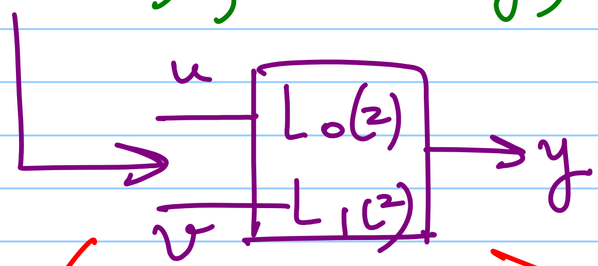
ECE 615- Lecture 18

Note Title 3/10/2016

NTF(z), stability, quantizer resolution



feedback topology (CFB)

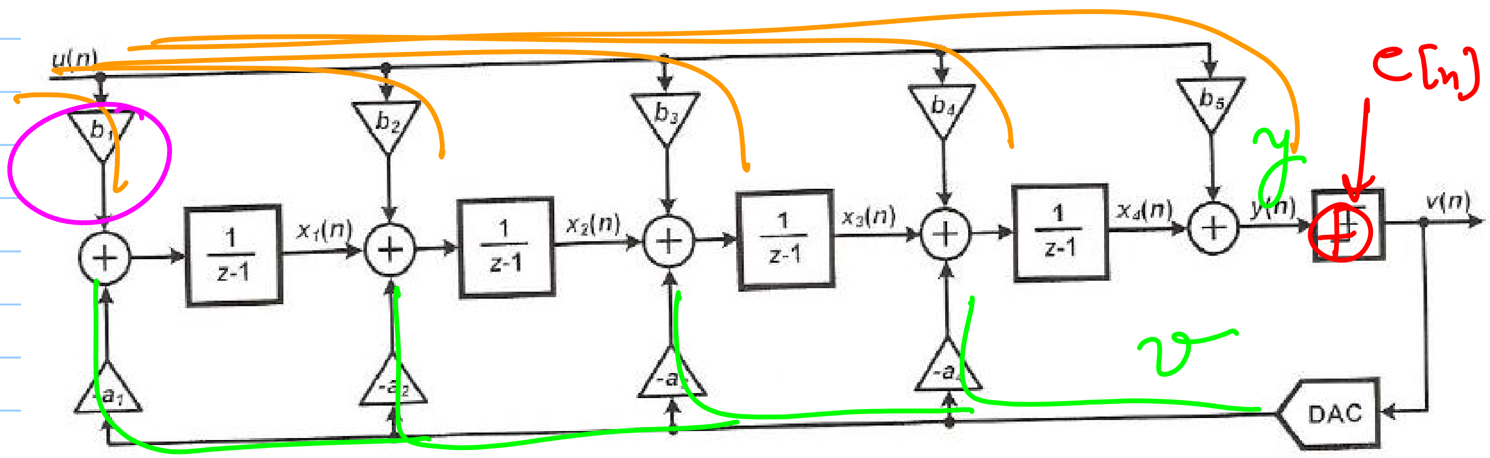
CIFB

Cascade of integrators with FF

feedforward topology (FF)

CIFF

CIFB



* Cascade of N delaying integrators $\frac{1}{z-1} = \frac{z^{-1}}{1-z^{-1}}$
* feedback from quantizer to each integrator inputs
* Multiple feed-in branches are possible $b_1 \to b_5$

$$L_0(z) = \frac{b_1 + b_2(z-1) + \cdots + b_{N+1}(z-1)^N}{(z-1)^N}$$

$$L_1(z) = -\frac{a_1 + a_2(z-1) + \cdots + a_N(z-1)^{N-1}}{(z-1)^N}$$

$a_1, b_1 > 0$

$$NTF(z) = \frac{1}{1-L(z)} = \frac{(z-1)^N}{D(z)}$$

coeff = 1

where $D(z) = a_1 + a_2(z-1) + \cdots + a_N(z-1)^{N-1} + (z-1)^N$

all NTF zeros lie at $z=1$

$NTF(z=\infty) = 1 \Rightarrow$ realizable

a_i's introduce finite non-zero poles into the NTF(z)

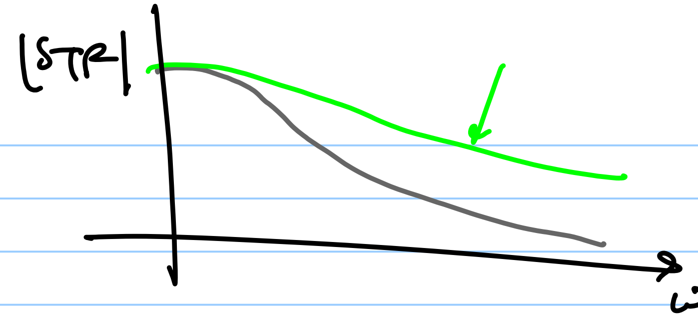
↳ also determine the zeros of L(z)

$$STF(z) = \frac{L_0(z)}{1 - L(z)} = \frac{b_1 + b_2(z-1) + \cdots + b_{N+1}(z-1)^N}{D(z)}$$

b_i's determine the zeros of the STF while a_i's determine the poles of the STF

* The poles of NTF & STF are shared

* if $b_1 \neq 0$, $b_{2:N+1} = 0$



$$STF(z) = \frac{b_1}{D(z)}$$

* This NTF can be implemented using Butterworth response

$\because$ all NTF zeros are at $z=1$

$\Rightarrow$ need all a_i's branches

↳ no flexibility of removing a_i

* More flexibility in choosing b_i's but $b_1 \neq 0$

Ex.

$$b(2:\text{end}) = 0$$

$$STF(z) = \frac{b_1}{D(z)}$$

for $|STF| = 1 \Rightarrow b_1 = D(1)$

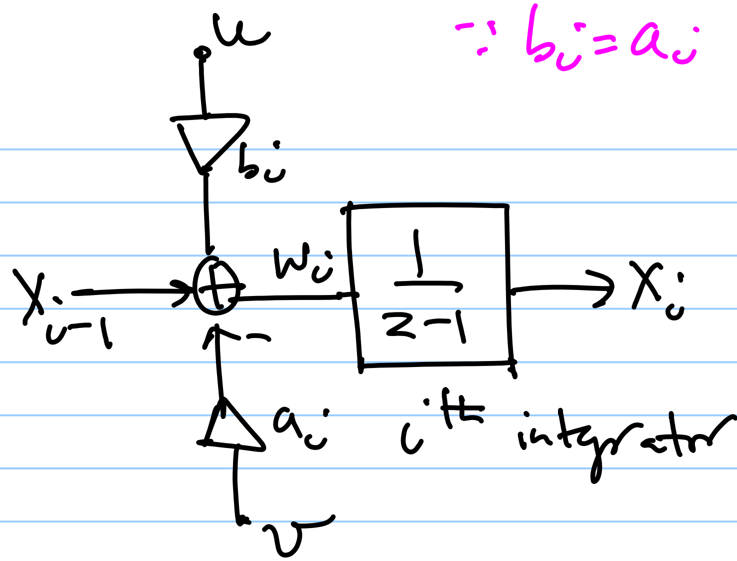
Ex. Low distortion CIFB case

$b_i = a_i$ for $i \le N$ and $b_{N+1} = 1$

$V(z) = U(z) + NTF(z) \cdot E(z)$ ← 1

$STF(z) = 1$ for all frequencies ← low distortion

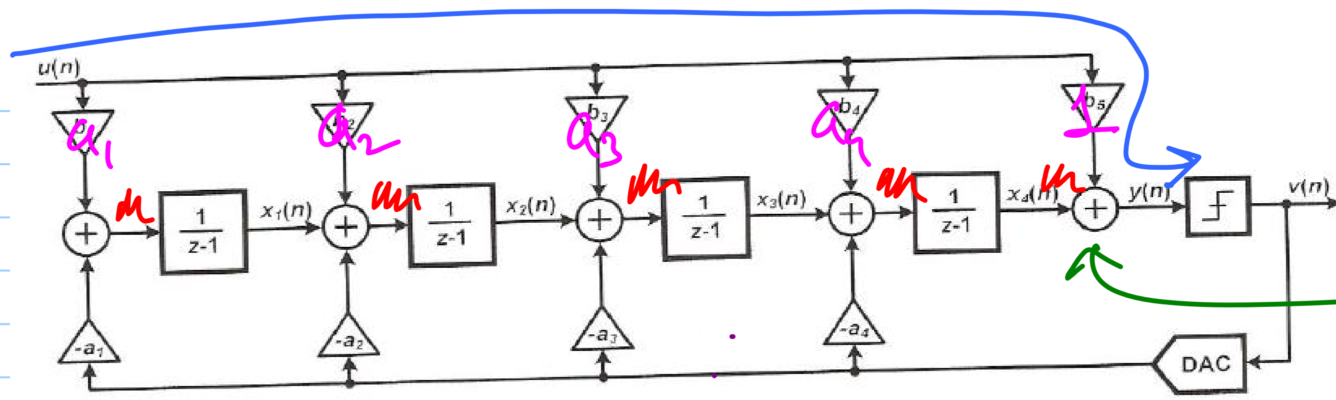
$\therefore\ b_i = a_i$



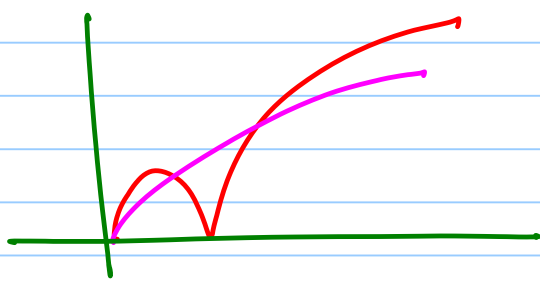
$$W_i(z) = X_{i-1}(z) - a_i V(z) + b_i U(z)$$
$$= X_{i-1}(z) + a_i\left(U(z) - V(z)\right)$$
$$= X_{i-1}(z) + a_i\left(NTF(z)\cdot E(z)\right)$$

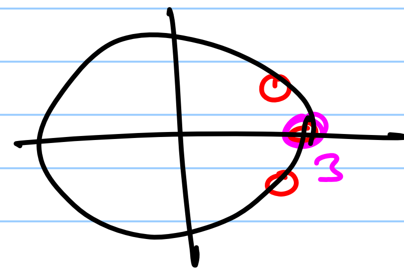
No signal here!

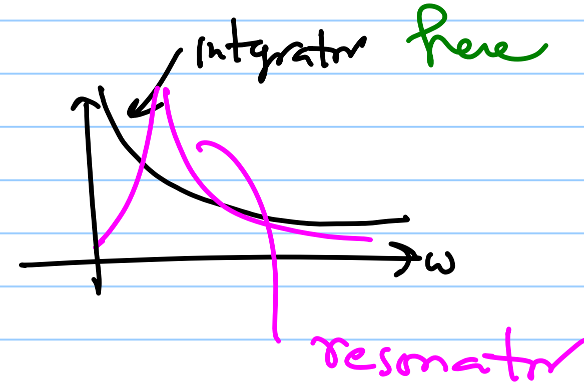
* The input signal is not present at any integrator input
* The loop-filter only processes the quantization noise & not the signal
  ↳ reduced opamp output swings, esp for multibit quantizer



Extra summer needed here

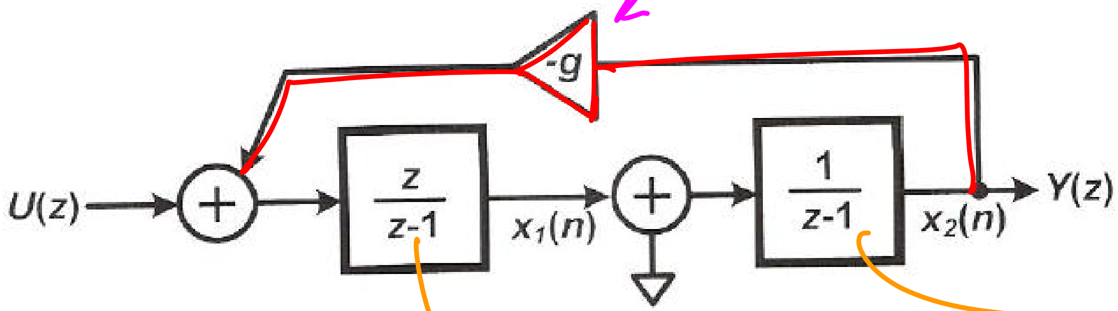






* Use feedback around pairs of integrators to form resonators

IoT



delaying

non-delaying

$$u\frac{z}{(z-1)^2} - \frac{gz}{(z-1)^2}y = y$$

$$R(z) = \frac{y}{u} = \frac{z}{z^2 - (2-g)z + 1}$$

$$= \frac{z}{z^2 - (2\cos\alpha)z + 1}$$

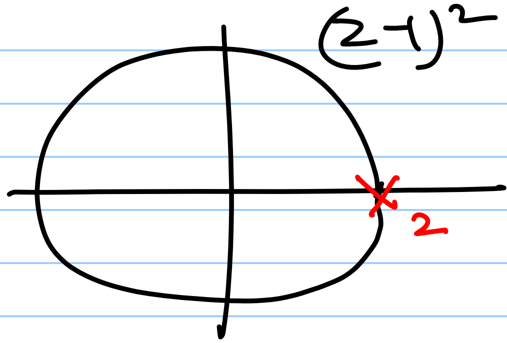
let $\cos\alpha \triangleq 1 - g/2$

$$= \frac{z}{(z - e^{j\alpha})(z - e^{-j\alpha})}$$

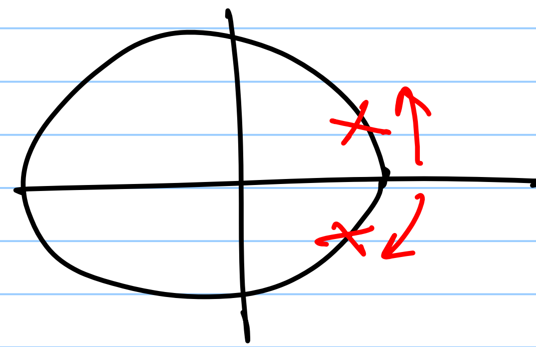
complex roots at

$\omega = \pm\alpha = \cos^{-1}(1 - g/2)$

$$\frac{z}{(z-1)^2}$$



$\Rightarrow$



$$\frac{z}{z^2 - (2-g)z + 1}$$

we had

$$\cos\alpha = 1 - g/2$$

$$\Rightarrow \quad 1 - 2\sin^2\frac{\alpha}{2} = 1 - g/2$$

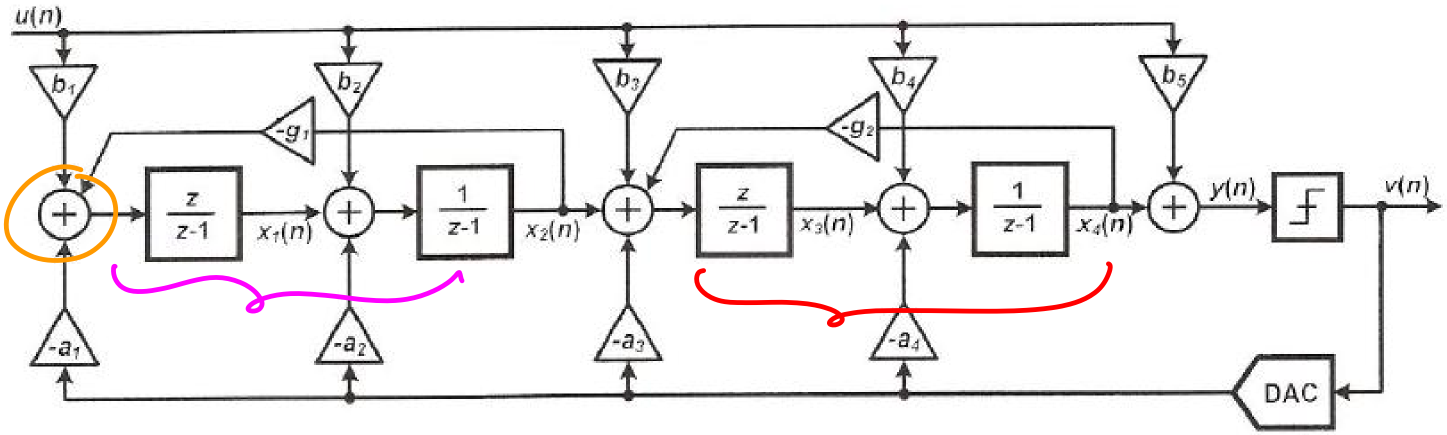
$$\Rightarrow \quad \sin\left(\frac{\alpha}{2}\right) = \pm\frac{\sqrt{g}}{2}$$

for $\alpha \ll \pi$ — for OSR >> 1

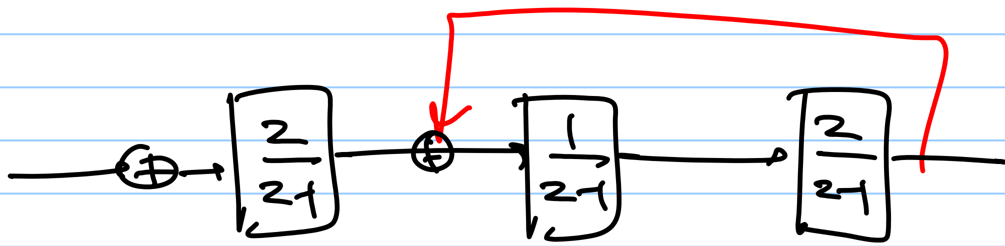
$$\Rightarrow \quad \sin\left(\frac{\alpha}{2}\right) \simeq \frac{\alpha}{2} = \pm\frac{\sqrt{g}}{2} \quad \Rightarrow \quad \boxed{\alpha \simeq \pm\sqrt{g}}$$

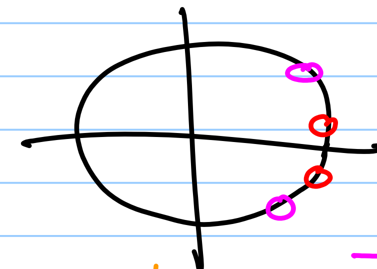
Resonator poles are located at $z = e^{\pm j\sqrt{g}}$

CRFB



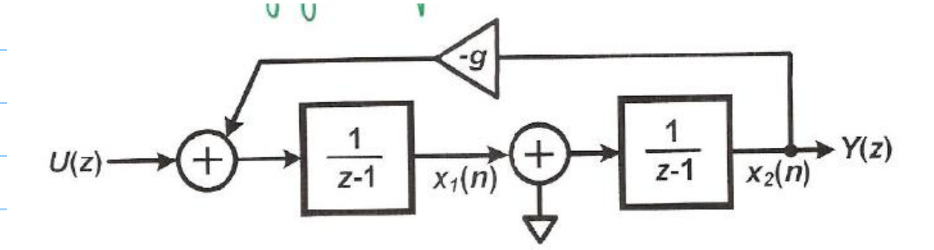
Odd order





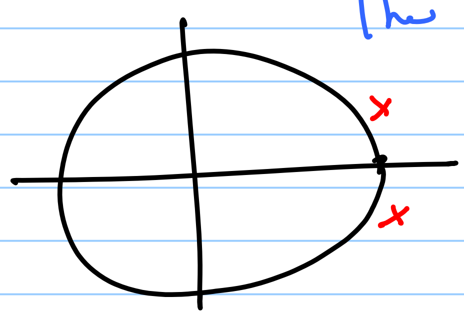
$$\frac{z}{z-1} = \frac{1}{1-z^{-1}} \Rightarrow \frac{z^{-1/2}}{1-z^{-1}}$$

Resonator with delaying integrators

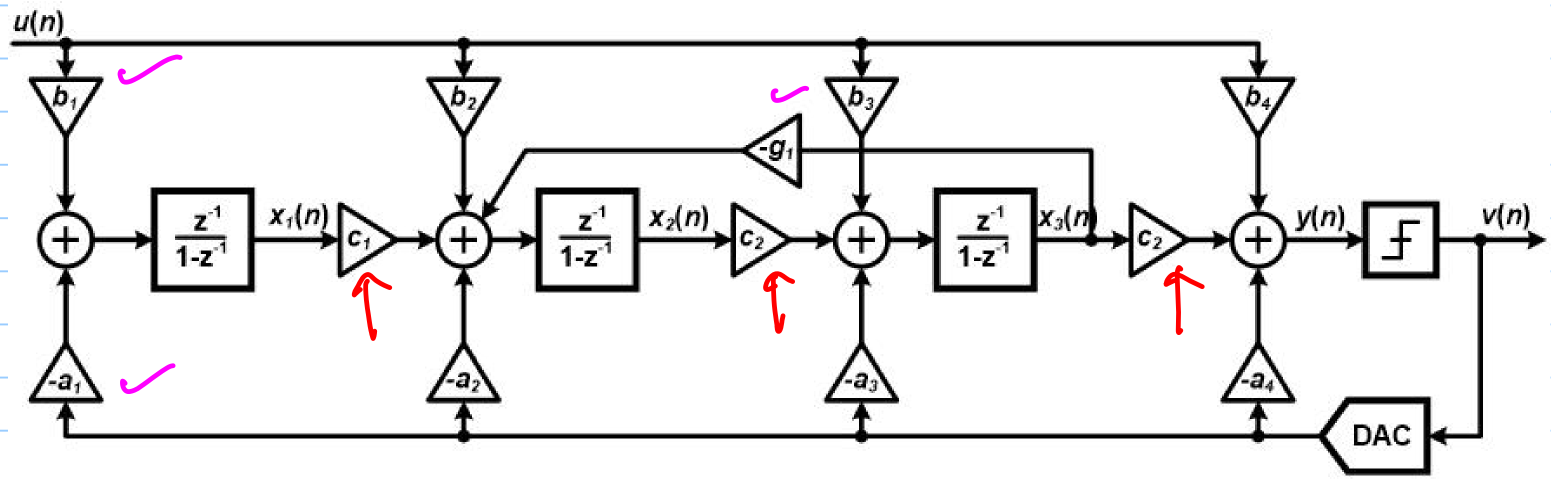


$$R_1(z) = \frac{Y(z)}{U(z)} = \frac{1}{z^2 - 2z + (g+1)}$$

the poles are at $z_i = 1 \pm j\sqrt{g}$



CIFB topology with resonator



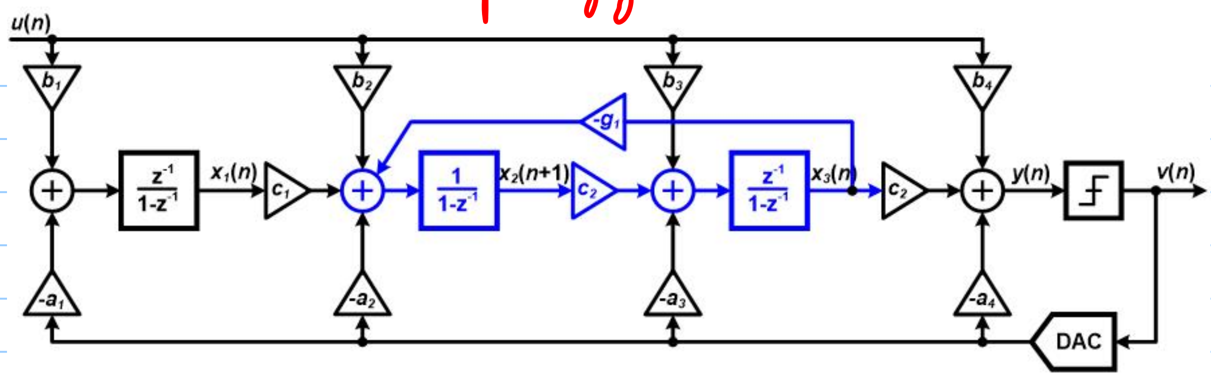
ΔΣ Toolbox function

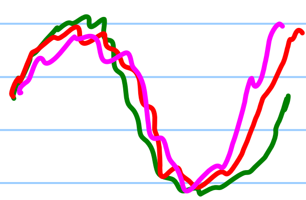
⇒ realizeNTF( ) function

NTF(z) ⇒ a's, b's, c's, g's

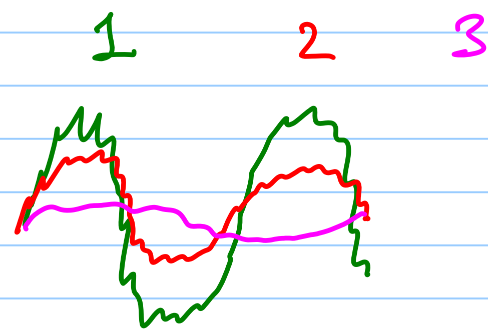
↓ used in DRS.

CRFB Topology





DRS



* c's are used for state scaling using Dynamic Range Scaling