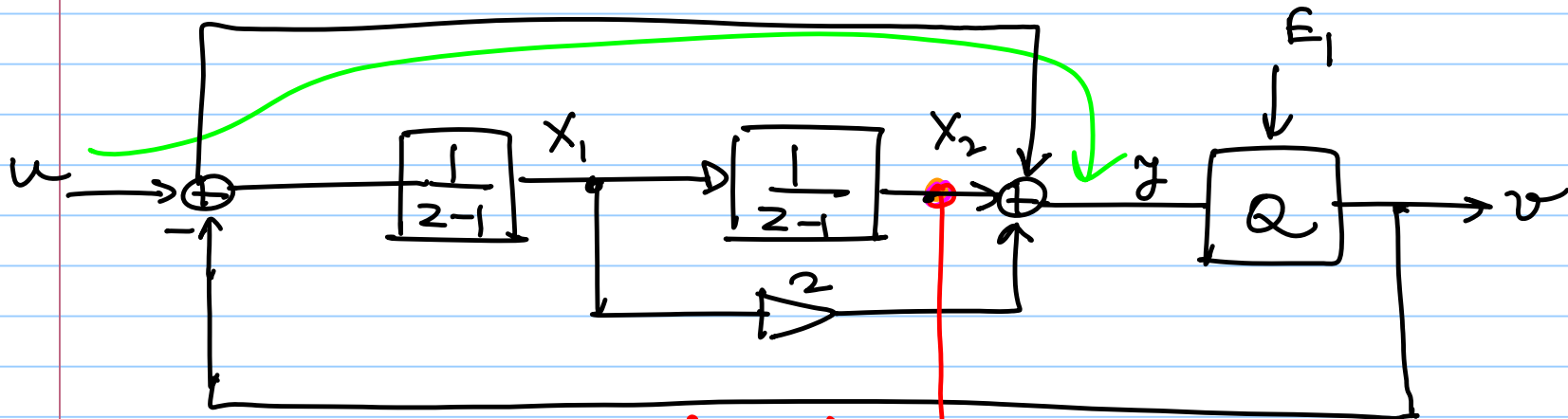


ECE 615 - Lecture 22



$$-z^{-2} E_1(z)$$

feedforward modulator \Rightarrow 2nd order

$$NTF = (1-z^{-1})^2$$

$$STF = 1$$

$$\rightarrow x_2 = -z^{-2} \cdot E_1(z) \rightarrow \text{low distortion topology}$$

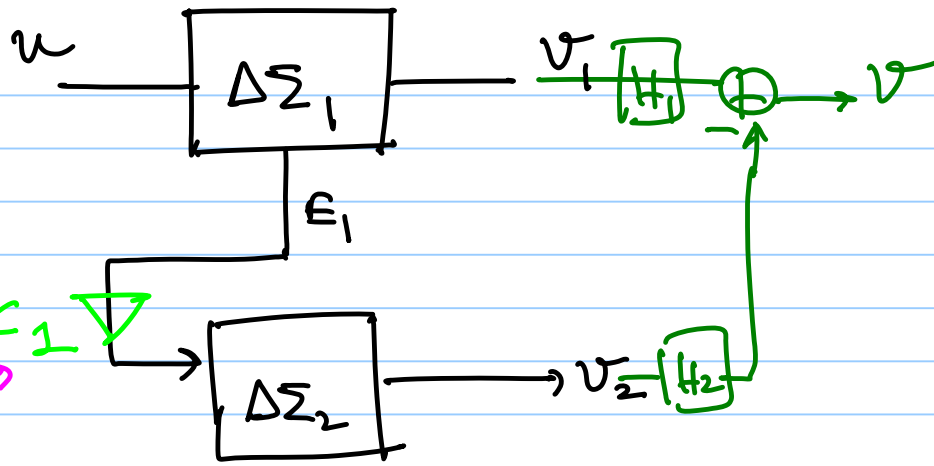
ϵ_x . 2-2 MASH (Soso)

$$STF_1 = STF_2 = z^{-2}, \quad NTF_1 = NTF_2 = (1-z^{-1})^2$$

$$V(z) = z^{-4}U(z) - \boxed{(1-z^{-1})^4}E_2(z) \rightarrow E_1(z) \text{ is cancelled (ideally)}$$

$$NTF(z) = (1-z^{-1})^4$$

↳ stability is still governed by
OBC = 4.



if $|E_1|_{\max} > MSA_2$
 \rightarrow instability or SNDR drop

interstage coupling coeff.

$|E_1|_{\max} \ll MSA_2$
 \rightarrow low SNDR in the 2nd modulator

* for a 2nd-order "single-bit" first stage $\Delta\Sigma$, usually $C_1 = \frac{1}{4}$

* for multi-bit quantizer in the first-stage $C_1 \geq 1$

* The inverse of scaling factor (c_i) needs to be included in H_2 to cancel $E_1(z)$

$$H_1 = STF_{2,\alpha}$$

$$H_2 = \frac{1}{c_1} NTF_{1,\alpha}$$

$$V(z) = STF_1 \cdot STF_{2,\alpha} V(z) + \boxed{\frac{1}{c_1} NTF_{1,\alpha} NTF_2} E_2(z)$$

NTF

in general.

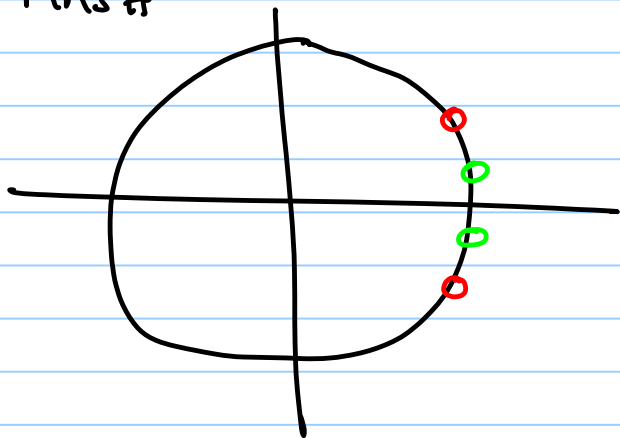
$$NTF_{\text{case}} = \frac{(1 - \bar{z}^1)^{N_{\text{case}}}}{\prod_{i=1}^n c_i}$$

Inband noise

$$IBN = \frac{\sigma_c^2}{\pi} \int_0^{\pi/\omega_{SR}} \frac{|NTF_1(e^{j\omega})|^2 |NTF_2(e^{j\omega})|^2 d\omega}{\pi c_i^2} = \frac{IBN_0}{\pi c_i^2}$$

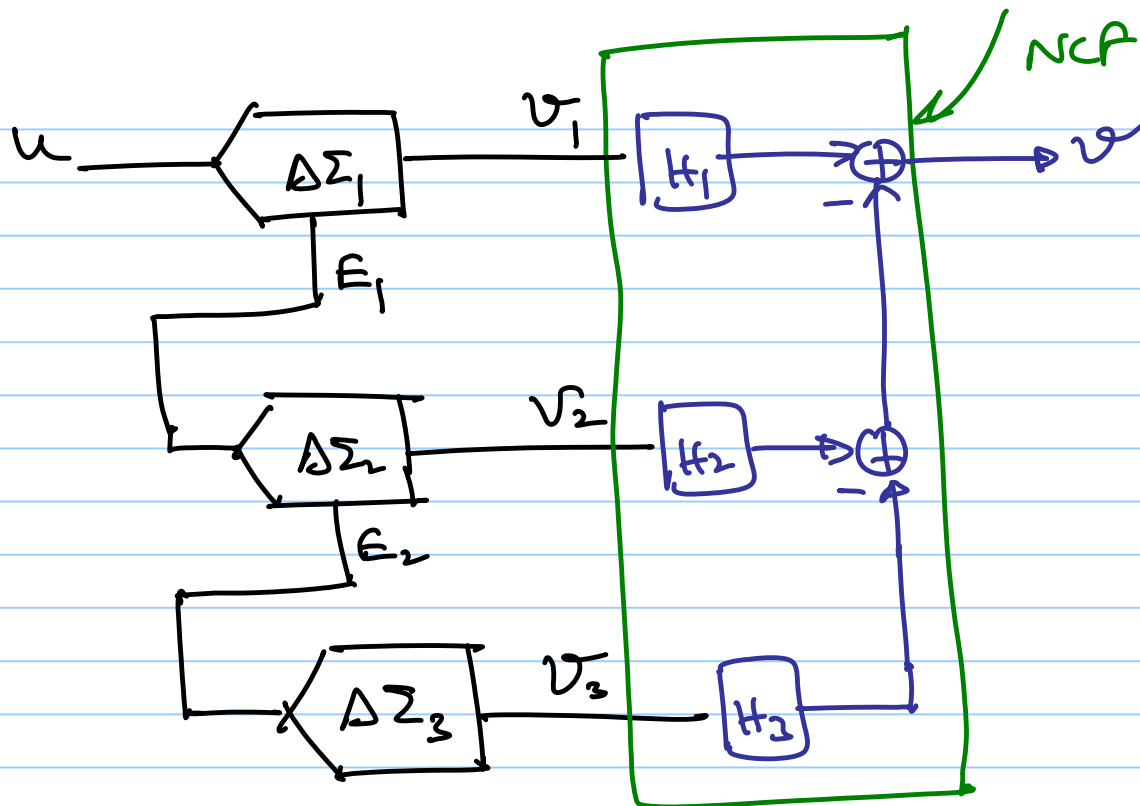
for single-bit MASH $c_i < 1 \Rightarrow IBN \uparrow \Rightarrow SQNR \downarrow$

2-2 MASH



zero pairing in MASH

\hookrightarrow NCF may be more complicated.



Goal: Cancel E_1 & E_2

$$\left. \begin{aligned} H_1 \cdot NTF_1 - H_2 \cdot STF_2 &= 0 \\ H_2 \cdot NTF_2 - H_3 \cdot STF_3 &= 0 \end{aligned} \right\} \rightarrow \textcircled{1}$$

If E_1 & E_2 are perfectly cancelled

$$V = STF_1 \cdot H_1 \cdot U + NTF_3 \cdot H_3 \cdot E_3$$

$$H_1 = STF_2, \quad H_2 = NTF_1$$

$$H_3 = \frac{NTF_1 \cdot NTF_2}{STF_3}$$

$$STF_3 = z^{-k}$$

$$NTF = \frac{NTF_1 \cdot NTF_2 \cdot NTF_3}{STF_3}$$

2-1-1 MASH

2-2-1 MASH

⚡ didn't consider C_i 's

Analysis of Noise Leakage:

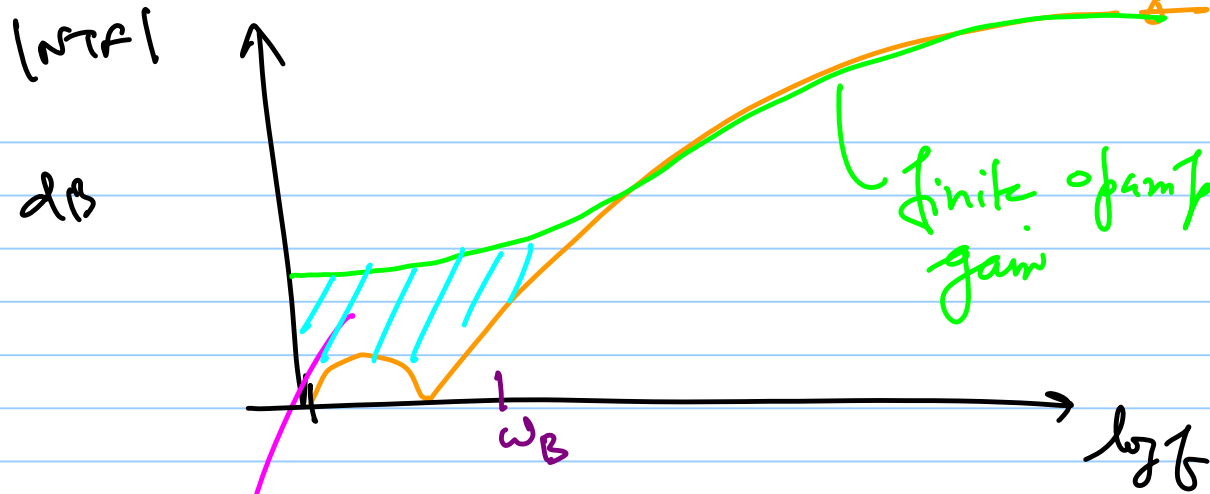
In a single-loop $\Delta\Sigma$ modulator, the issues are:

↳ imperfect matching of capacitors (C 's)

↳ finite-gain of opamps

↳ incomplete-settling & slewing in opamps

→ These anomalies change the NTF & STF, and thus the SQNR



$$|NTF| \approx \frac{1}{|L|}$$

loop gain



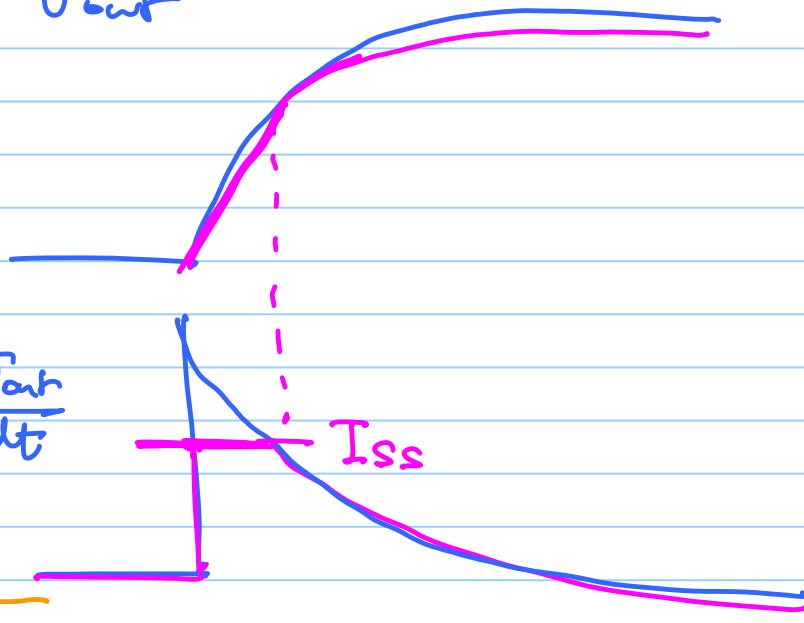
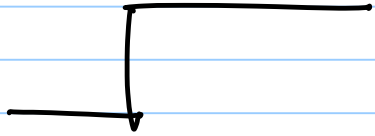
"notch filling"
 IBN \uparrow
 SNR \downarrow

More SNR \Rightarrow opamp DC gain \uparrow

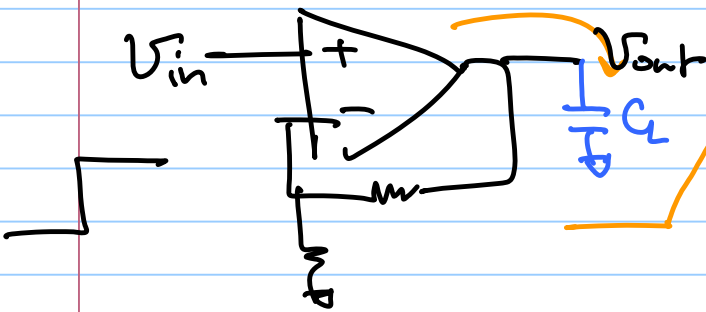
(First order settling)
Linear settling.

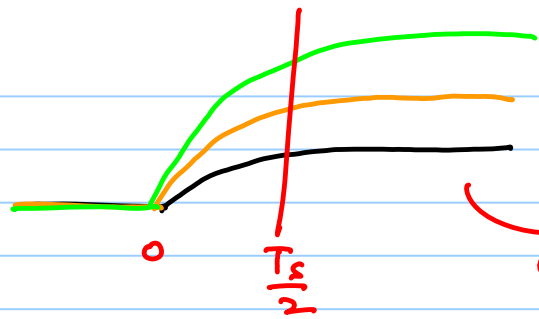
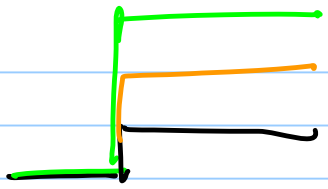
v_{out}

$$A_{CL}(1 - e^{-t/\tau})u(t)$$



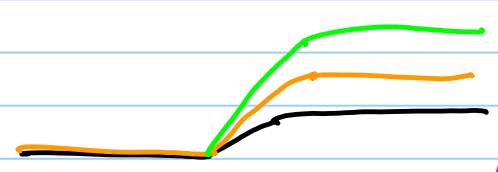
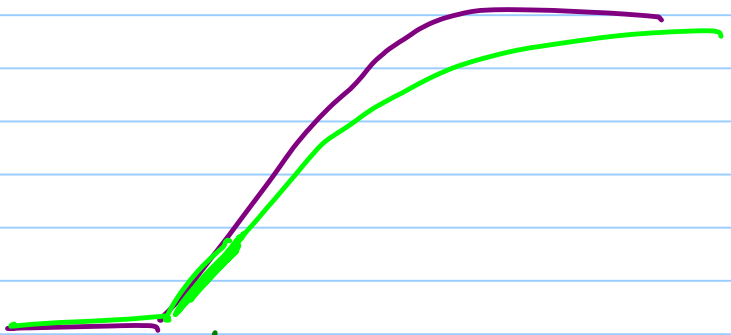
$$i_{out} = C_L \frac{dv_{out}}{dt}$$





gain = 1
1 - ε

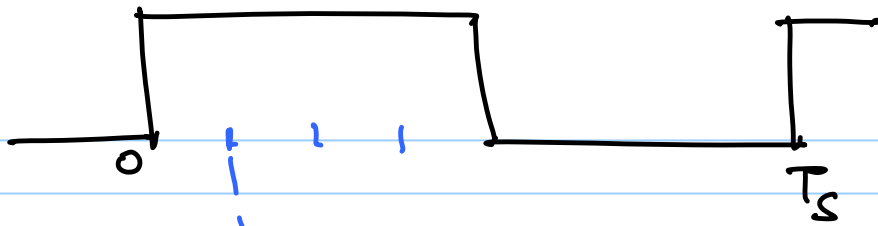
$\epsilon = (1 - e^{-T_s/2})$
incomplete settling error



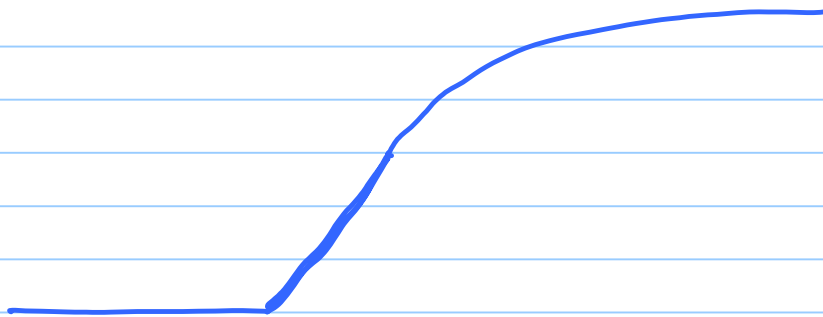
} Slowing leads to non-linearity

In switched capacitor, the opamp shouldn't slow for more than 25% of the settling period!

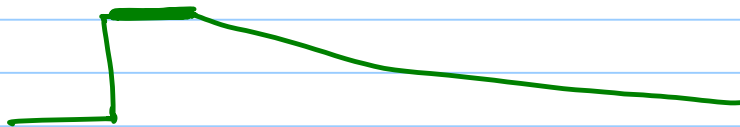
CLK



Can only slew of $\leq T_s/8$



$\frac{dV_{sat}}{dt}$



In a MASH to cancel E_1

$$H_{1i} NTF_{1a} - H_{2i} STF_{2a} \stackrel{!}{=} 0$$

$$\Rightarrow H_{e1} = \underline{H_1} \cdot NTF_1 - H_2 \cdot \underline{STF_2}$$

$H_{e1} \neq 0$ due to imperfect analog components

$$|H_{e1}| = ?$$

$$H_1 = STF_2 \leftarrow \text{flat in the signal band}$$

$$H_2 = NTF_1$$

$$|H_{e1}| = |NTF_1 - H_2| = |NTF_{1a} - NTF_{1i}|$$

ideal

leakage

$$|H_{e,1}| \approx |NTF_{1,a} - NTF_{1,i}|$$

first-stage should be
as close to ideal response
as possible
↳ E_1 leakage

$$I_i(z) = \frac{a}{z-1}$$

← integrator gain = C_1/C_2

$$I_a(z) = \frac{a'}{z-p'}$$

→ model for imperfect integrator

$$a' = a(1+D)$$

$$|H_{e_1}| \approx \frac{1}{A} + z^{-1} (1-z^{-1}) \left[\frac{D}{a} + \left(1 + \frac{1}{a}\right) \frac{1}{A} \right]$$

direct noise
feedthrough
from E_1 to V

1st-order
noise-shaper