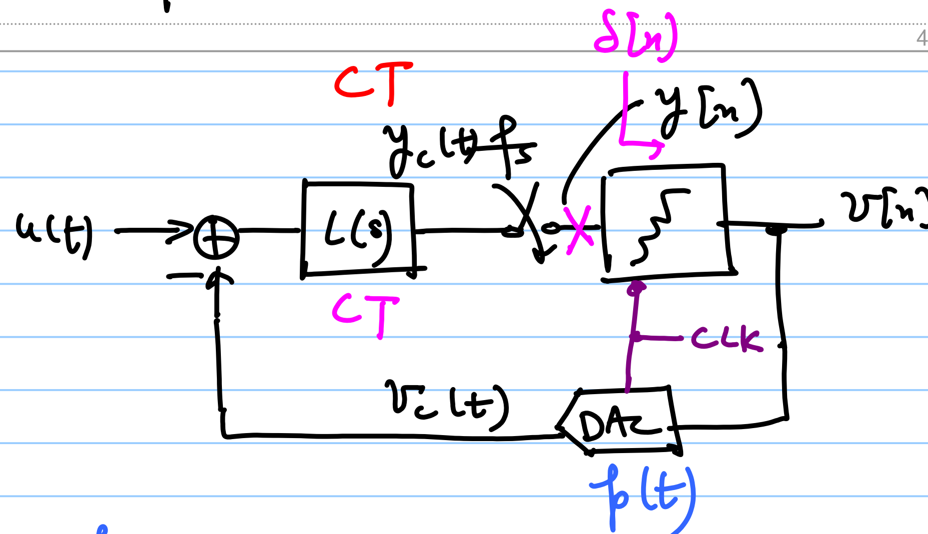
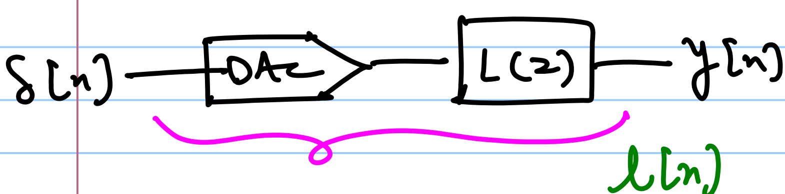
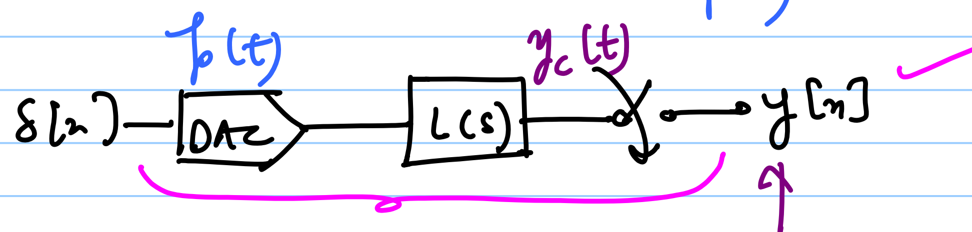
ECE 615 - Lecture 24

Note Title 4/7/2016









Impulse invariant Transformation (IIT)

$$y[n] \triangleq y_c(t)\Big|_{t=nT_s}$$

if $c[n] = \delta[n]$

$$l[n] \triangleq y(t)\Big|_{t=nT_s,\ v[n]=\delta[n]}$$

I. I. T.

$$\Rightarrow \quad \mathcal{Z}^{-1}\{L(z)\} = \left\{\mathcal{L}^{-1}\left(L(s) \cdot R_{DAC}(s)\right)\right\}\Big|_{t=nT_s}$$

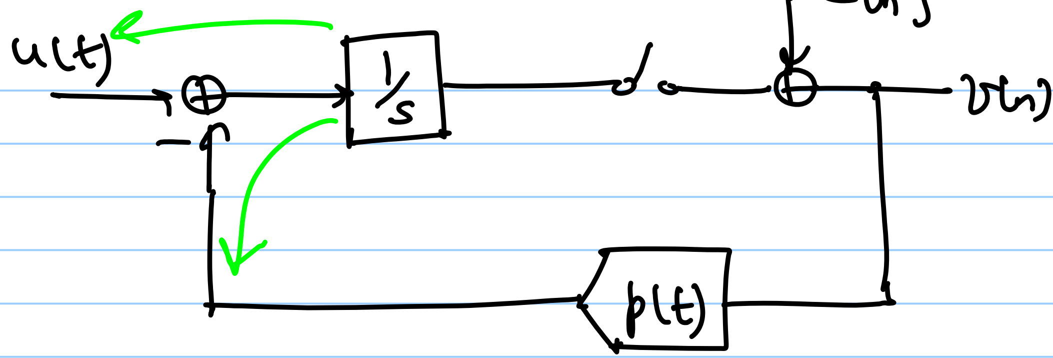
DT prototype

loop filter

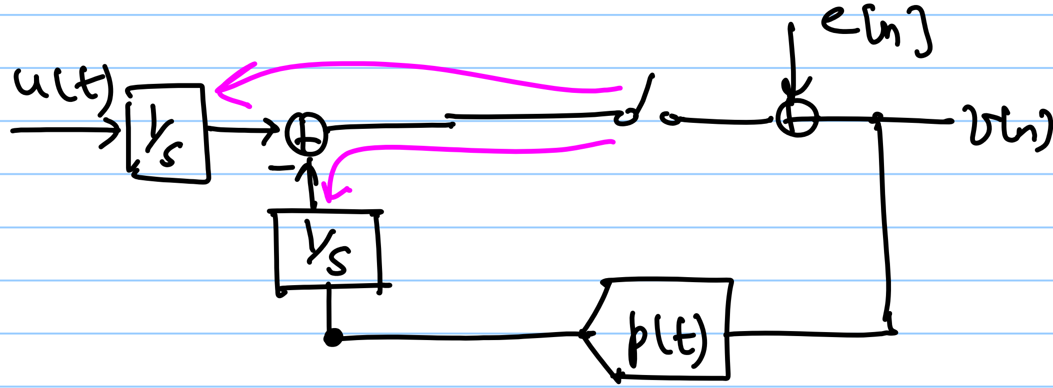
$\mathcal{L}(p(t))$

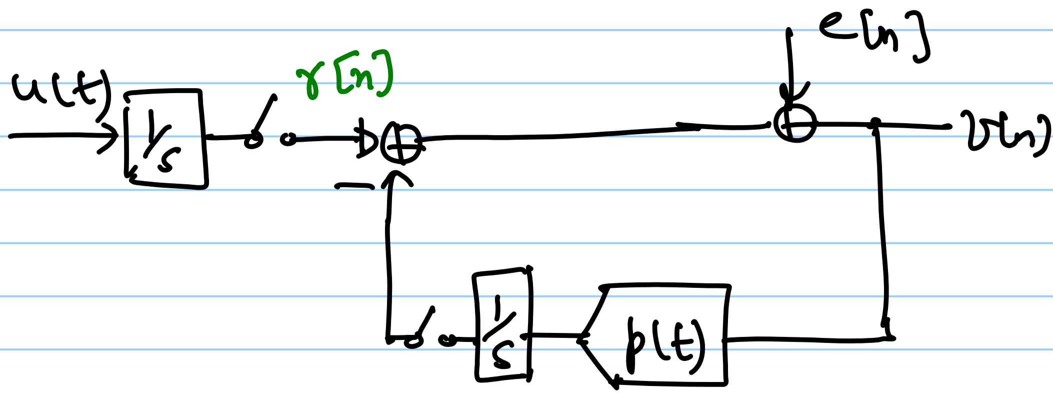
DAC pulse shape

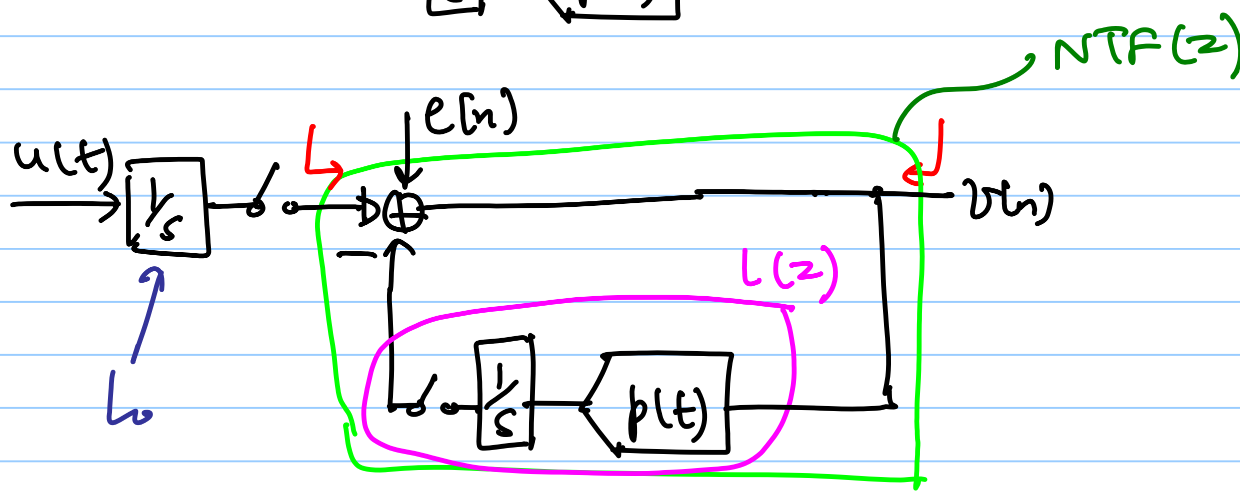
$$NTF(z) = \frac{1}{1+L(z)}$$



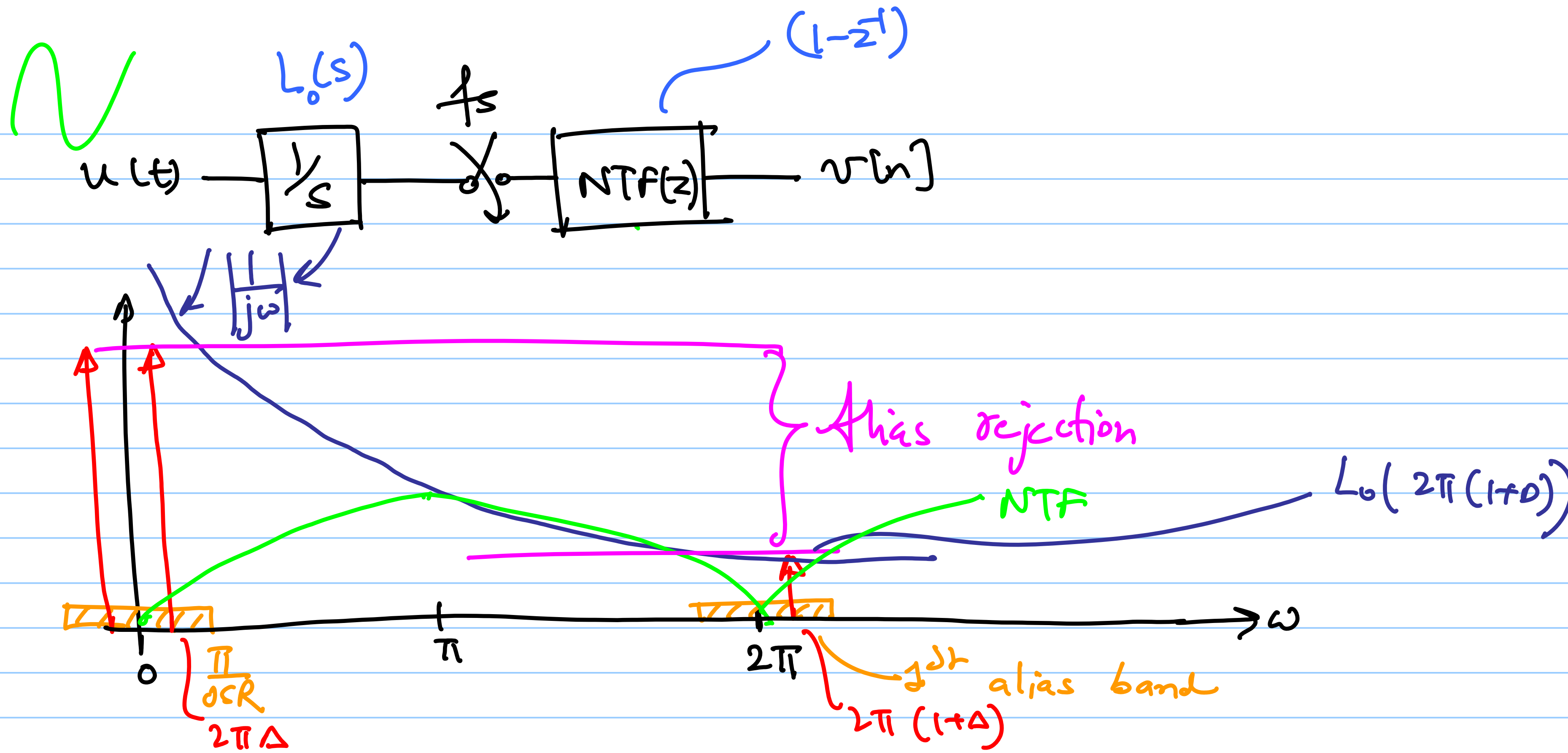
$f_s = 1\ Hz$



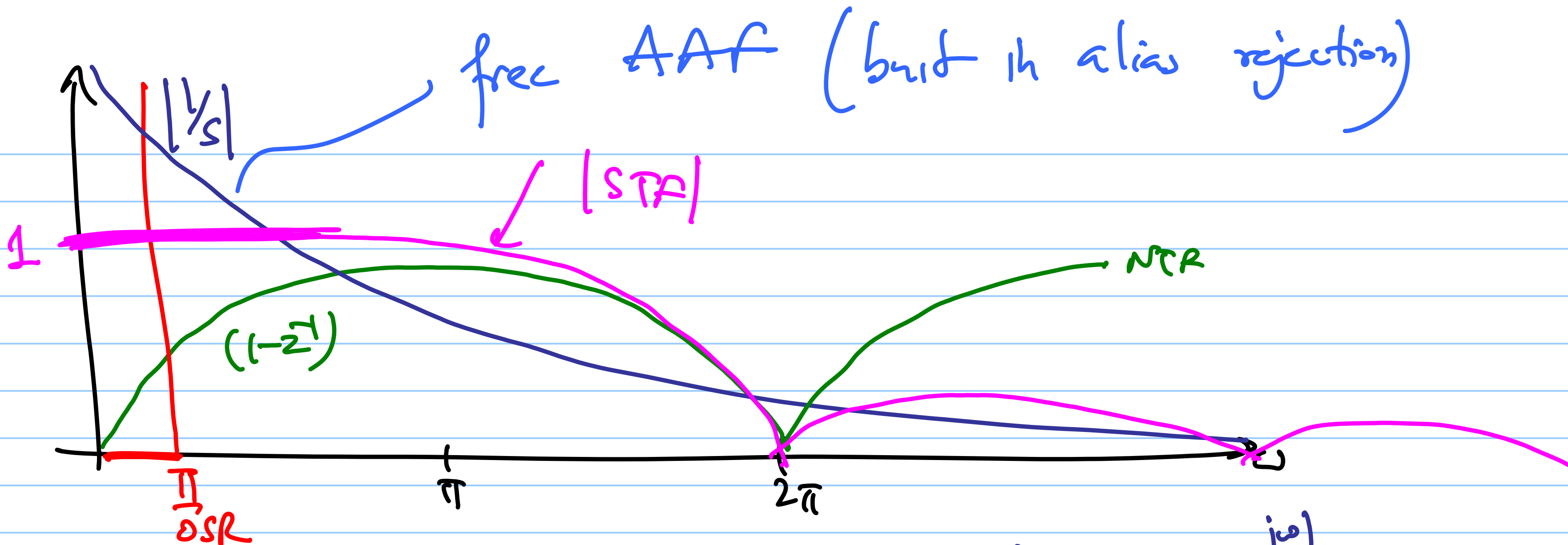




$$\frac{1}{1+L(z)} = NTF(z)$$



$L_0(s)$
f_s
$(1-z^{-1})$
$u(t)$
$1/s$
$NTF(z)$
$v[n]$
$\left|\frac{1}{j\omega}\right|$
Alias rejection
NTF
$L_0(2\pi(1+\Delta))$
ω
0
π
2π
$\frac{\pi}{OSR}$
$2\pi\Delta$
$2\pi(1+\Delta)$
1st alias band

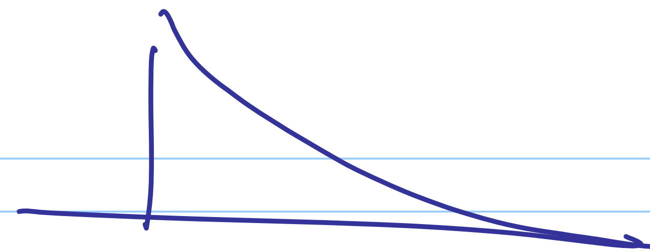


In general $STF(j\omega) = L_0(j\omega) \cdot NTF(e^{j\omega})$

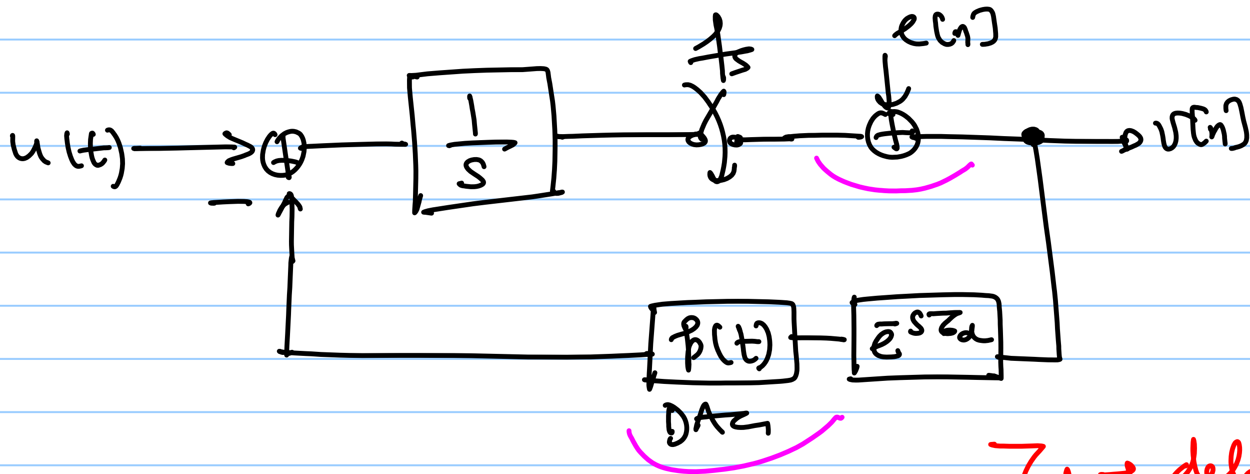
$AAF \longrightarrow L_0(j\omega)$

↳ implicit anti-aliasing in CT-ΔΣ Modulators this alias rejection is

$$\frac{|L(2\pi(1+\Delta))|}{|L(2\pi\Delta)|}$$



Excess loop-delay in CT-ΔΣ:

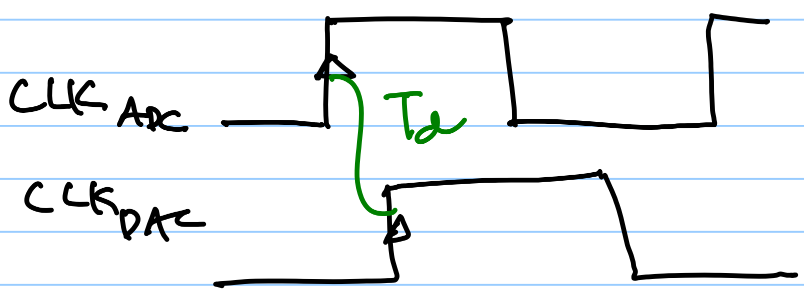


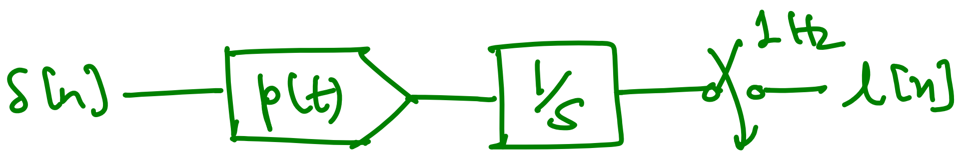
$f_s = 1\,\text{Hz}$

ADC & DAC will have finite delay.

$\tau_d \to$ delay in quantizer

$0 < \tau_d < T_s$

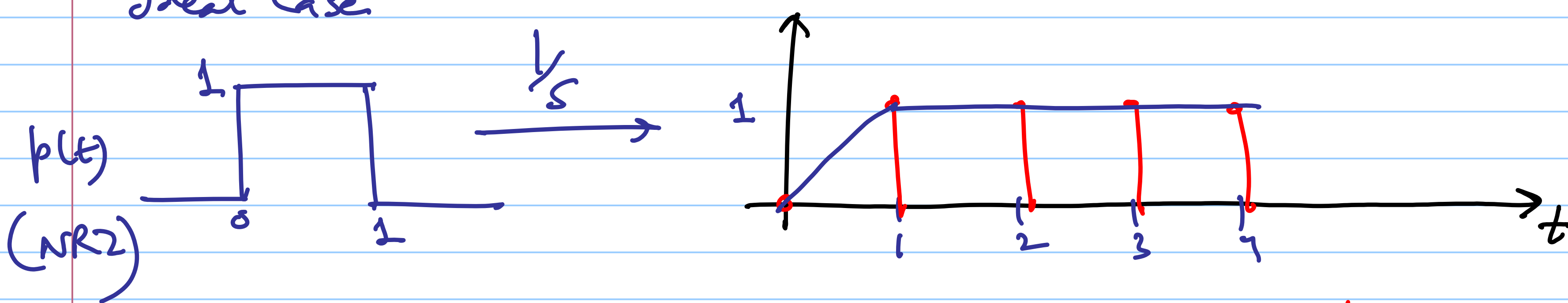




$l[n] \overset{Z}{\longleftrightarrow} L(z)$

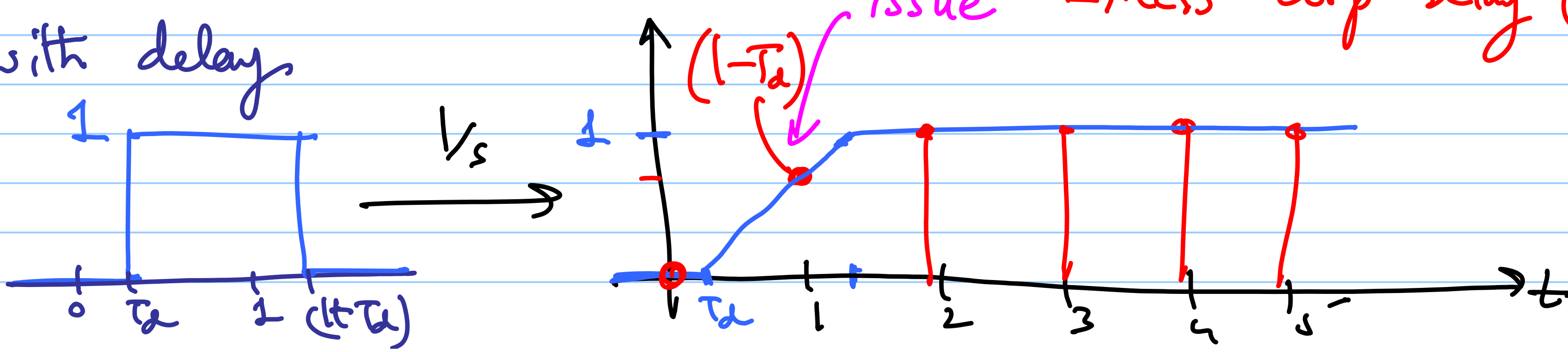
$l(n) = \{0, 1, 1, \dots\}$

Ideal Case



Excess Loop Delay (ELD)

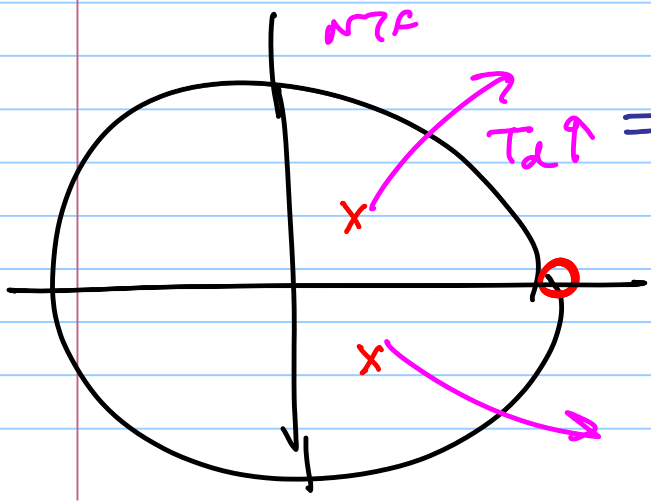
with delay



$$l[n] = \{0,\ 1-T_d,\ 1,\ 1,\ \ldots\}$$

$$L(z) = \frac{z^{-1}}{1-z^{-1}} - T_d z^{-1}$$

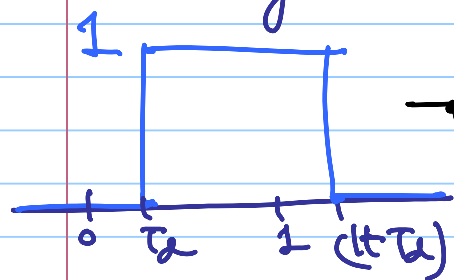
$$NTF(z) = \frac{1}{1+L(z)} = \frac{1}{1+\frac{z^{-1}}{1-z^{-1}} - T_d z^{-1}} = \frac{(1-z^{-1})}{1-T_d z^{-1}(1-z^{-1})}$$

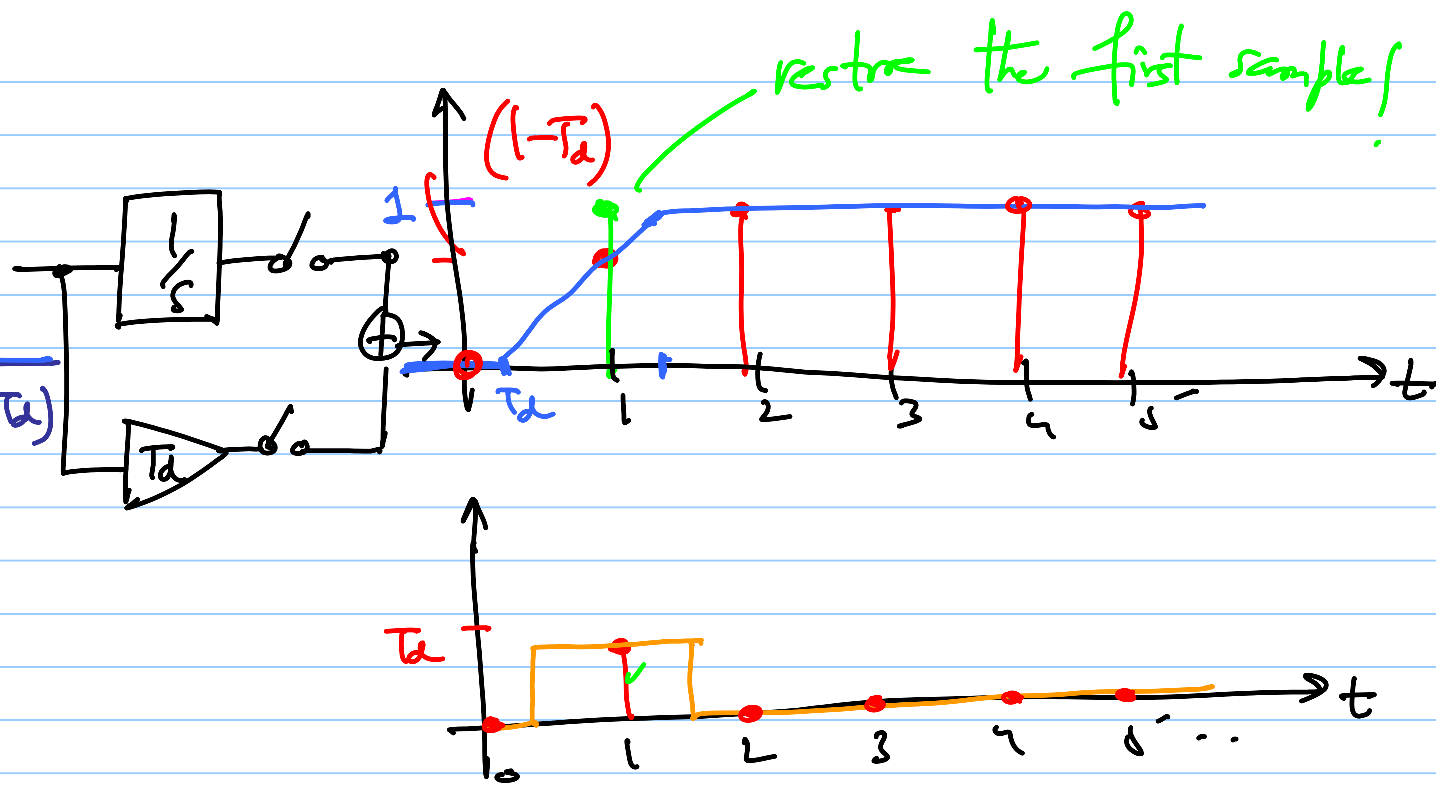


$$T_d \uparrow = \frac{(1-z^{-1})}{1 - T_d z^{-1} + T_d z^{-2}}$$

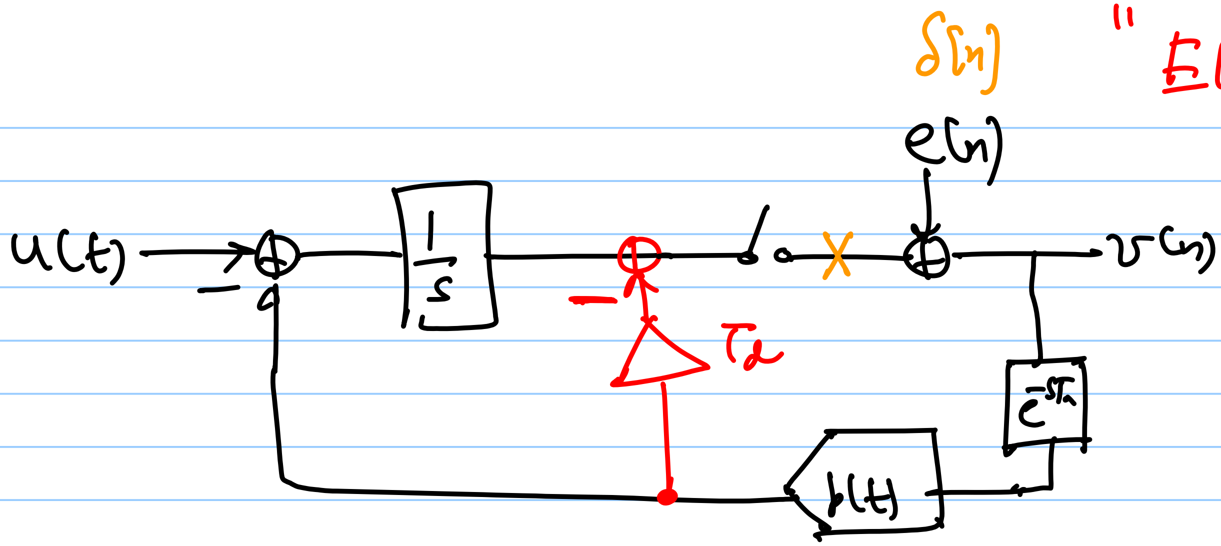
ideally $\rightarrow (1-z^{-1})$

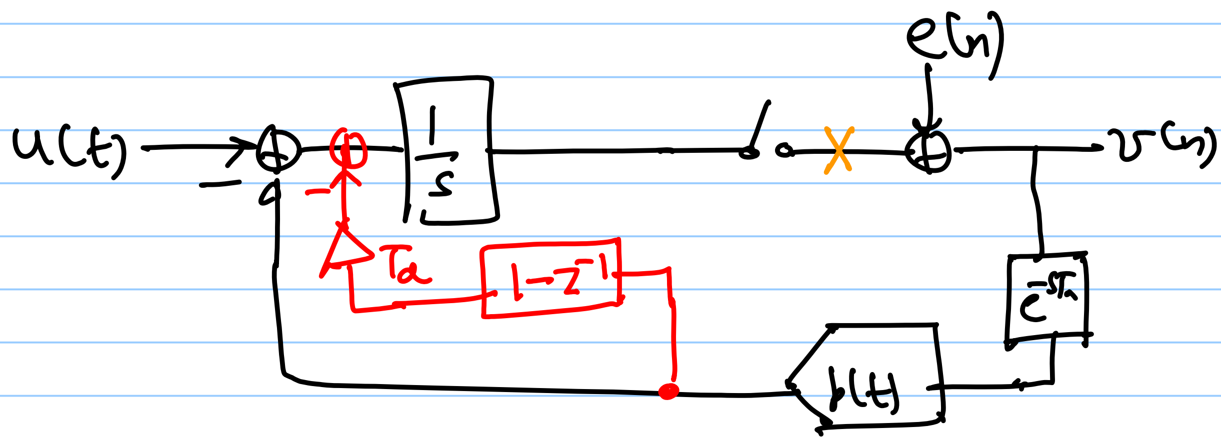
with delay

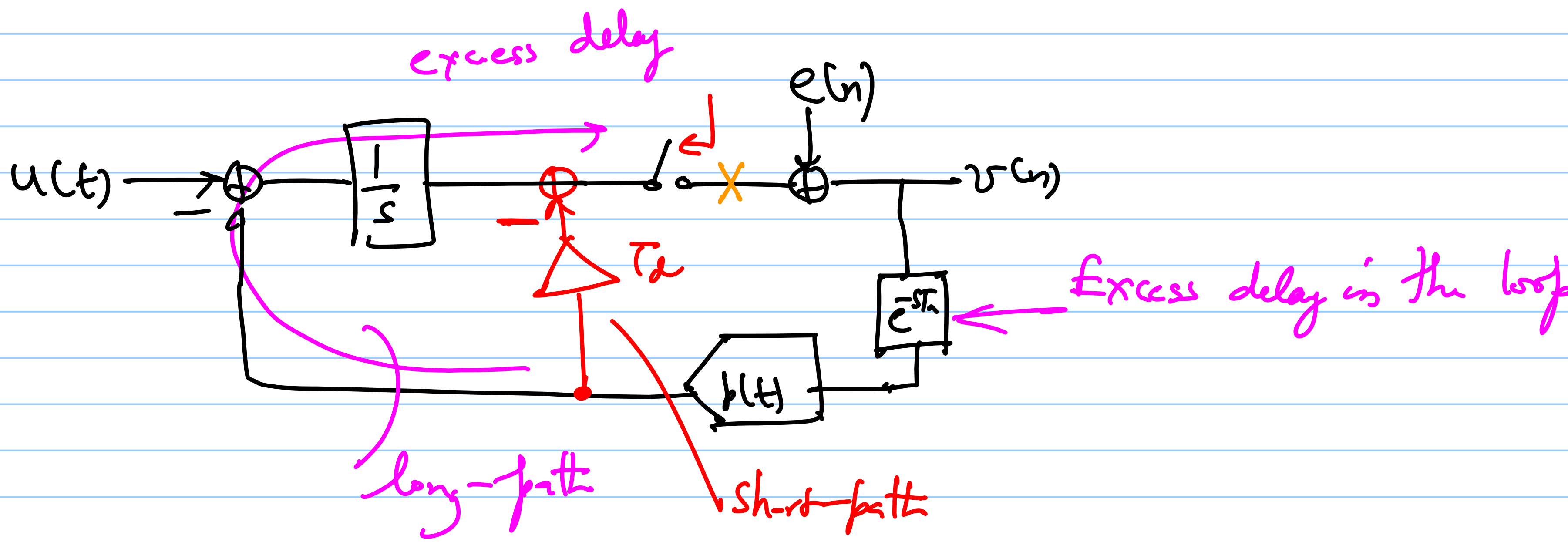

1
0
T_d
1
$(1+T_d)$


$\frac{1}{s}$
T_d
$(1-T_d)$
restore the first sample!
1
T_d
1
2
3
4
5
t
T_d
0
1
2
3
4
5 ...
t

"ELD Compensation"






excess delay
u(t)
1/s
e(n)
v(n)
T_d
e^{-sT_n}
Excess delay in the loop
h(t)
long-path
short-path