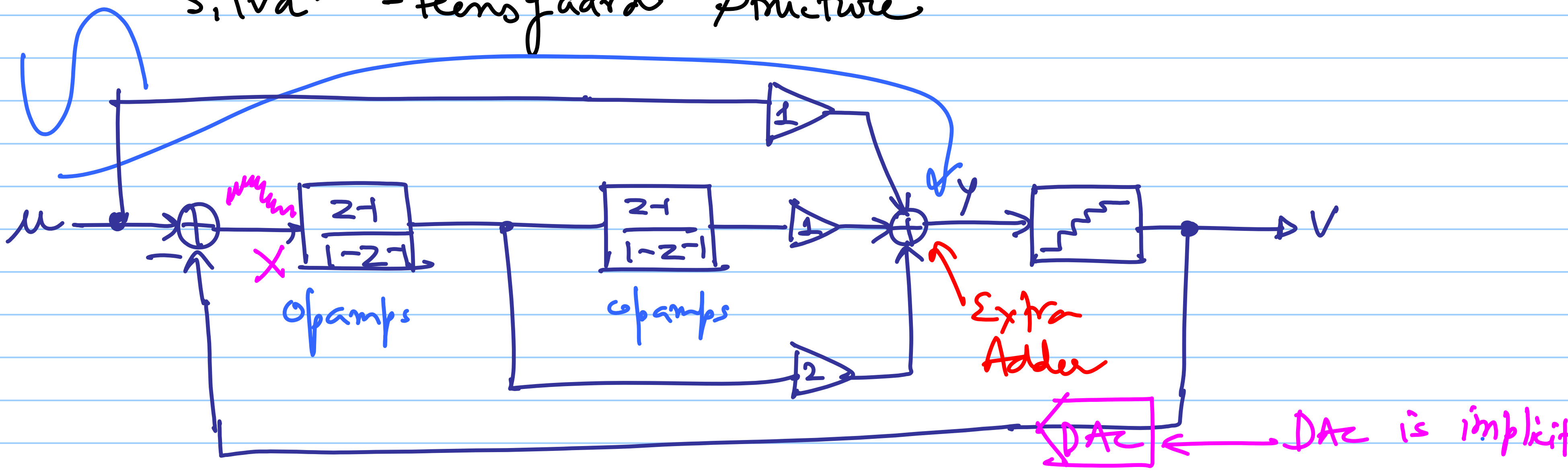
ECE 615 - Lecture 12

Note Title 2/18/2016

Silva - Steensgaard Structure



u(z) sees no delay

$$V(z) = U(z) + (1-z^{-1})^2 E(z)$$

$X(z) = U(z) - V(z)$ ← loop filter input

$= -(1-z^{-1})^2 E(z)$ ~ containing only shaped noise term

(+) ① loop filter only process Quantization Noise

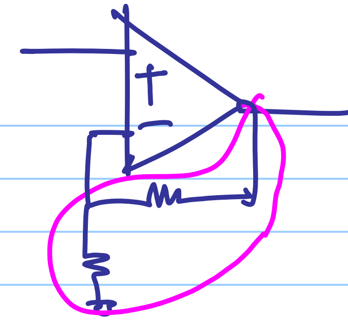
↳ signal swing at X is reduced

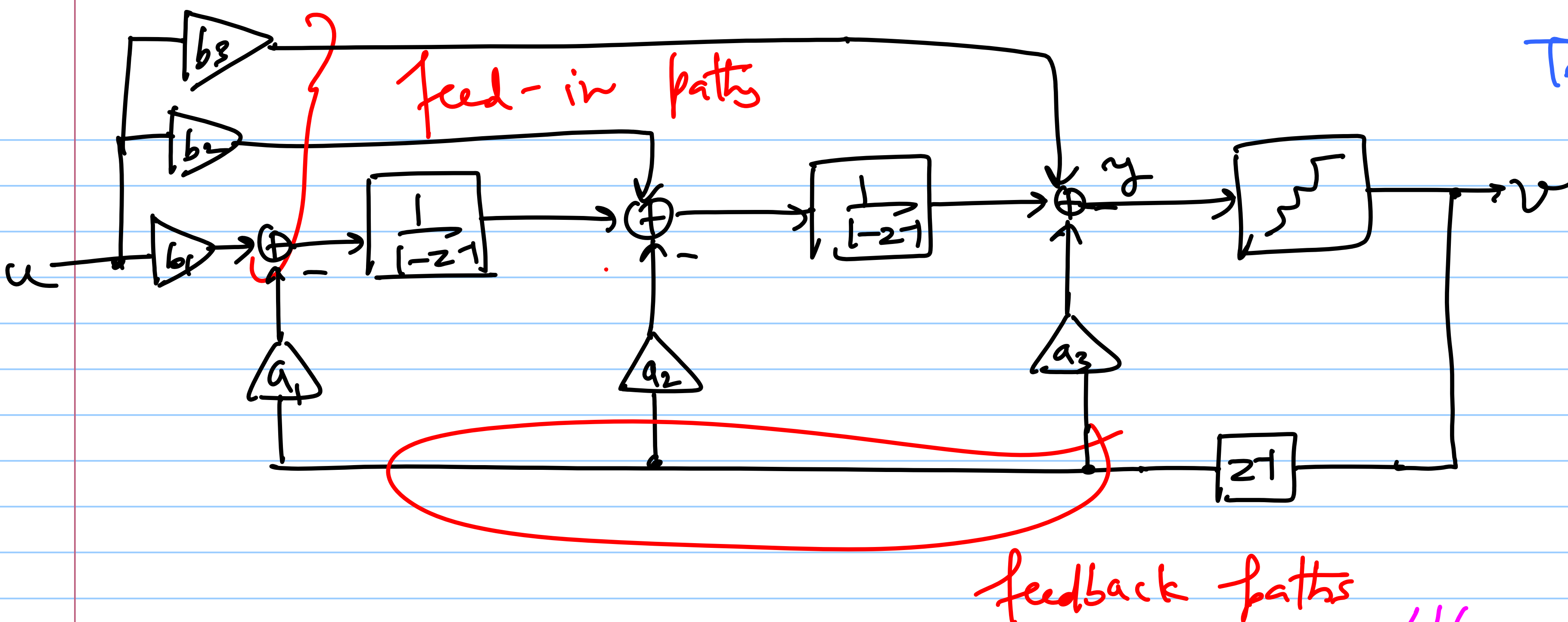
↳ lower SR (∴ linearity) requirement from the opamps

(−) Extra ADDER before the quantizer

↳ passive switched capacitor adder

* The linearity of the feedback DAC is very critical





Textbook
pg 82-85

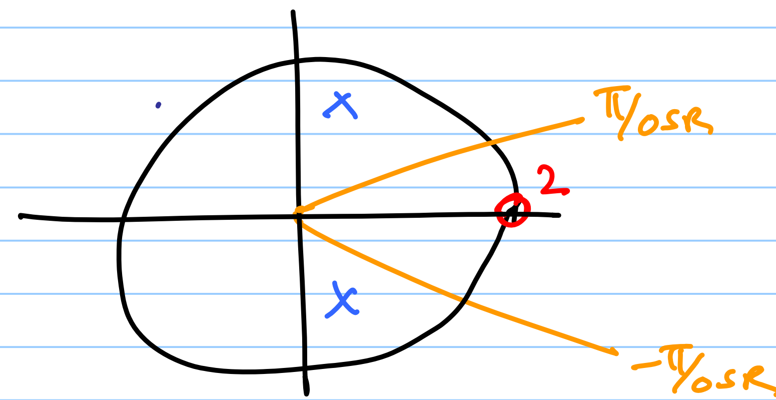
$$NTF(z) = \frac{(1-z^{-1})^2}{\underset{\uparrow \text{ a's}}{A(z)}} \; ; \qquad STF(z) = \frac{\overset{\text{b's}}{B(z)}}{\underset{\text{a's}}{A(z)}}$$

$$B(z) = b_1 + b_2 (1 - z^{-1}) + b_3 (1 - z^{-1})^2$$

$$A(z) = 1 + (a_1 + a_2 + a_3 - 2) z^{-1} + (1 - a_2 - 2a_3) z^{-2} + a_3 z^{-3}$$

$a_3 = 0$ is DT-ΔΣ

$$NTF(z) = \frac{(1 - z^{-1})^2}{A(z)}$$



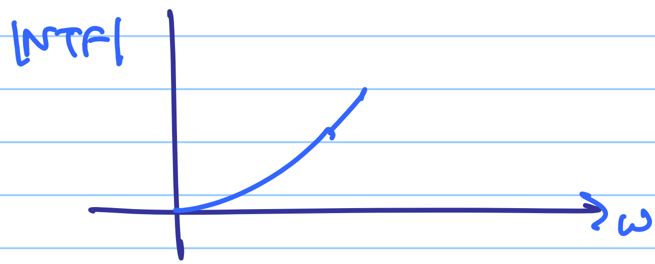
so far

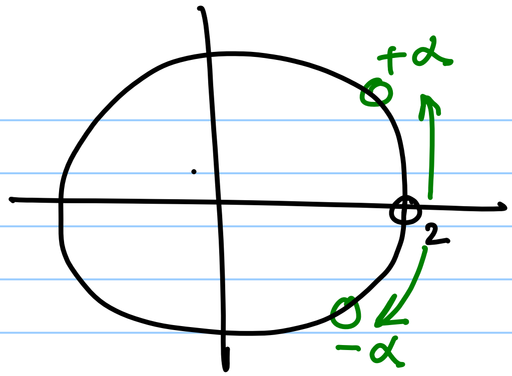
$$|NTF(z)| = \frac{|(1-z^{-1})^2|}{|A(z)|} \simeq \frac{\omega^2}{|A(1)|} \quad \text{for} \quad \omega \ll \pi$$

$|A(1)|$ is the DC gain of $A(z)$

$$= k\omega^2$$

$$\Rightarrow k = \frac{1}{|A(1)|}$$

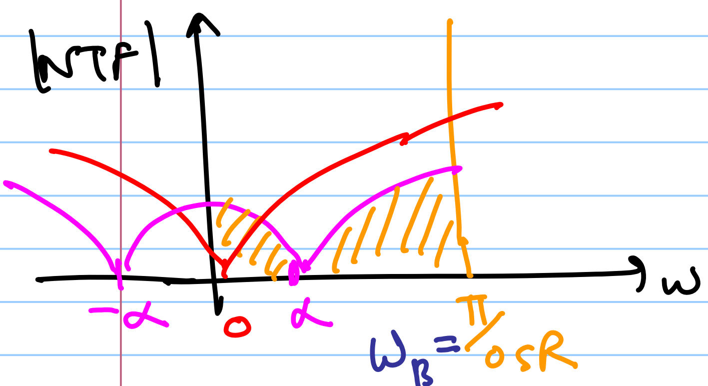




Move the NTF zeros. from $z=1$ to $z = e^{\pm j\alpha}$

$|NTF(z)|$ in the signal band $\because \omega \ll \pi$ $\simeq k(\omega+\alpha)(\omega-\alpha)$

$= k(\omega^2-\alpha^2)$



$$\Rightarrow IBN = \frac{\Delta^2}{12\pi}\int_0^{\omega_B} \left|k(\omega^2-\alpha^2)\right|^2 d\omega$$

$$= \frac{\Delta^2 k^2}{12\pi}\int_0^{\omega_B} (\omega^2-\alpha^2)^2 d\omega = \frac{\Delta^2 k^2}{12\pi}\cdot I(\alpha)$$

The Integral

$$I(\alpha) = \int_0^{\omega_B} (\omega^2 - \alpha^2)^2 \, d\omega$$

For the least IBN, $I(\alpha)$ must be minimized

$$\Rightarrow \frac{d}{d\alpha} I(\alpha) = 0$$

$$\Rightarrow \frac{d}{d\alpha} \int_0^{\omega_B} (\omega^2 - \alpha^2)^2 \, d\omega = 0$$

$$\Rightarrow \int_0^{\omega_B} \frac{d}{d\alpha} (\omega^2 - \alpha^2)^2 \, d\omega = 0$$

$$= \quad 4\alpha \int_0^{\omega_B} (\omega^2 - \alpha^2)\, d\omega = 0$$

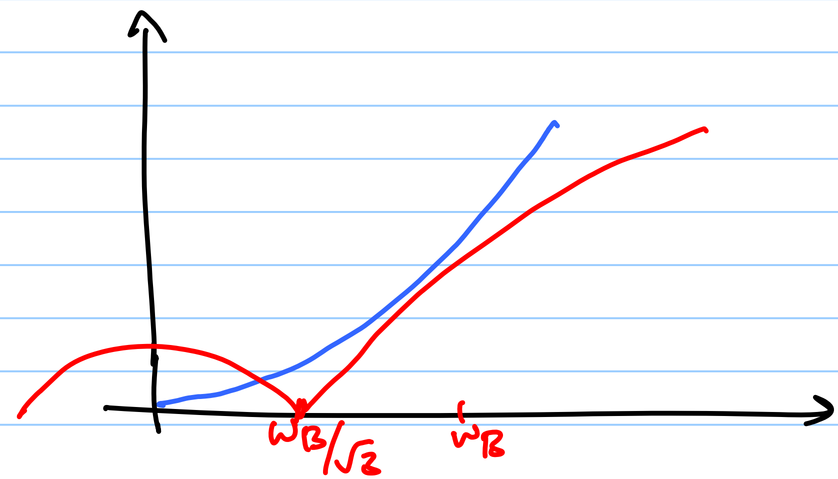
$$\Rightarrow \quad \frac{\omega_B^3}{3} - \alpha^2 \omega_B^2 = 0$$

$$\Rightarrow \quad \boxed{\alpha^* = \frac{\omega_B}{\sqrt{3}}} \qquad \Rightarrow \quad \alpha = \frac{\pi}{OSR} \cdot \frac{1}{\sqrt{3}}$$

$$\frac{I(0)}{I(\alpha^*)} = \frac{9}{4} \Rightarrow$$

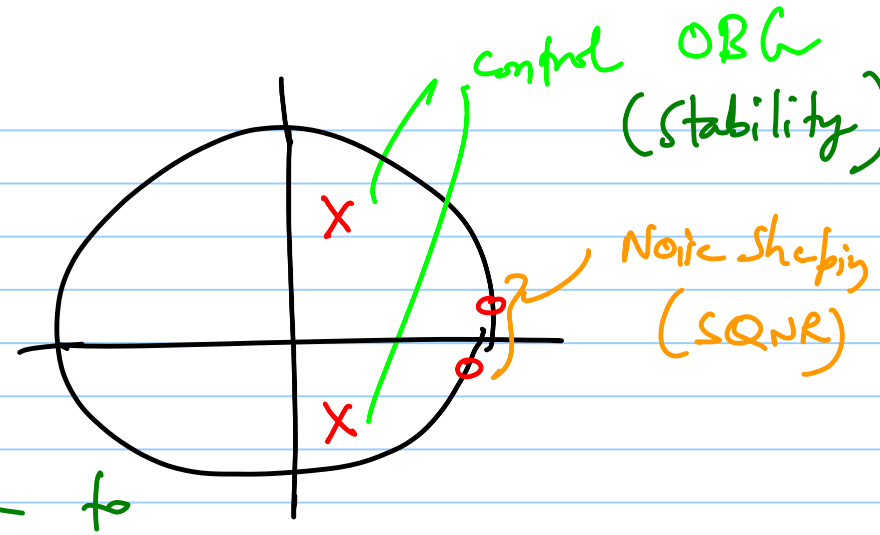
SQNR improvement of $\log_{10}(9/4)$

$= 3.5$ dB



pole locations determine OBG

↳ determines stability → will come back to this later



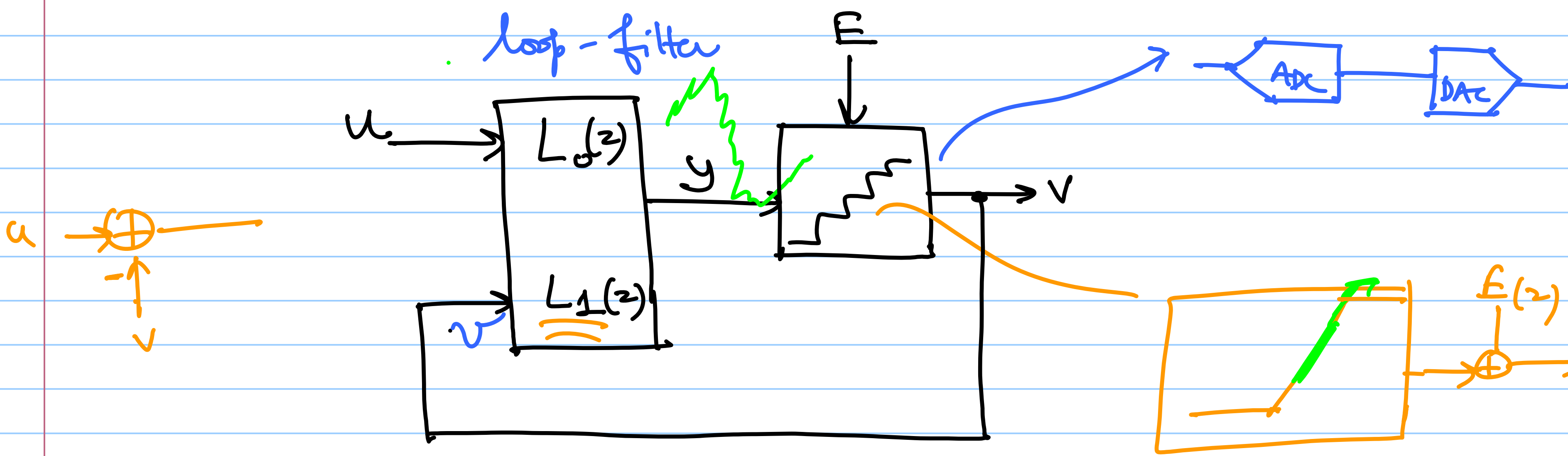
$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

Lets say

$$NTF(z) = (1-z^{-1})^3$$

$$OBG = \sum |h(n)| = 8$$

Describing function Analysis



$$Y(z) = L_0(z)\,U(z) + L_1(z)\,V(z)$$

$$\therefore \quad V(z) = Y(z) + E(z)$$

$$\Rightarrow \quad V(z) = L_0(z)\,U(z) + L_1(z)\,V(z) + \underline{\underline{E(z)}}$$

Linearized model

$$\Rightarrow \quad V(z) = \underbrace{\frac{L_0(z)}{1-L(z)}}_{STF(z)}\,U(z) + \underbrace{\frac{L_1(z)}{1-L_1(z)}}_{NTF(z)} \cdot E(z)$$

minus?

absorbed by $L_1(z)$

$$= STF(z)\,U(z) + NTF(z)\,E(z)$$

$$L_0(z) = \frac{STF(z)}{NTF(z)}, \qquad L_1(z) = \frac{NTF(z)-1}{NTF(z)}$$

Ex. for 2nd order modulator

$$L_0(z) = \frac{1}{(1-z^{-1})^2}$$

$$L_1(z) = \frac{-z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{(1-z^{-1})^2}$$