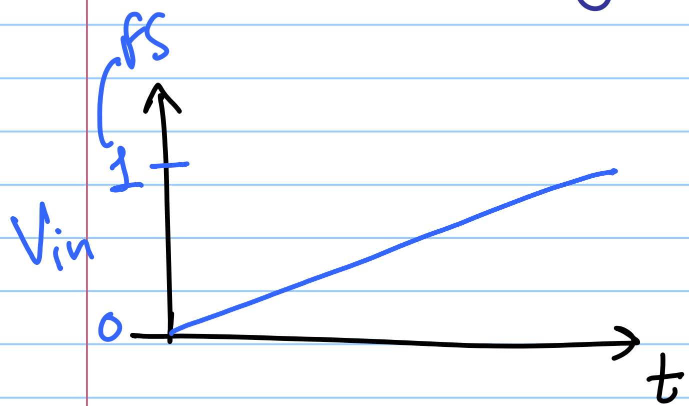
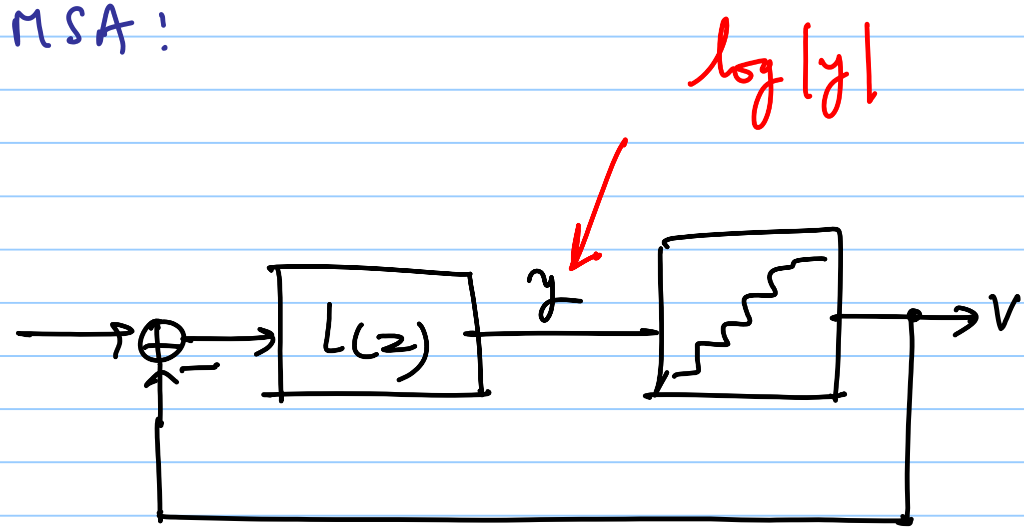
ECE 615 - Lecture 17

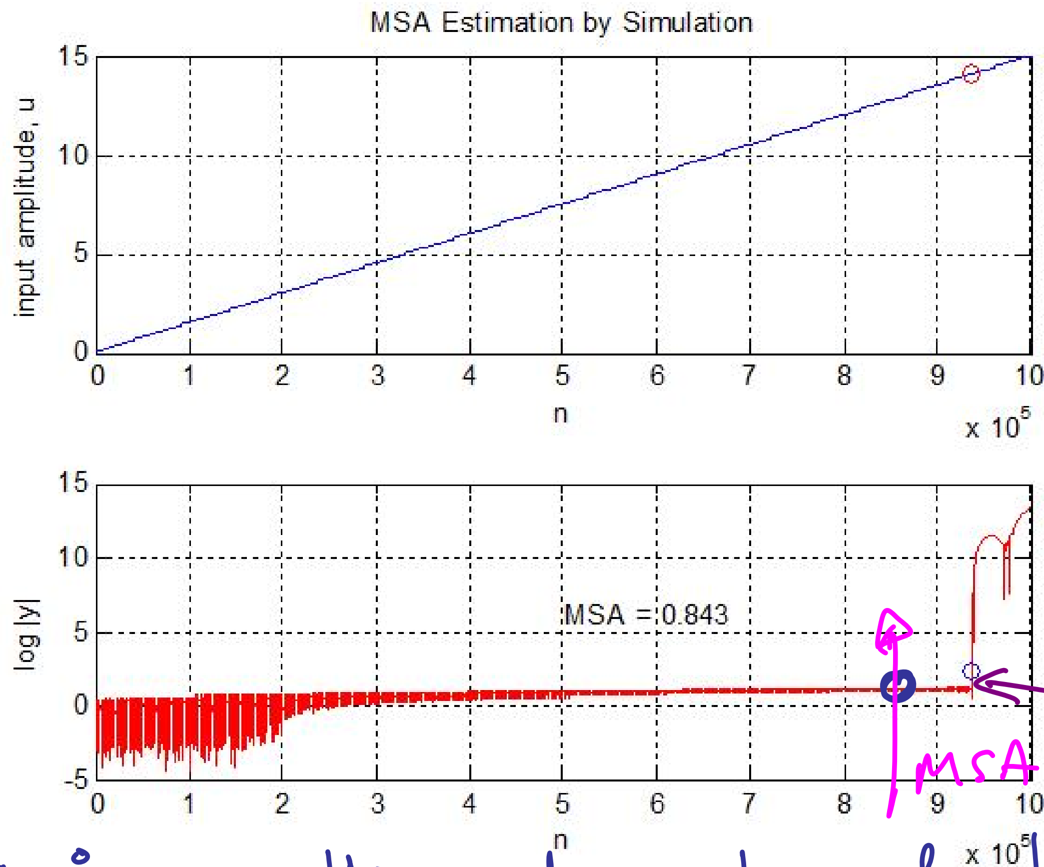
Note Title 3/8/2016

Estimating MSA:





apply very slow ramp

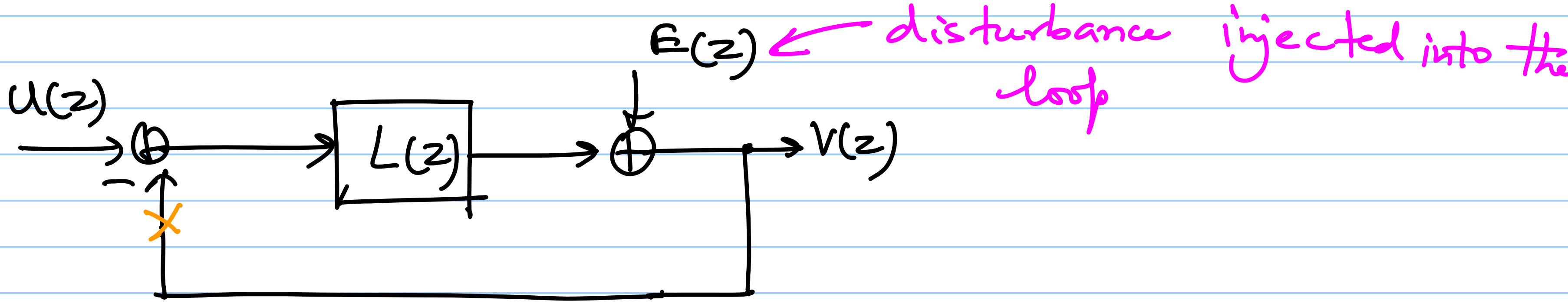


log|y|

log|y| blows up

⇒ Use 90% of this value where log|y| blows up as a conservative estimate for the MSA.

Sensitivity of a feedback loop:



$$V(z) = U(z)\cdot\frac{L(z)}{1+L(z)} + E(z)\cdot\frac{1}{1+L(z)}$$

* The loop rejects the disturbance E at frequencies where the loop-gain is high

loop-gain = L(z)

→ The sensitivity of the loop $\triangleq \dfrac{1}{1+L(e^{j\omega})}$

↳ how effectively the disturbance is suppressed is called the sensitivity of the loop.

↳ the loop is insensitive to the disturbance at low frequencies ∵ $|L(e^{j\omega})|$ is high at low frequencies

* for $\Delta\Sigma$ Modulator sensitivity is the same as the NTF.

* $h[0] = NTF(\infty) = 1$ for realizability

The NTF can be expanded as

complex zeros pair

$$NTF(z) = \frac{(1+a_1 z^{-1})(1+a_2 z^{-1}+a_3 z^{-2})\cdots}{(1+b_1 z^{-1})(1+b_2 z^{-1}+b_3 z^{-2})\cdots}$$

complex pole pair

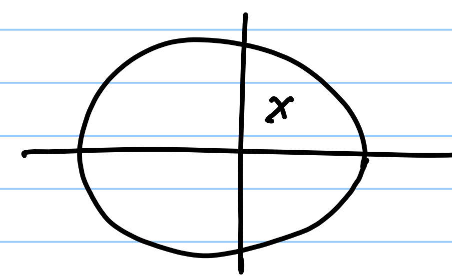
* poles must be within the unit circle

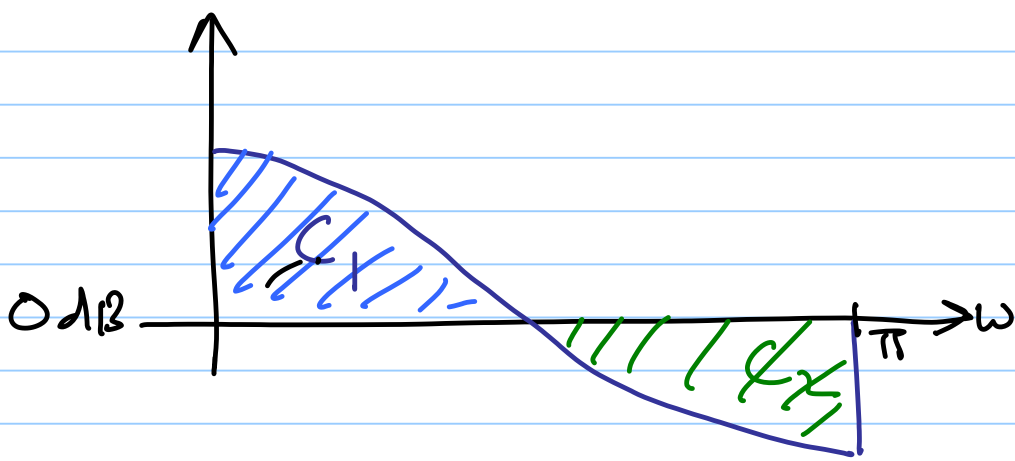
* zeros are inside (on) the unit circle

① It can be shown that

$(1+a_1 z^{-1})$ log magnitude

$$\int_0^{\pi} \log\left|1+a_1 e^{-j\omega}\right| d\omega = 0 \quad \text{if} \quad |a_1| \le 1 \rightarrow ①$$





$C_1 = C_2$

Area above 0dB line = Area below 0dB line

Using ① we can show that

$$\int_0^{\pi} \log \left| 1 + a_2 e^{-j\omega} + a_3 e^{-j2\omega} \right| d\omega = 0 \longrightarrow ②$$

if the roots of $(1 + a_2 z^{-1} + a_3 z^{-2})$ lie within or on the unit circle

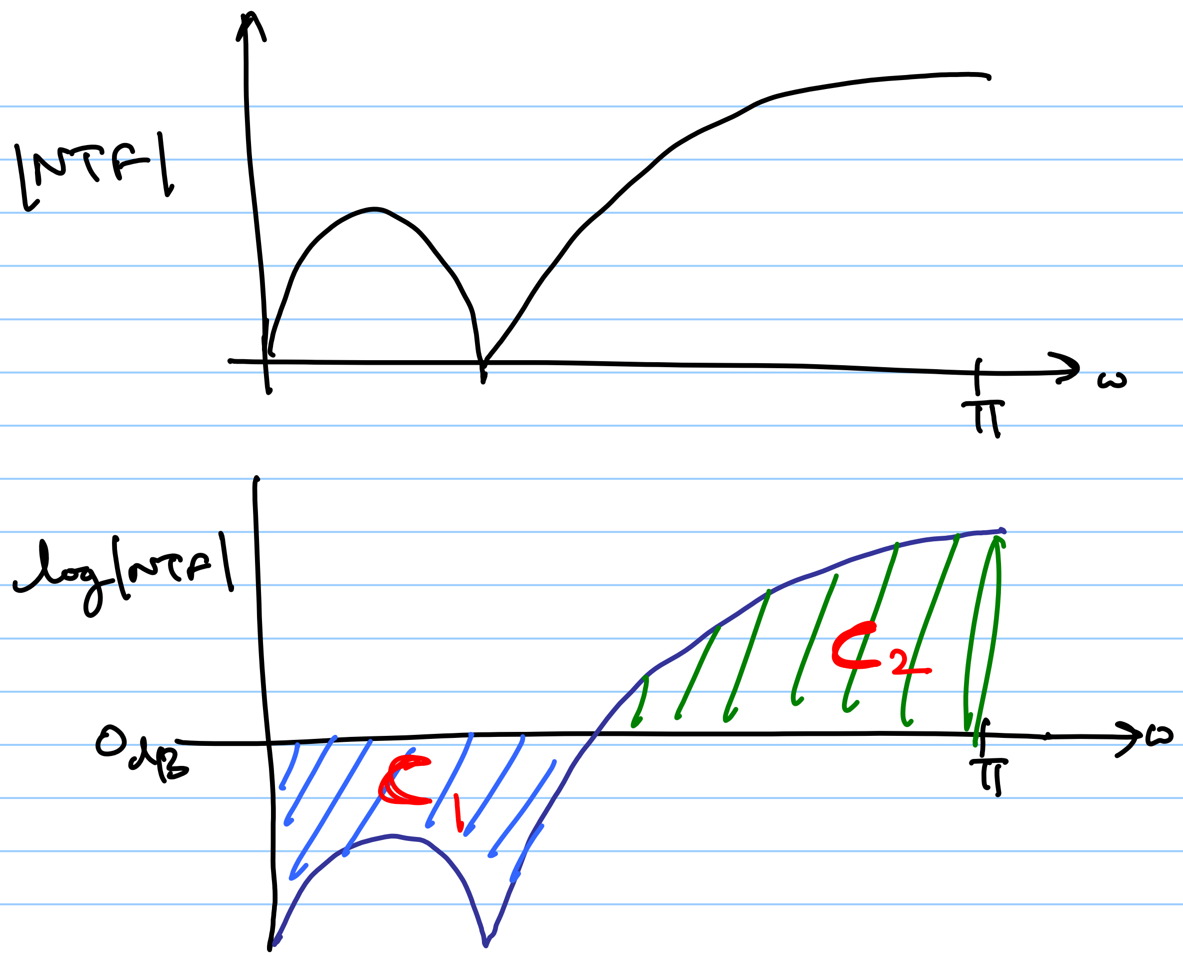
Using ① & ②

$$\int_0^{\pi} \log \left| NTF(e^{j\omega}) \right| d\omega = \int_0^{\pi} \log \left| \frac{(1+a_1 e^{-j\omega})(1+a_2 e^{-j\omega} + a_3 e^{-j\omega}) \cdots}{(1+b_1 e^{-j\omega})(1+b_2 e^{-j\omega} + b_3 e^{-j2\omega}) \cdots} \right| d\omega$$

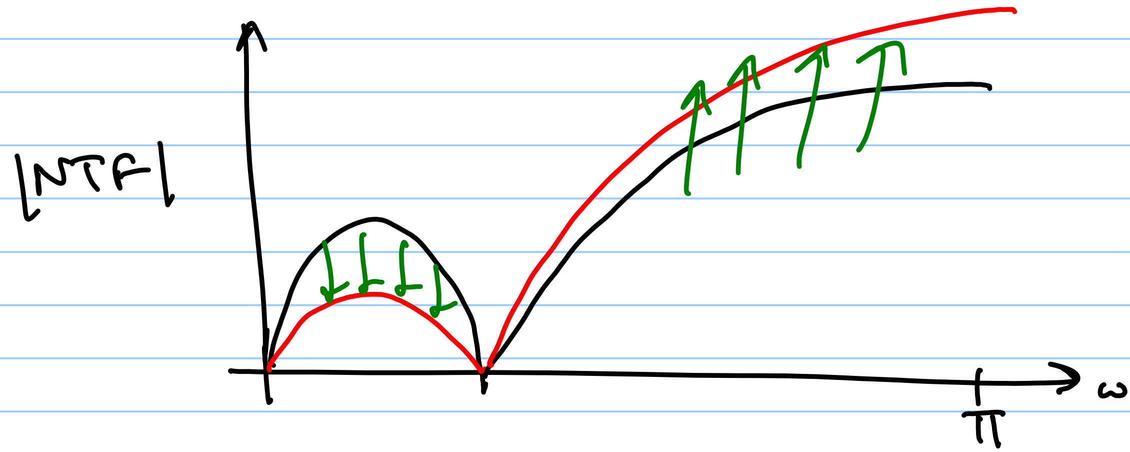
$$= ?$$

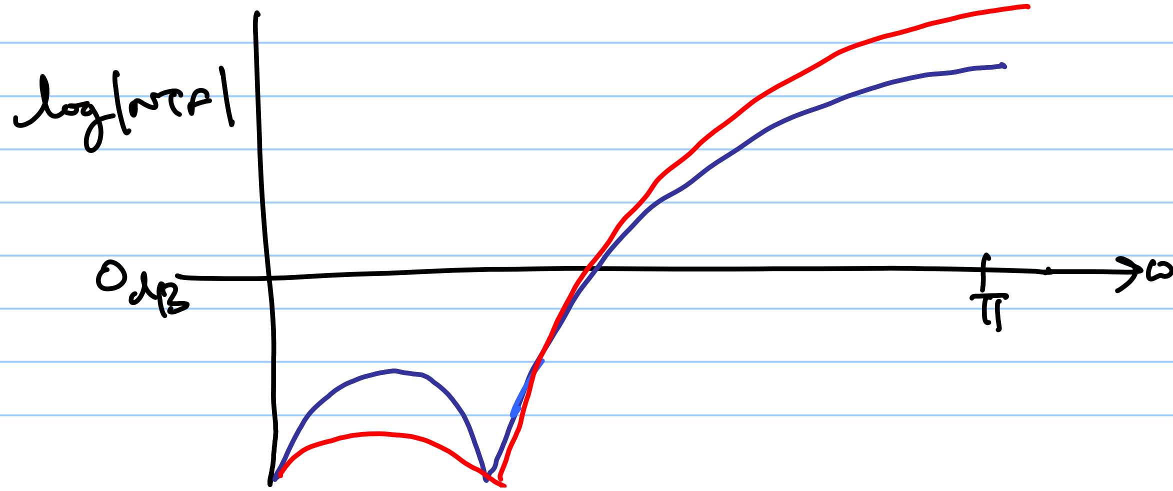
$$\boxed{\int_0^{\pi} \log \left| NTF(e^{j\omega}) \right| d\omega = 0}$$ ← Bode's Sensitivity Theorem

⇒ The integral of the log magnitude of a stable NTF = 0



$C_1 = C_2$



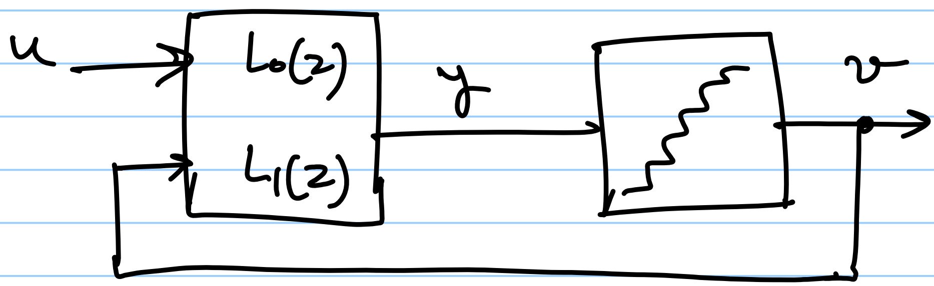


$C_1 = C_2$

"good inband performance comes at the expense of poor out of band performance"

⇒ tradeoff between IBN and OBG.

$\Delta\Sigma$ Modulator Architectures:



$$NTF(z) = \frac{1}{1 - L_1(z)}$$

$$STF(z) = \frac{L_0(z)}{1 - L_1(z)}$$

* $L_1(z)$ has high gain in the signal band

  $|NTF| \simeq \frac{1}{|L_1|}$ in the signal band

* $L_0(z)$ must also be large in the signal band in order to keep $|STF| \simeq 1$

  $|STF| \simeq \frac{|L_0|}{|L_1|} \simeq 1$ in the signal band

$$L_0(z) \triangleq \frac{N_0(z)}{D_0(z)}$$

$$L_1(z) \triangleq \frac{N_1(z)}{D_1(z)}$$

$$NTF(z) = \frac{1}{1 - L_1(z)} = \frac{1}{1 - \frac{N_1(z)}{D_1(z)}}$$

$$= \frac{D_1(z)}{D_1(z) - N_1(z)}$$

* poles of $L_1(z)$ $\Rightarrow$ zeros of $NTF(z)$ $\leftarrow$ Important

* $NTF(z)$ and $STF(z)$ share the same poles
  - $\hookrightarrow$ i.e. the roots of $1 - L_1(z) = 0$
  - $\hookrightarrow$ zeros of $L_0(z)$ may cancel some poles in $D_0(z)$

Contrived Example:

* $STF(z) = z^{-k}$ $\qquad$ $NTF(z) = (1-z^{-1})^N$

$|STF| = 1$

$$L_0(z) = \frac{STF(z)}{NTF(z)} = z^{-k}(1-z^{-1})^{-N} = \frac{z^{N-k}}{(z-1)^N}$$

$\rightarrow$ N poles at $z=1$

$(N-k)$ zeros at $z=0$

$$L_1(z) = \frac{1}{NTF(z)} - 1 = (1-z^{-1})^{-N} - 1 = \frac{1 - (1-z^{-1})^N}{(1-z^{-1})^N}$$

$$= \frac{z^N - (z-1)^N}{(z-1)^N}$$

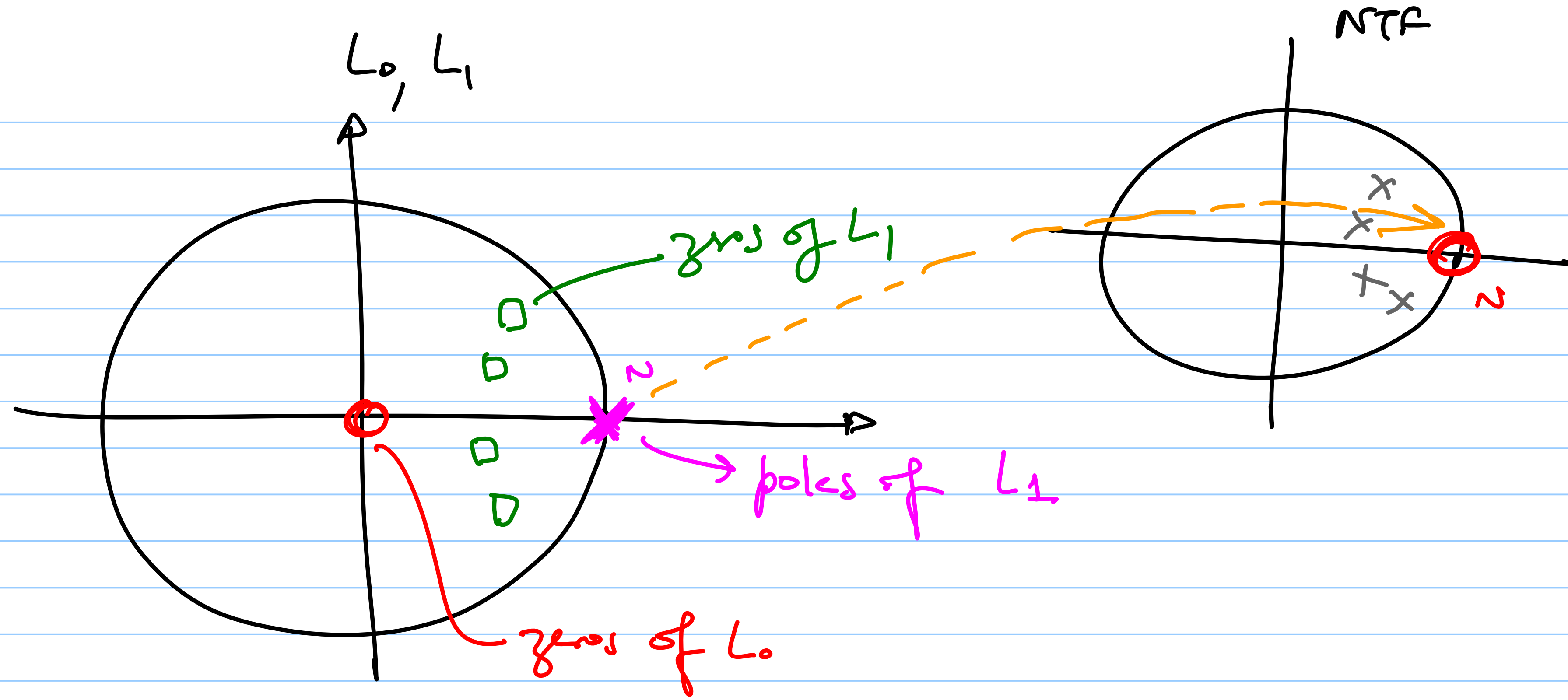
$\hookrightarrow$ N-poles at $z=1$ $\Longrightarrow$ N NTF zeros at $z=1$

zeros of $L_1(z)$: roots of

$$(1-z^{-1})^N = 1 = e^{j2\pi}$$

$$\Rightarrow \quad z_i = \frac{1}{1-e^{j\frac{2\pi i}{N}}} = \frac{1}{2}\left[1 + j\cot\left(\frac{\pi i}{N}\right)\right]$$

$i = 1, 2, \cdots N-1$



L_0, L_1
zeros of L_1
poles of L_1
zeros of L_0
NTF