ECE 615 - Lecture 1

Note Title 1/12/2016

Fourier Series: Periodic signal $g(t)$

$$g(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k f_0 t} \qquad f_0 = \frac{1}{T}$$

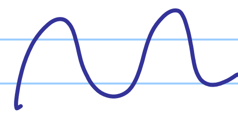
$$a_k = \frac{1}{T_s}\int_0^{T_s} g(t)\, e^{-j2\pi k f_0 t}\, dt$$

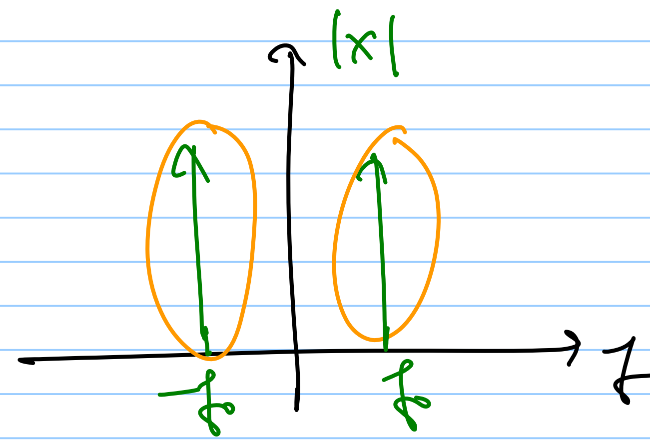
Fourier Transform:

$$X(t) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{+j2\pi f t}\, dt$$

$$x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(f)$$





$$a\,x(t) + b\,y(t) \overset{\mathcal{F}}{\longleftrightarrow} a\,X(f) + b\,Y(f)$$

$$x(t - t_0) \overset{\mathcal{F}}{\longleftrightarrow} X(f)\, e^{-j2\pi f t_0}$$ ← time delay

$$e^{j2\pi f_0 t}\, x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(f-f_0)$$

frequency translation

$$\cos(2\pi f_0 t) + j\sin(2\pi f_0 t)$$

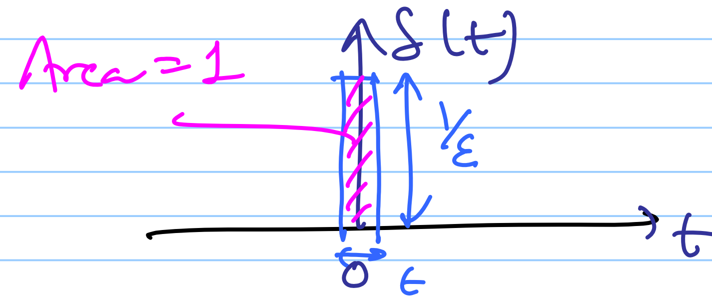
$$x(t) \circledast y(t) \overset{\mathcal{F}}{\longleftrightarrow} X(f)\cdot Y(f)$$

$$x(t)\cdot y(t) \overset{\mathcal{F}}{\longleftrightarrow} X(f) \circledast Y(f)$$

Dual

Delta function

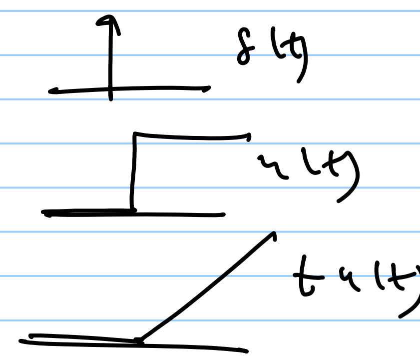
$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

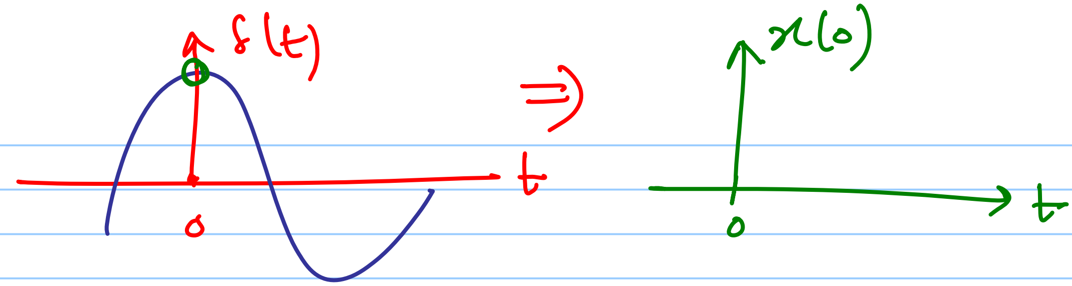


Lt $\epsilon \to 0$

$$x(t) \cdot \delta(t) = x(0)\, \delta(t)$$ ← picks value at $t=0$

$$x(t) \cdot \delta(t-t_0) = x(t_0) \cdot \delta(t-t_0)$$



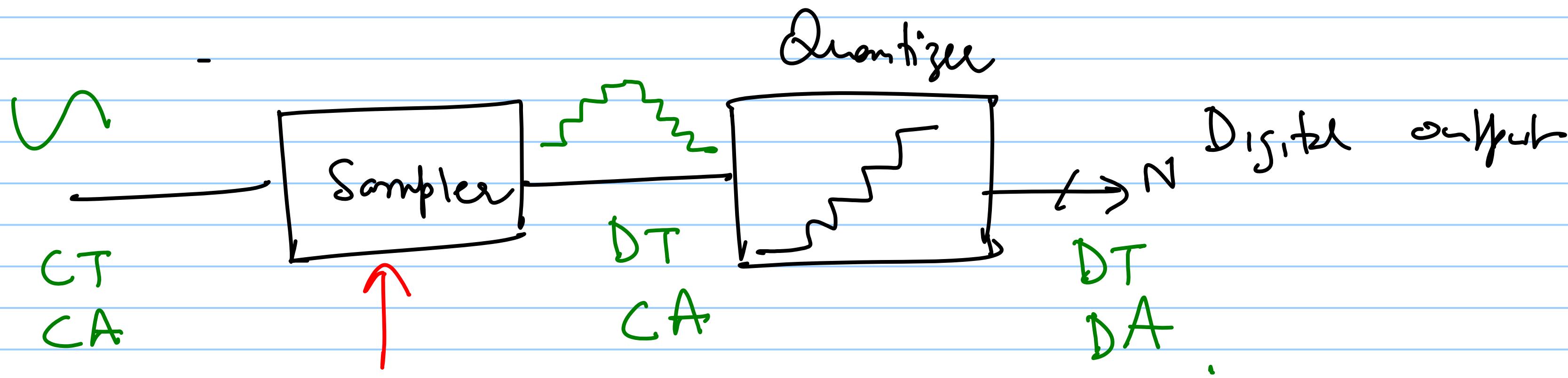


$$x(t) \circledast \delta(t) = x(t)$$

$$x(t) \circledast \delta(t - t_0) = x(t - t_0)$$

$$\delta(t) \xrightarrow{\mathcal{F}} 1$$

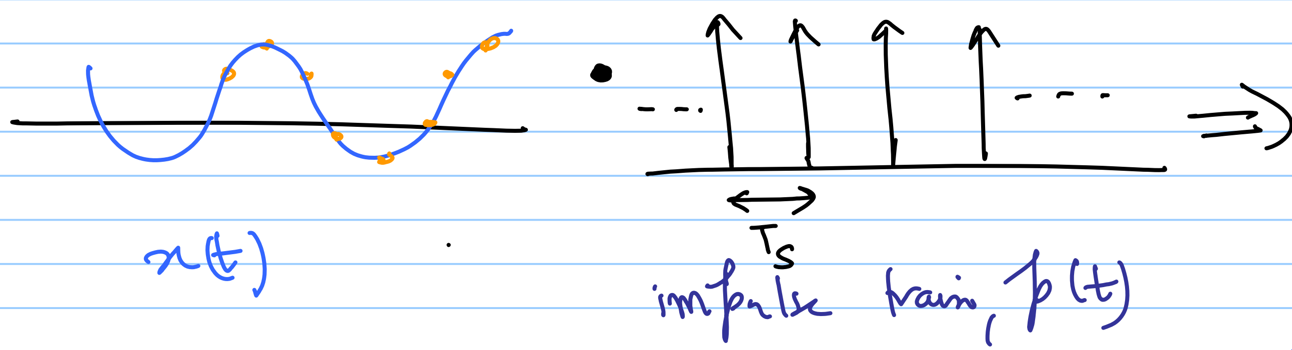
ADC (Analog-to-Digital Converter)

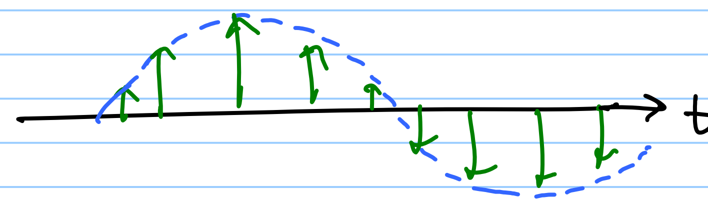


Ideal Sampling :

$T_s \Rightarrow$ Sample period

$f_s = \frac{1}{T_s} \Rightarrow$ sampling rate

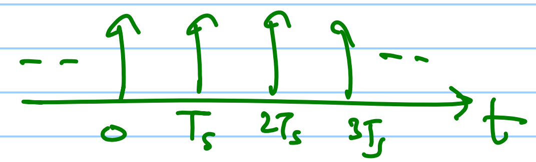




Sampled signal $\quad y(t) = x(t) \cdot p(t)$

$y[n] = y(nT_s)$

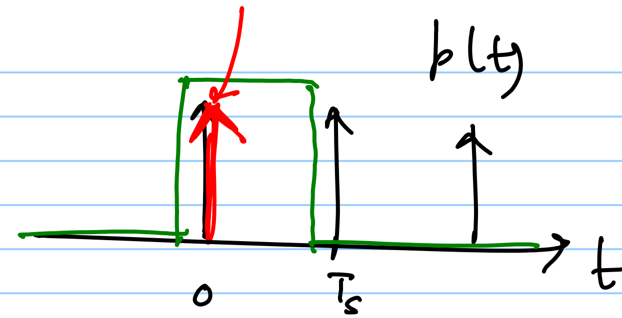
$$y(t) = x(t) \cdot \overbrace{\sum_{n=-\infty}^{\infty} \delta(t - nT_s)}^{p(t)} = x(t) \cdot p(t)$$



$$Y(f) = X(f) \circledast P(f)$$

??

$p(t)$ ← periodic signal



$$p(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k f_s t}$$

Fourier series coefficient

$$a_k = \frac{1}{T_s}\int_{0}^{T_s^-} p(t)\, e^{-j2\pi k f_s t}\, dt$$

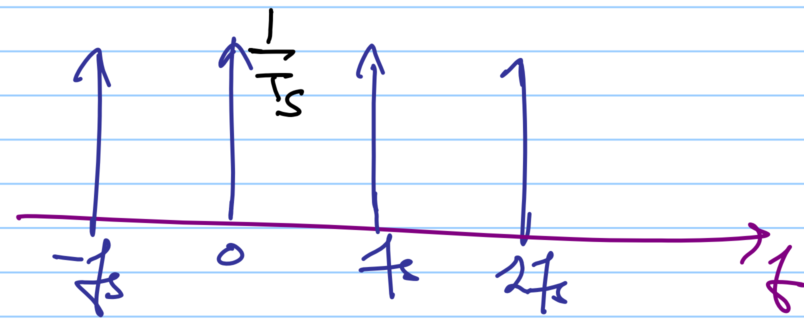
$$= \frac{1}{T_s}\int_{0}^{T_s^-} \delta(t)\, \underbrace{e^{-j2\pi k f_s (0)}}_{1}\, dt = \frac{1}{T_s}$$

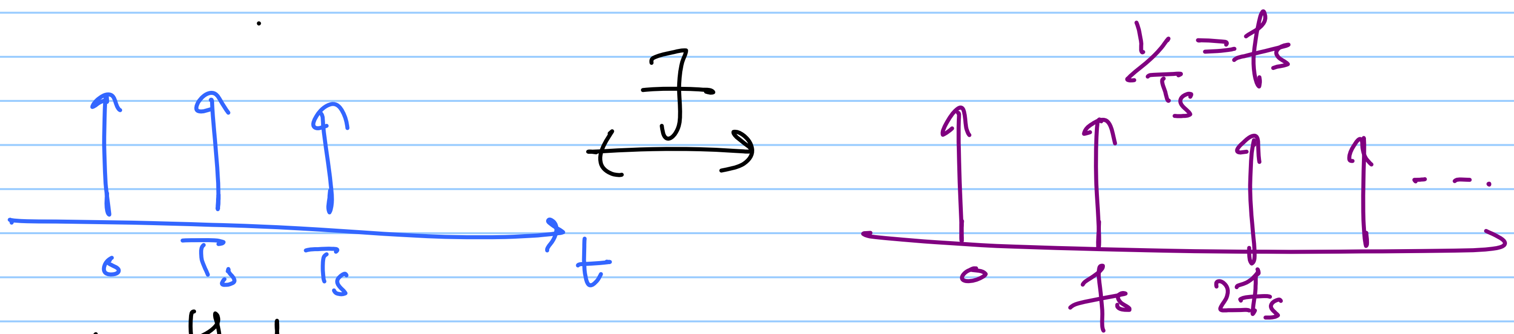
$a_k = \frac{1}{T_s}, \quad -\infty < k < \infty$

$$p(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T_s} e^{j2\pi \boxed{k f_s} t}$$

Take fourier transform

$$P(f) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} \delta(f - k f_s)$$





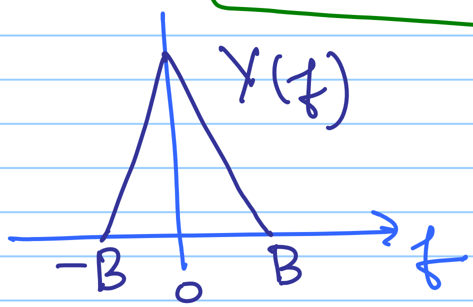
Recall that

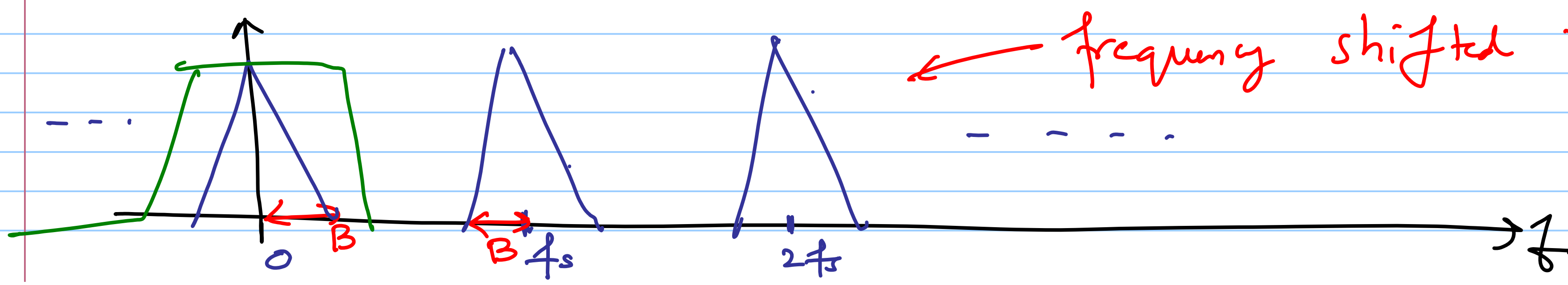
$$Y(f) = X(f) \circledast P(f)$$

$$= X(f) \circledast \frac{1}{T_s} \sum_{k=-\infty}^{\infty} \delta(f - kf_s)$$

$$= \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(f) \circledast \delta(f - kf_s)$$

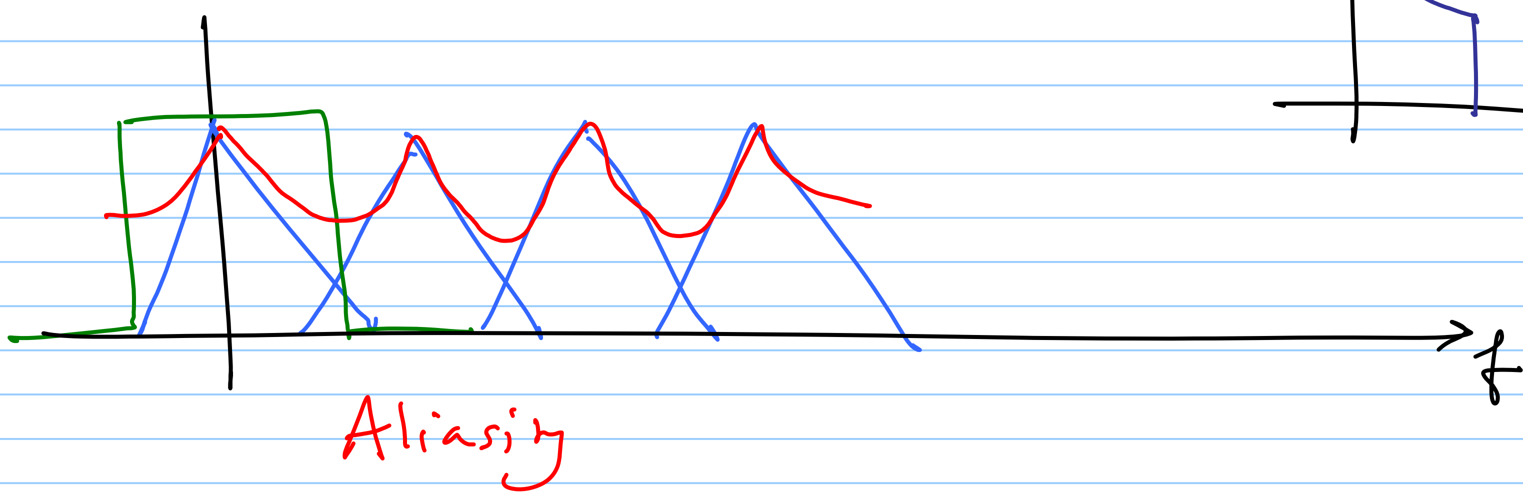
$$Y(f) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(f - kf_s)$$





$f_s > 2B$ Nyquist Sampling theorem

If $f_s < 2B$



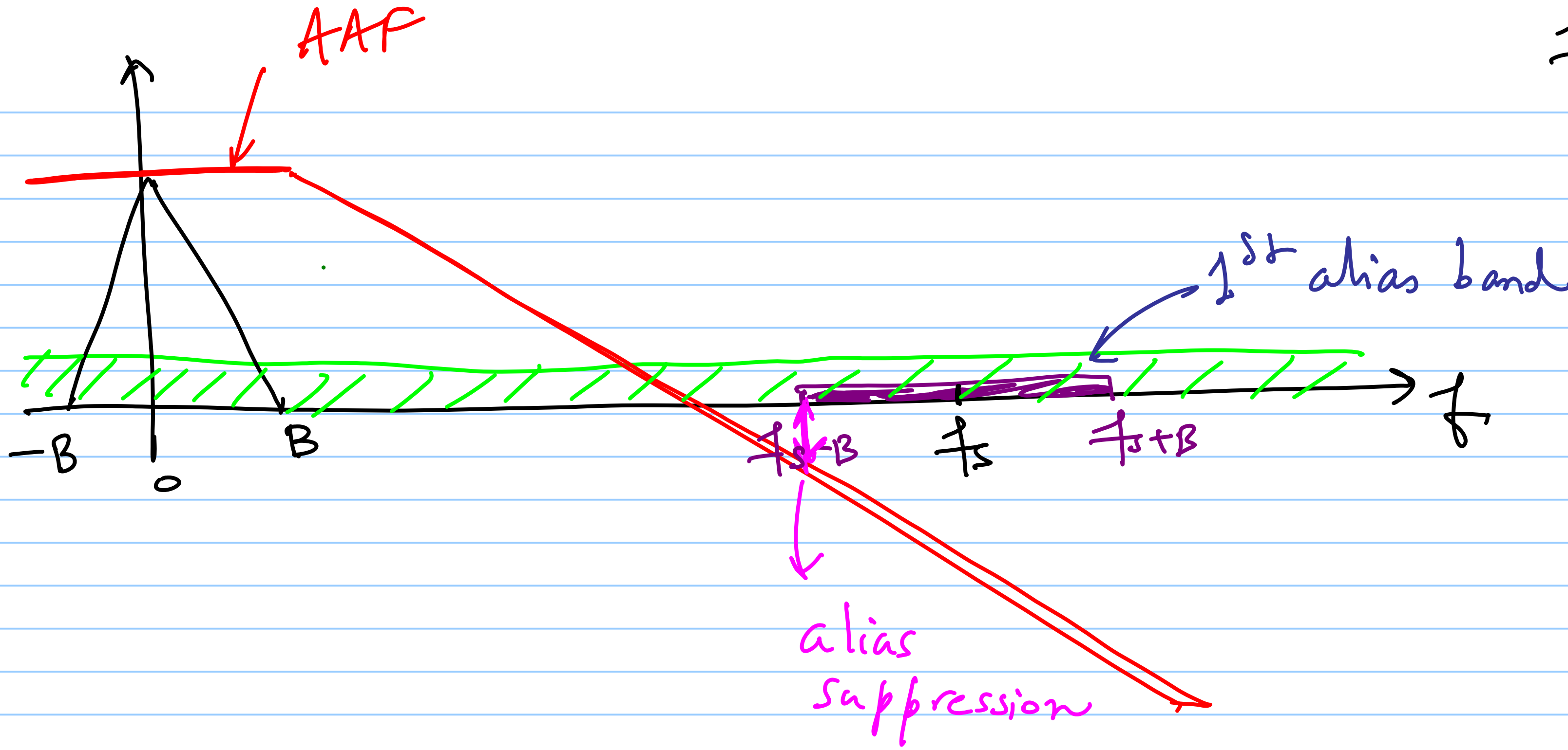
Bandlimit the input signal before the sampler

AAF

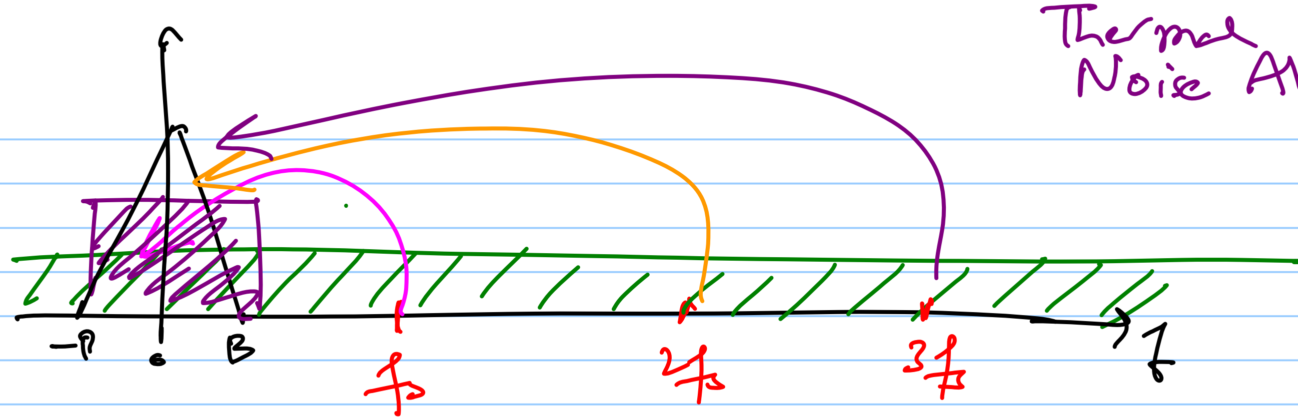
↑ antialiasing filter

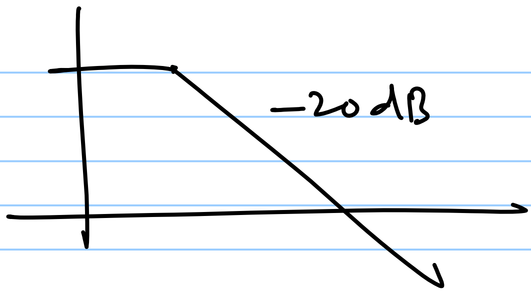
↳ Always a CT filter!

f_s

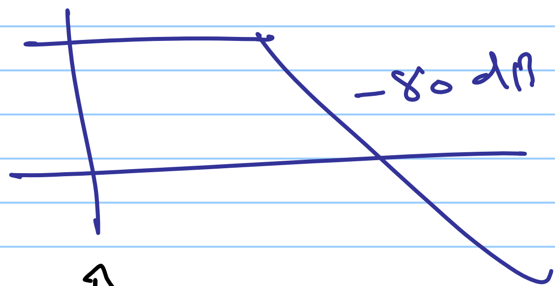


Thermal Noise Aliasing

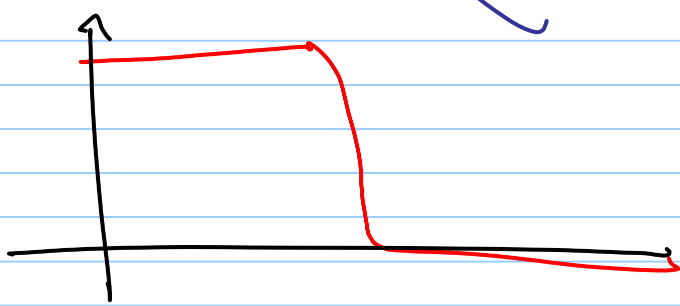




$n = 1$



$n = 4$



$n > 20$