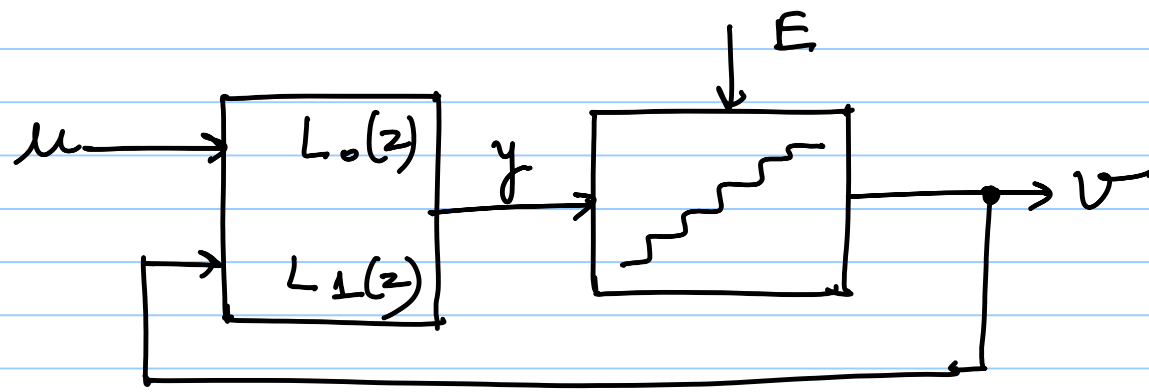
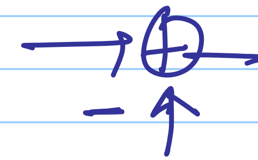
ECE 615 - Lecture 13

Note Title 2/23/2016

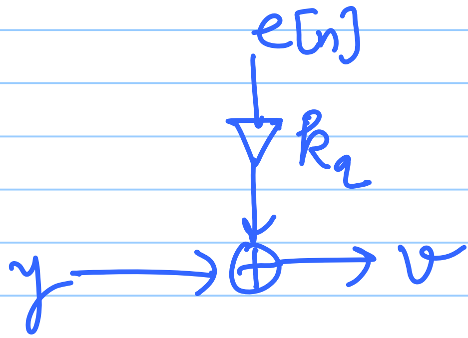


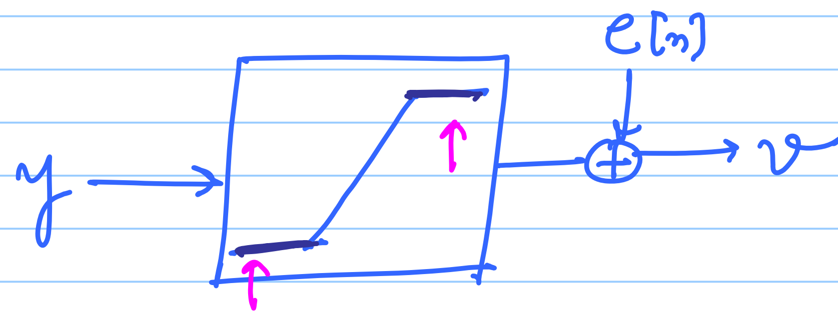


$$V(z) = \underbrace{\frac{L_0(z)}{1-L_1(z)}}_{STF(z)} U(z) + \underbrace{\frac{L_1(z)}{1-L_1(z)}}_{NTF(z)} E(z)$$

minus is absorbed in $L_1(z)$

* We need to understand the effect of quantizer non linearity





* Overload (saturation) of the quantizer causes instability
  - ↳ when 'y' exceeds the range of the quantizer
    - ↳ the output 'v' doesn't change at all
      - ↳ feedback 'breaks down'.

Definition of $\Delta\Sigma$ modulator instability

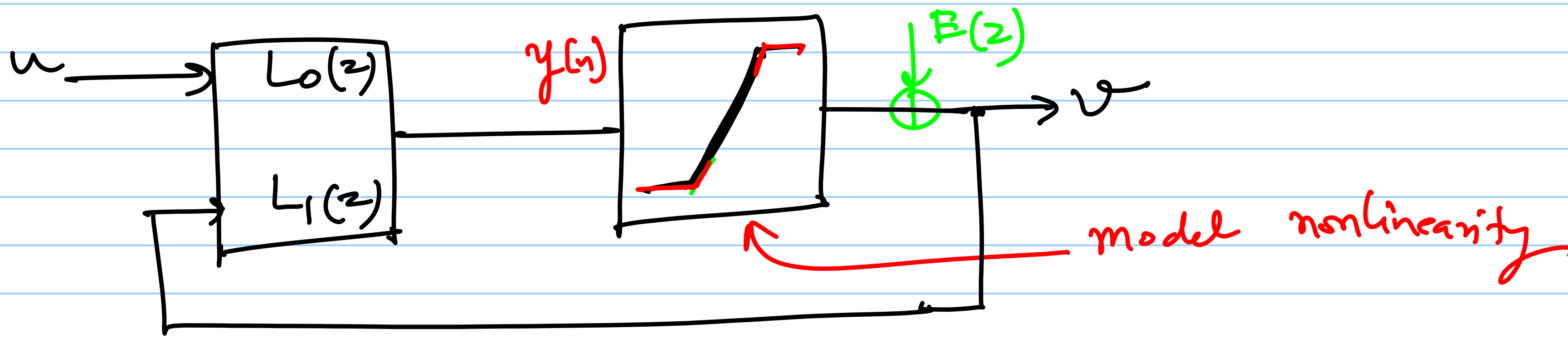
* If the state variables become unbounded for a bounded input, the system is unstable

State variables $\hat{=}$ integrator outputs in the loop-filter.

higher order modulators

* for small input amplitude $\Rightarrow$ system is stable

* when 'u' is increased in amplitude,
quantizer output gets 'clipped' at the max
or the min
↳ or it wildly oscillates b/w the two
↳ loop-filter output y[n] blows
up when unstable.



$$h[n] \xleftrightarrow{\mathcal{Z}} NTF(z)$$

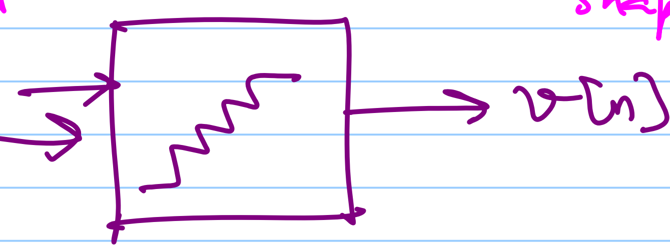
$$V(z) = Y(z) + E(z)$$

$$Y(z) = V(z) - E(z)$$

$$= NTF(z)\cdot E(z) - E(z) + STF(z)\cdot U(z)$$

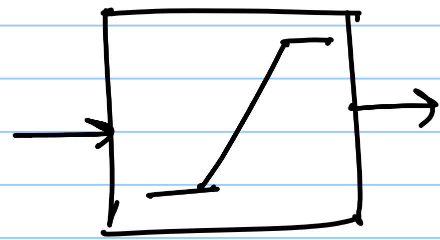
$$= \left(NTF(z) - 1\right)E(z) + \underbrace{STF(z)}_{\simeq 1}\cdot U(z)$$

$$y[n] = \underbrace{u[n]}_{\text{input}} + \underbrace{e[n] \circledast [h[n] - 1]}_{\text{shaped noise}}$$



Describing function method (by Ardalan & Paulos)

$u_1 + n_1$



$v_1 = f(u_1, n_1)$

$\triangleq k_0 u_1 + k_1 n_1$

Approx. the quantizer by a two input linear system

$$v_1 = k_0 u_1 + k_1 n_1$$

find gains k_0 & k_1 from simulation data

$$v_1[n] = k_0 u_1[n] + k_1 n_1[n] \quad \text{for 'N' samples}$$

$$\underbrace{[u_1 \quad n_1]}_{N\times 2} \underbrace{\begin{bmatrix} k_0 \\ k_1 \end{bmatrix}}_{2\times 1} = \underbrace{V_1}_{N\times 1}$$

Let $A \triangleq [u_1 \quad n_1]$ be $\mathbb{R}^{N\times 2}$, then

$$A \begin{bmatrix} k_0 \\ k_1 \end{bmatrix} = V_1$$

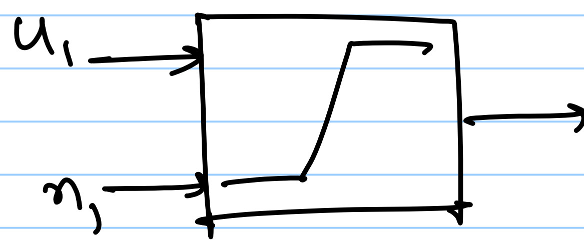
$$\Rightarrow \quad A^T A \begin{bmatrix} k_0 \\ k_1 \end{bmatrix} = A^T V_1$$

$$\Rightarrow \quad \underbrace{(A^T A)^{-1} (A^T A)}_{} \begin{bmatrix} k_0 \\ k_1 \end{bmatrix} = (A^T A)^{-1} A^T V_1$$

LMS fitting

$$\boxed{\begin{bmatrix}\hat{k}_0\\ \hat{k}_1\end{bmatrix} = \underbrace{(A^TA)^{-1}A^T}_{\text{pseudo inverse of } A}\, v_1}$$

error $\quad \left\| v - A \begin{bmatrix}\hat{k}_0\\ \hat{k}_1\end{bmatrix}\right\|_2 = 0 \quad$ if the system is linear



$v_1 = k_0 u_1 + k_1 n_1 +$ curve fitting error

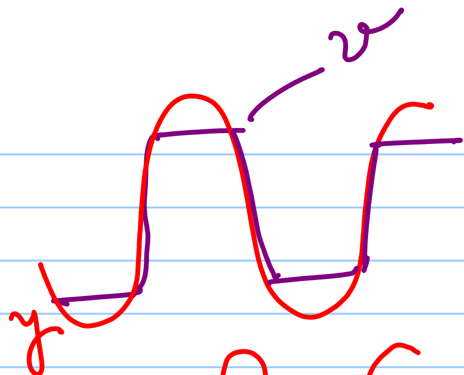
(a) input 'u' is within the quantizer range (no clipping)

$k_0 = k_1 = 1$

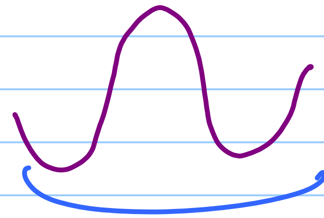
(b) Quantizer output clips

gains k_0 & k_1 are very different

gain for k_1 falls faster than the gain for signal (k_0)

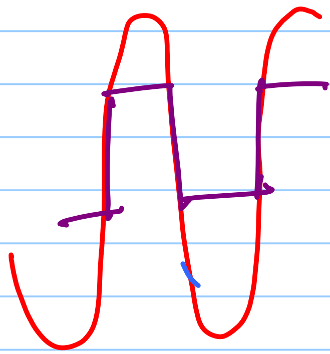


$\Rightarrow$

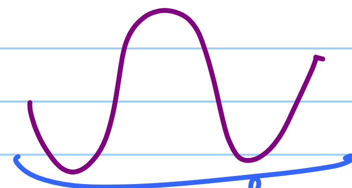


+ harmonics

k_0



$\Rightarrow$

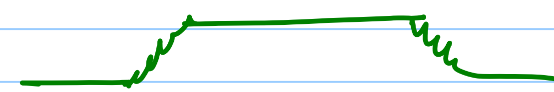


+ harmonics

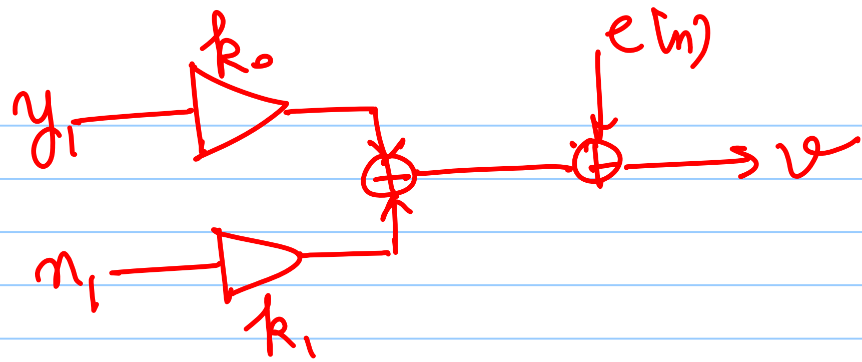
k_0 drops

$e_1[n]$ sees 0 gain

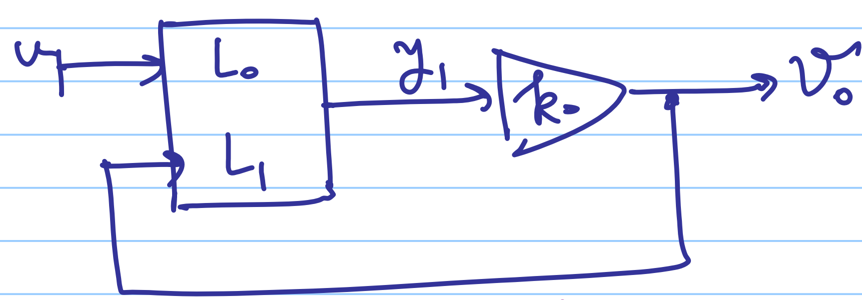
noise has gain on average



k_1 drops faster

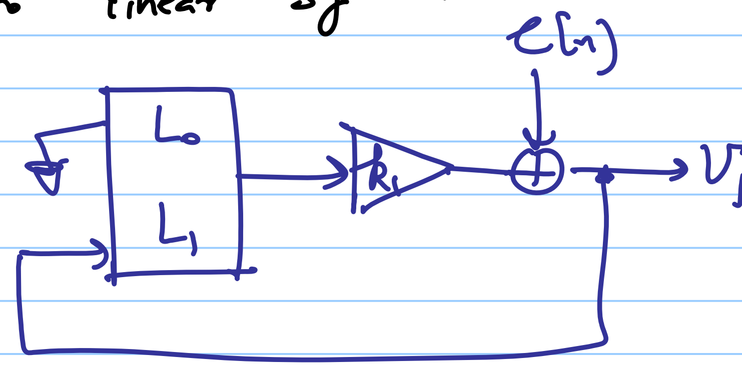


The modulator splits into two linear systems



signal loop

AND

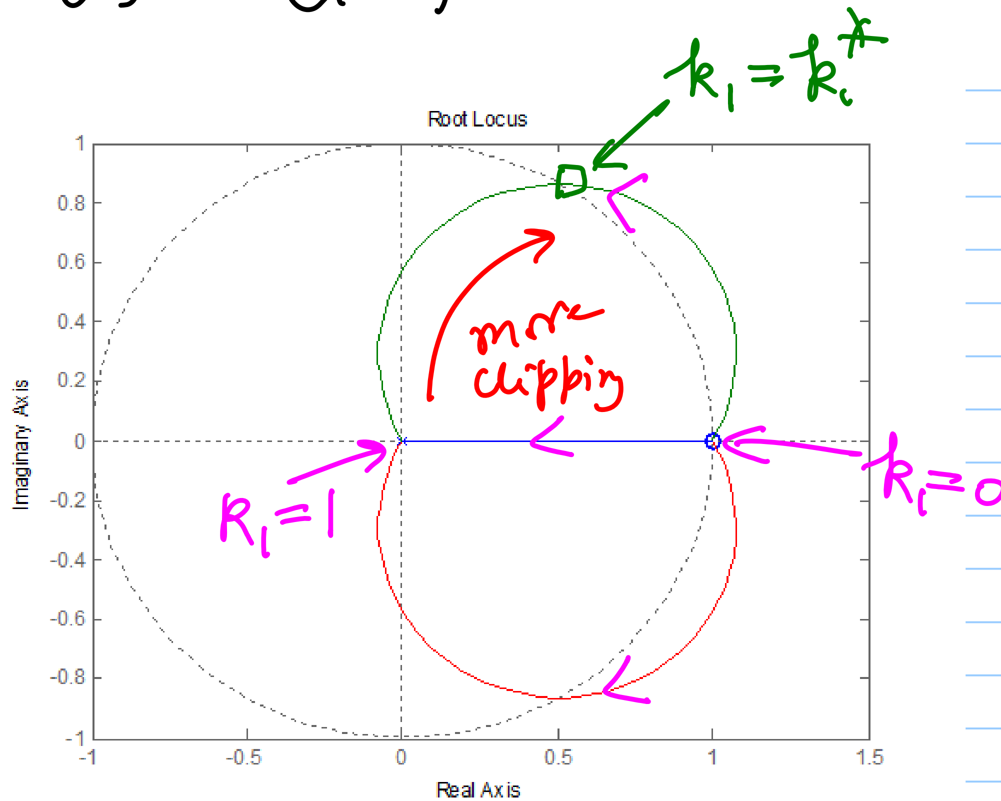


noise loop

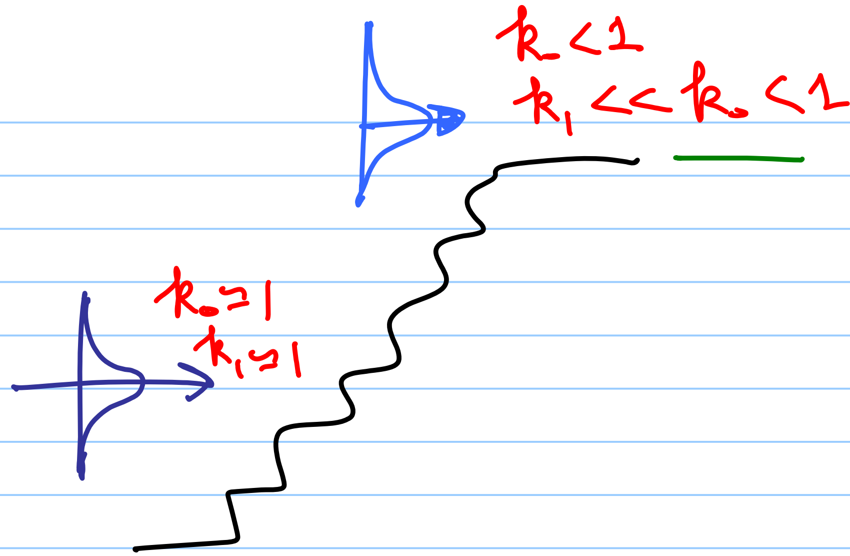
$$NTF'(z) = \frac{L_1(z)}{1 - k_1 L_1(z)} = \frac{NTF(z)}{k_1 + (1-k_1)NTF(z)}$$

$$STF'(z) = \frac{L_0(z)}{1 - k_0 L_1(z)}$$

Ex $NTF(z) = (1-z^{-1})^3$



root locus for 'k_1'


$k_o < 1$
$k_1 << k_o < 1$
$k_o \simeq 1$
$k_1 \simeq 1$