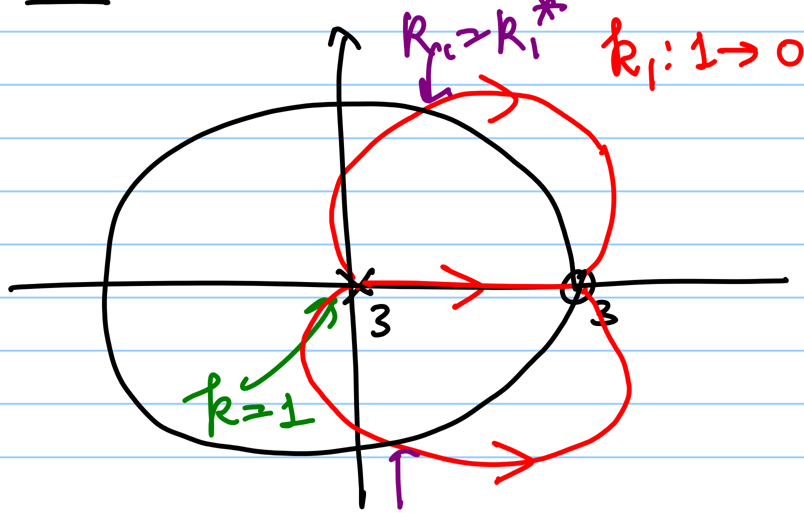
ECE 615 - Lecture 14

Note Title 2/25/2016

$$NTF'(z) = \frac{NTF(z)}{k_1 + (1-k_1)NTF(z)} = \frac{L_1(z)}{1-k_1 L_1(z)}$$

Ex $\quad NTF(z) = (1-z^{-1})^3 \Rightarrow$ 3 zeroes at $z=1$
3 poles at $z=0$



as quantizer overload increases

$k_1 : 1 \longrightarrow 0$

at $k_1 = 0$; poles move to $z=1$

$\Rightarrow$ complete instability

As $k_1 = k_1^*$, poles move out of $|z|=1$ and NTF becomes unstable

* poles leave the unit circle only for NTF of order 3 or higher

Higher-order $\Delta\Sigma$ modulators

* signal dependent stability
* gain falls off as the quantizer overloads

Noise at the quantizer input $= (NTF(z) - 1)\, E(z)$

↳ mean square value of noise $= \dfrac{\Delta^2}{12}\left[\| h[n] \|_2 - 1 \right]$

$\sum_n h^2[n]$

Aside

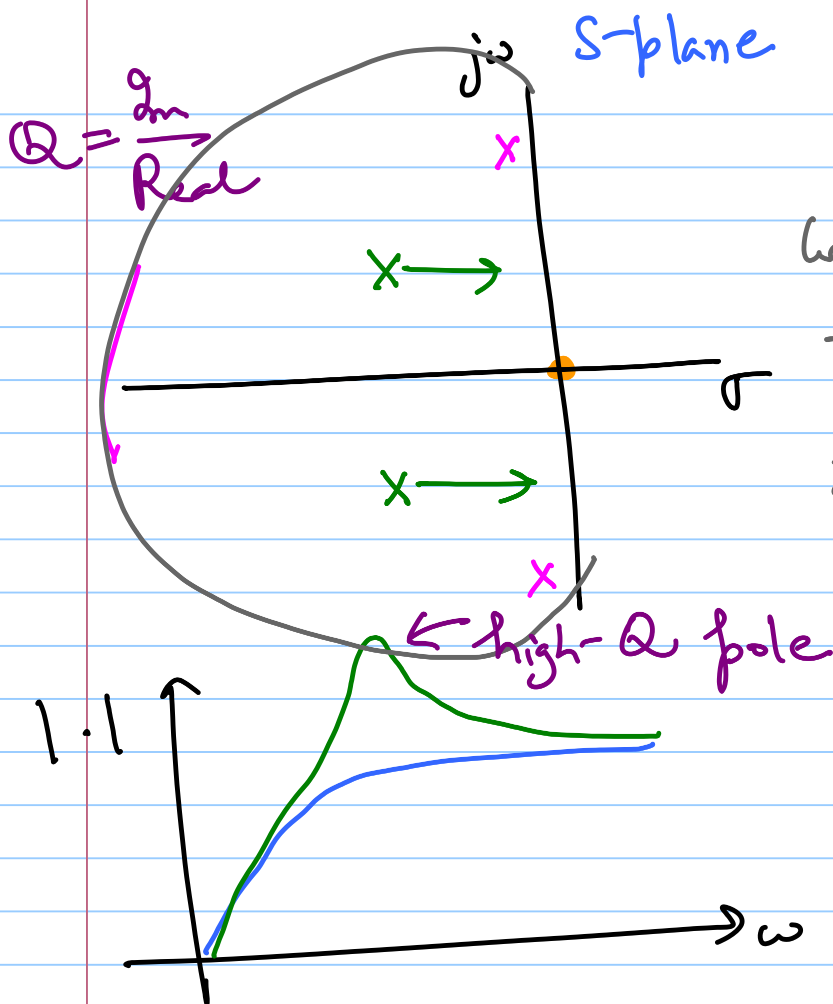
$$\|h[n]\|_1 = \sum_n |h[n]|$$

$$\|h[n]\|_2 = \sum_n h^2[n]$$

The $\Delta\Sigma$ modulator 'destabilization' sequence:

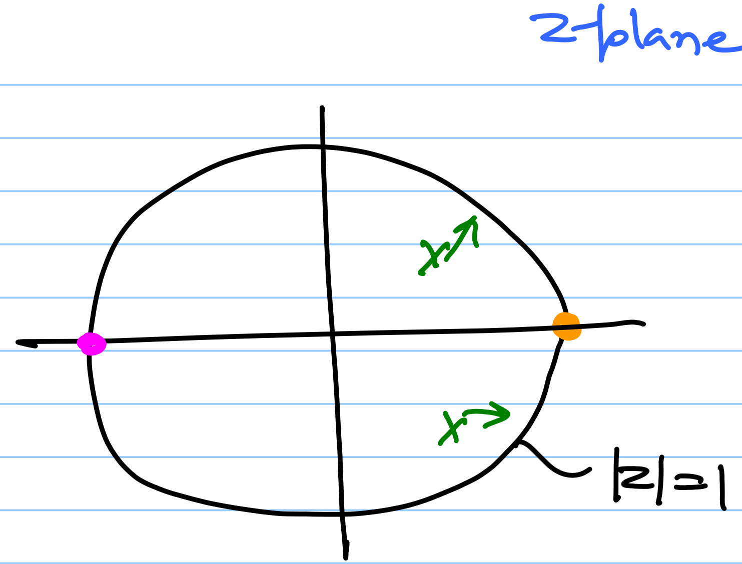
① As the input DC level is increased, the gain (k_1) for the noise falls

② As 'k_1' drops, poles of the NTF start moving towards the $j\omega$ axis (unit-circle)



Conformal Mapping

$z = e^{jsT_s}$

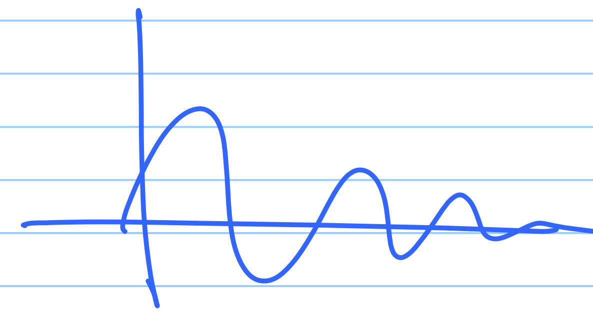


As k_1 drops, Q-factor of the poles increases

③ As the Q-factor of the poles increases

↳ length of $h[n]$ increases

↳ $\sigma_q^2 = \frac{\Delta^2}{12}(\|h\|_2 - 1)$ increases



⇒ quantization noise variance at 'y' increases

④ Clipping of the output waveform occurs more frequently ⇒ R_1 drops further

⇒ poles move faster towards unit circle

⇒ noise variance increases further

⑤ System becomes unstable

↳ state variables become unbounded

↳ SNR drops down

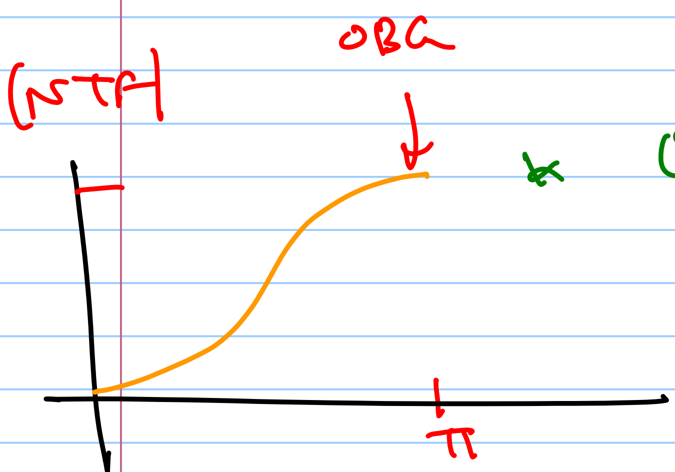
* Very rapid runaway to instability.

As OBG increases, the variance of noise is higher

↳ variance of noise is higher

↳ Quantizer overloads more often

$\Rightarrow$ Maximum Stable Amplitude (MSA) decreases at higher OBG



$\propto$ $OBG = |NTF(z=-1)|$ $= \sum_n |h[n]| = \|h\|_1$

$|NTF(e^{j\omega})|_{\omega=\pi}$

naturally $\|h(n)\|_2$ will be large for OBG $\uparrow$

* Variance of quantization noise at the quantizer input

order

$$= \frac{\Delta^2}{12}\left(\sum_n h^2[n] - 1\right) = \begin{cases} \frac{5\Delta^2}{12} & \text{for } N=2 \\ \frac{19\Delta^2}{12} & \text{for } N=3 \\ \frac{69\Delta^2}{12} & \text{for } N=4 \end{cases}$$

$$\underline{NTF(z) = (1-z^{-1})^N}$$

* Maximum stable Input Amplitude (MSA) ⇒ Umax in the Toolbox

MSA < FS of the quantizer

As the NTF order ↑ ⇒ OBG ↑ ⇒ MSA ↓↓

Multi bit Modulators

* If the number of levels in the quantizer is increased
  - ⇒ LSB size decreases $\left(\Delta = \frac{FS}{2^n}\right)$
  - ⇒ amplitude and variance of the quantizer noise at "y" goes down
  - ⇒ can allow larger input signal amplitude without overloading the quantizer
  - ⇒ MSA increases ! (better stability for the same order)

* A binary (single-bit) $\Delta\Sigma$ modulator with $NTF(z)$ is likely to be stable if $\leftarrow$ Lee Criterion

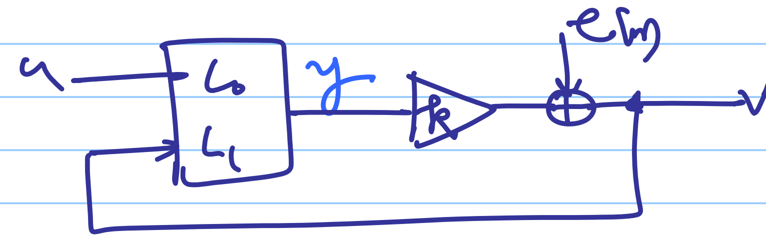
$$\underbrace{\max_{\omega} \left| NTF(e^{j\omega}) \right|}_{OBG} < 1.5 \quad \text{for single-bit modulators}$$

OBG < 1.5 for single-bit

↳ criterion is neither sufficient nor necessary

* $\max_{\omega} |H(e^{j\omega})|$ is also called the infinity-norm denoted by $\|H\|_\infty$

Note:



$$k = \frac{E[|y|]}{E[y^2]}$$

curve-fit for single-bit quantizer, $k \neq 1$

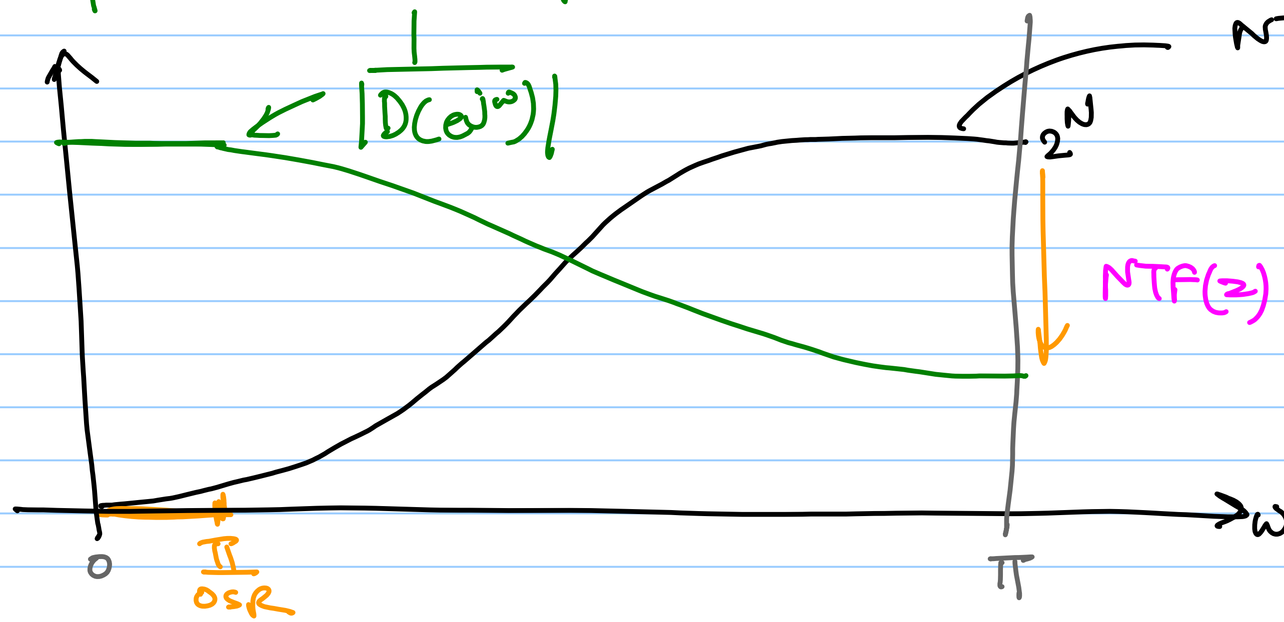
'k' is found by simulation.

Systematic NTF Design Procedure

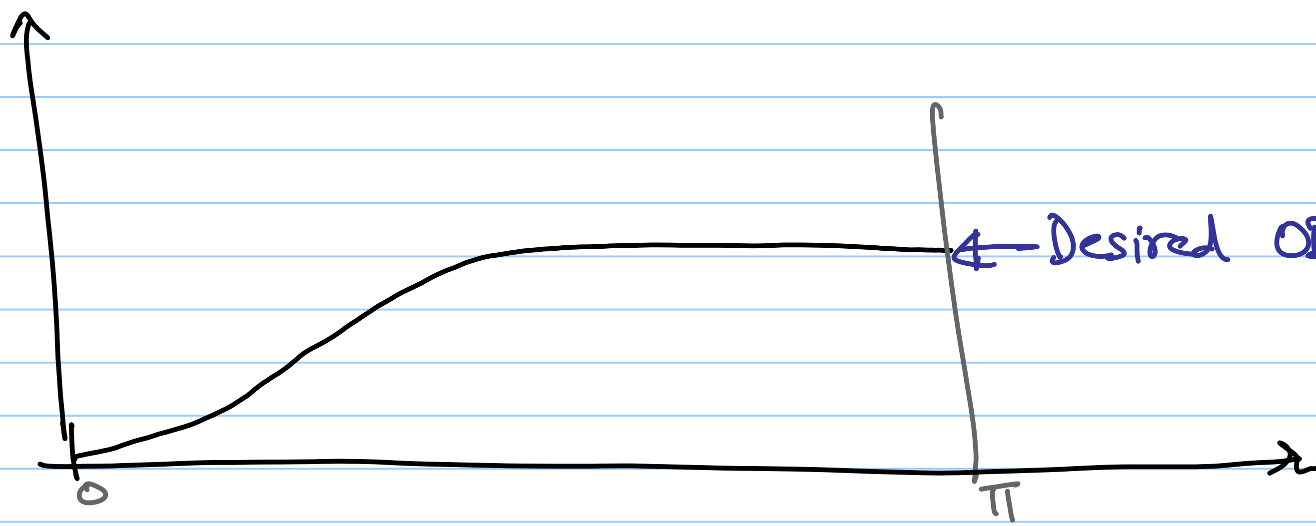
for higher-order NTF, MSA reduces drastically as $OBG = 2^N$

* But we want to limit MSA to 75% to 80% of the full scale amplitude

$$NTF(z) = (1-z^{-1})^N$$



$$NTF(z) = \frac{(1-z^{-1})^N}{D(z)}$$

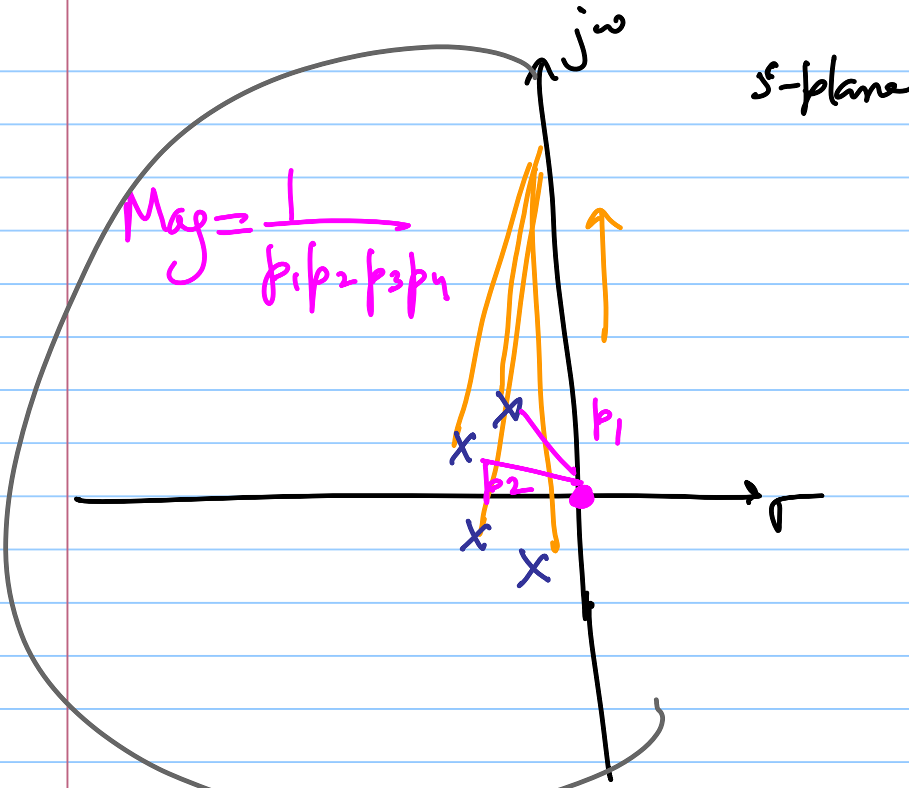


← Desired OBG = 1.5 to 3

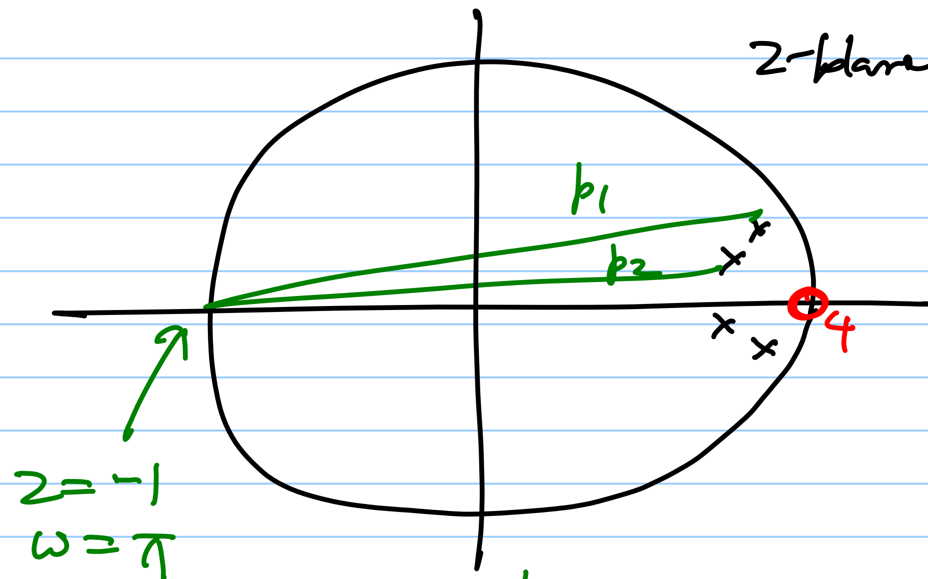
⇒ Introduce poles in the signal band to reduce OBG

↳ Try to keep the same IBN

↳ OBG $\downarrow$ ⇒ $\sigma_q^2 \downarrow$ ⇒ MSA $\uparrow$



$$\text{Mag} = \frac{1}{p_1 p_2 p_3 p_4}$$



$$\text{OBG} = \frac{1}{p_1 p_2 p_3 p_4}$$