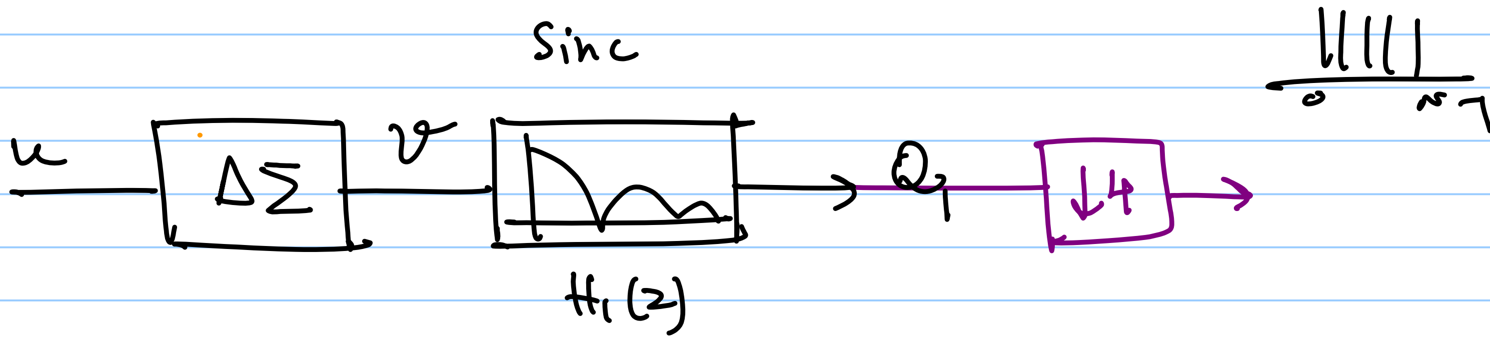
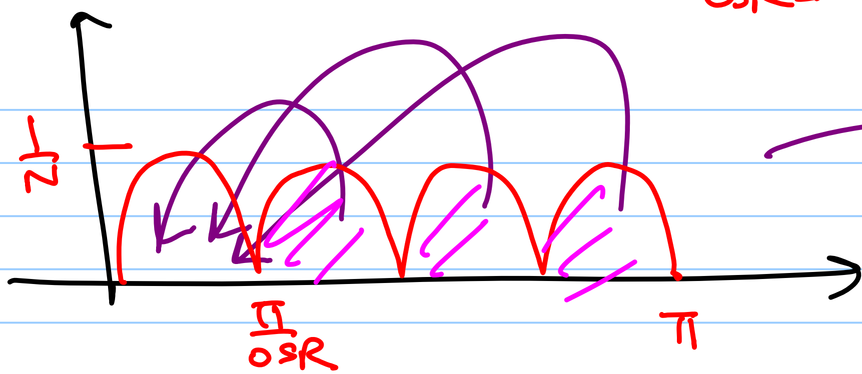
ECE 615- Lecture 11

Note Title 2/16/2016



$$Q_1(z) = H_1(z)\cdot NTF(z)\cdot E(z)$$

$$= \frac{1}{N}\,\frac{(1-z^{-N})}{\cancel{(1-z^{-1})}}\,\cancel{(1-z^{-1})}\,E(z) = \frac{1}{N}\cdot(1-z^{-N})\,E(z)$$



All the noise will fold into baseband after decimation

total output quantization noise power

$$\sigma_{q_1}^2 = \int_0^{\pi} \frac{1}{N^2} \left|1 - e^{-j\omega N}\right|^2 \underline{S_e(\omega)} \cdot d\omega$$

$$= \frac{\Delta^2}{12\pi} \cdot \frac{1}{N^2} \int_0^{\pi} \left|1 - e^{-j\omega N}\right|^2 d\omega$$

$$= \frac{\Delta^2}{12\pi}\cdot\frac{1}{N^2}\underbrace{\int_0^{\pi} 4\sin^2\left(\frac{\omega N}{2}\right)d\omega}_{2\pi}$$

$$= \frac{\Delta^2}{12}\cdot\frac{1}{N^2}\cdot 2$$

$N = OSR$

$$= \frac{\Delta^2}{12}\cdot 2\cdot OSR^{-2} = \underbrace{\sigma_e^2}_{\frac{\Delta^2}{12}}\cdot 2\cdot OSR^{-2}$$

If we used ideal brickwall filter

$$\sigma_q^2 = \frac{\pi^2\Delta^2}{3\times 12}\, OSR^{-3} = \sigma_e^2\cdot\frac{\pi^2}{3}\times OSR^{-3}$$

MOD1 — Sinc filter $\Rightarrow$ OSR^{-2}

$\Rightarrow$ 6dB SQNR $\uparrow$ with 2x OSR

Sinc filter is nearly 'N' times less effective than the ideal LPF.

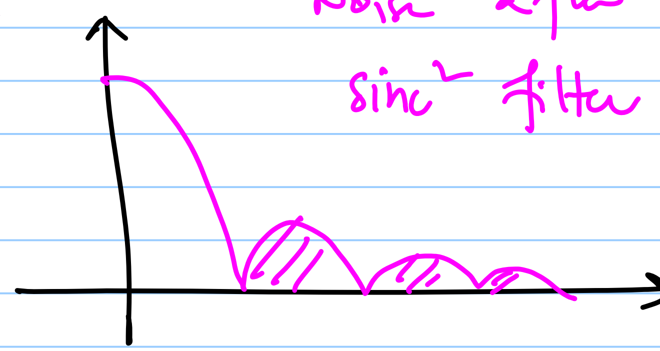
$\Rightarrow$ 1-bit increase for 2x OSR

Sinc² decimation filter

$$H_2(z) = \left[\frac{1}{N}\left(\frac{1-z^{-N}}{1-z^{-1}}\right)\right]^2$$

$$Q_2(z) = E(z)\cdot\cancel{(1-z^{-1})}\cdot\frac{1}{N^2}\frac{(1-z^{-N})^2}{(1-z^{-1})^{\cancel{2}}}$$

$$= \frac{1}{N^2}\frac{(1-z^{-N})^2}{(1-z^{-1})}E(z)$$

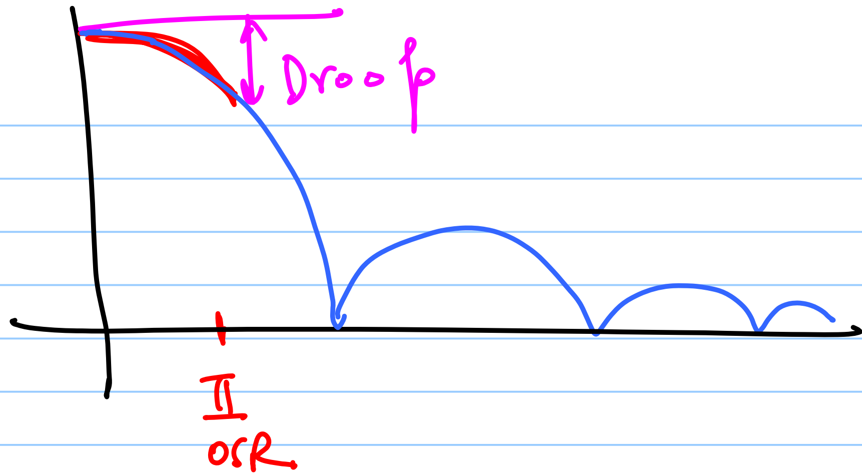


$$\sigma_{q2}^2 = \int_0^{\pi} |H_2(e^{j\omega})|^2 \cdot \frac{\Delta^2}{12\pi} \cdot |1-e^{-j\omega}|^2 \, d\omega = \frac{2N\sigma_e^2}{N^4} = \frac{2\sigma_e^2}{N^3} \qquad \frac{\Delta^2}{12}$$

$$= \underbrace{2\sigma_e^2 \cdot OSR^{-3}}$$

σ_{q2}^2 is still lower than the noise with ideal LPF but better than just a sinc filter

Sinc^2 is sufficient for 1st-order modulator

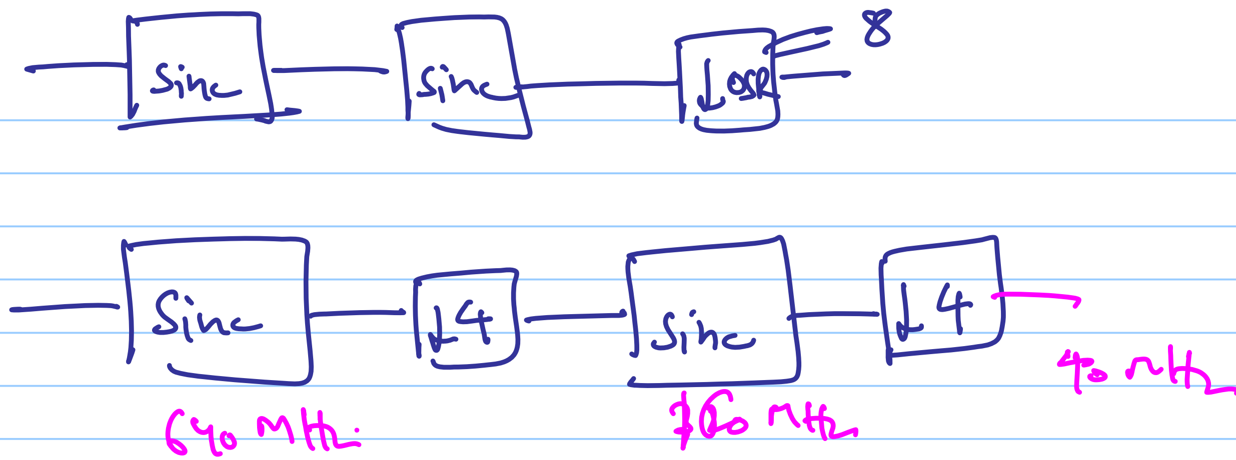


$$\text{Droop} = \frac{1}{OSR \cdot \sin\left(\frac{\pi}{OSR}\right)} \Rightarrow \frac{2}{\pi} \text{ @ } OSR \to \infty$$

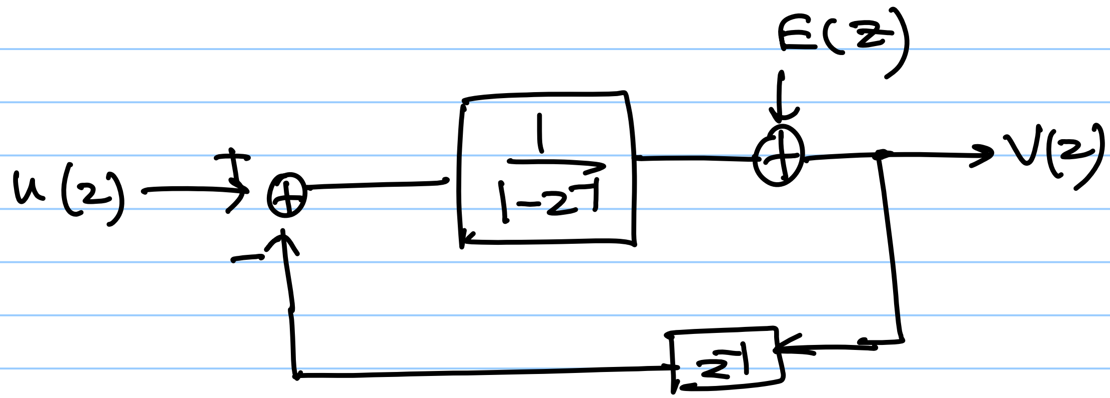
$\Rightarrow$

for L^{th}-order sinc filter

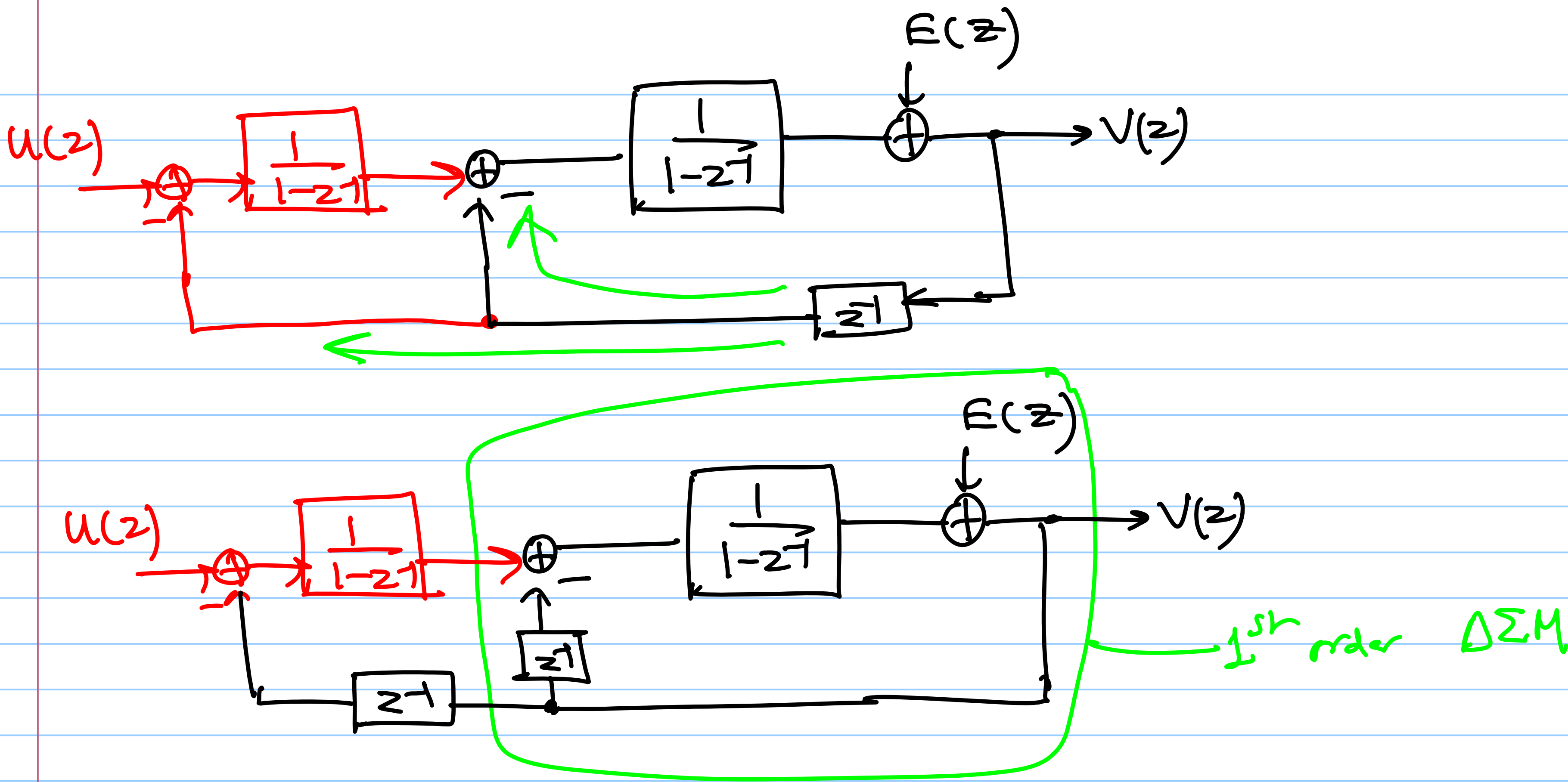
$$\text{Droop} = -3.92 \times L \text{ dB}$$


Sinc
Sinc
↓OSR
8
Sinc
↓4
Sinc
↓4
640 MHz
160 MHz
40 MHz

2nd-order DSM $\Delta\Sigma M$

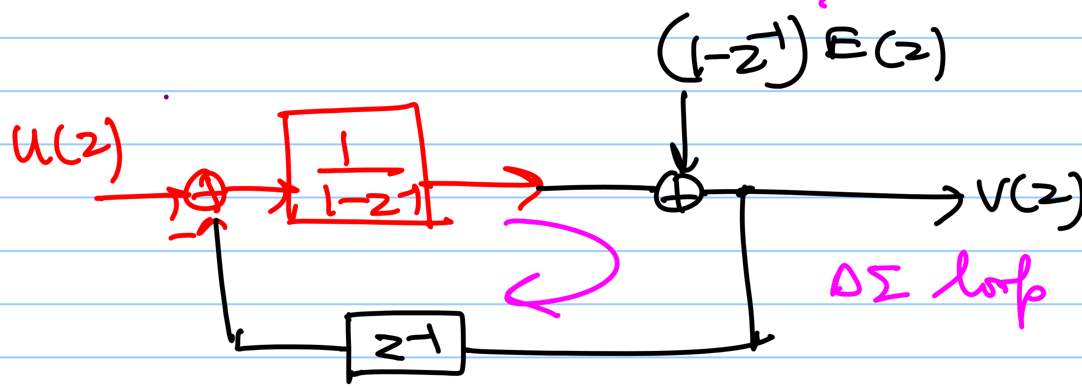


$$V(z) = U(z) + (1-z^{-1})E(z)$$



U(z)
$\frac{1}{1-z^{-1}}$
E(z)
V(z)
z^{-1}
1st order ΔΣM

1st-order shaped noise



$x(n)$

$x(n) - x(n-1)$

$\Rightarrow (1-z^{-1})X(z)$

$$V(z) = U(z) + (1-z^{-1})(1-z^{-1})E(z)$$

$$= U(z) + \underbrace{\boxed{(1-z^{-1})^2}}_{NTF(z)} E(z)$$

Double differentiation of the quantization noise

2nd-order noise shaping

$$\text{IBN} = \frac{\Delta^2}{12\pi} \cdot \int_0^{\pi/OSR} \left|1 - e^{-j\omega}\right|^2 d\omega \simeq \frac{\Delta^2}{12\pi} \cdot \int_0^{\pi/OSR} \omega^4 \, d\omega \quad \text{for } \omega \ll \pi$$

$$= \frac{\Delta^2}{12\pi} \left. \frac{\omega^5}{5} \right|_0^{\pi/OSR} = \boxed{\frac{\Delta^2}{60} \pi^4 \cdot OSR^{-5}}$$

$5 \times 3\text{dB} \uparrow$ in SQNR

$\therefore 15\text{dB} \uparrow$ in SQNR

$\frac{15}{6}$ = increase in ENOB

= 2.5 bits per 2x OSR

Example.

$N_0 = 4$-bit Quantizer $\qquad$ 2^{nd}-order $\Delta\Sigma$

$OSR = 64$

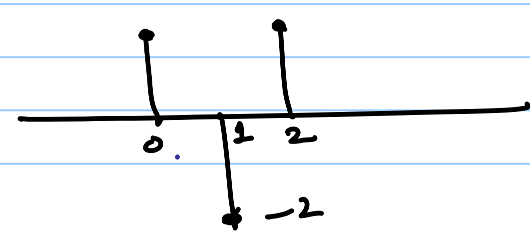
$$N_{inc} = \log_2 OSR \times 2.5 \text{ bits}$$

$$= 15 \text{ bits}$$

$$ENOB = 15 + 4 = 19 \text{ bits}$$

$$NTF(z) = (1-z^{-1})^2 \quad ; \quad \text{impulse response of the NTF}$$

$$= 1 - 2z^{-1} + z^{-2} \qquad h[n] = [1, -2, 1]$$



$$\sum_n h[n] = 0$$

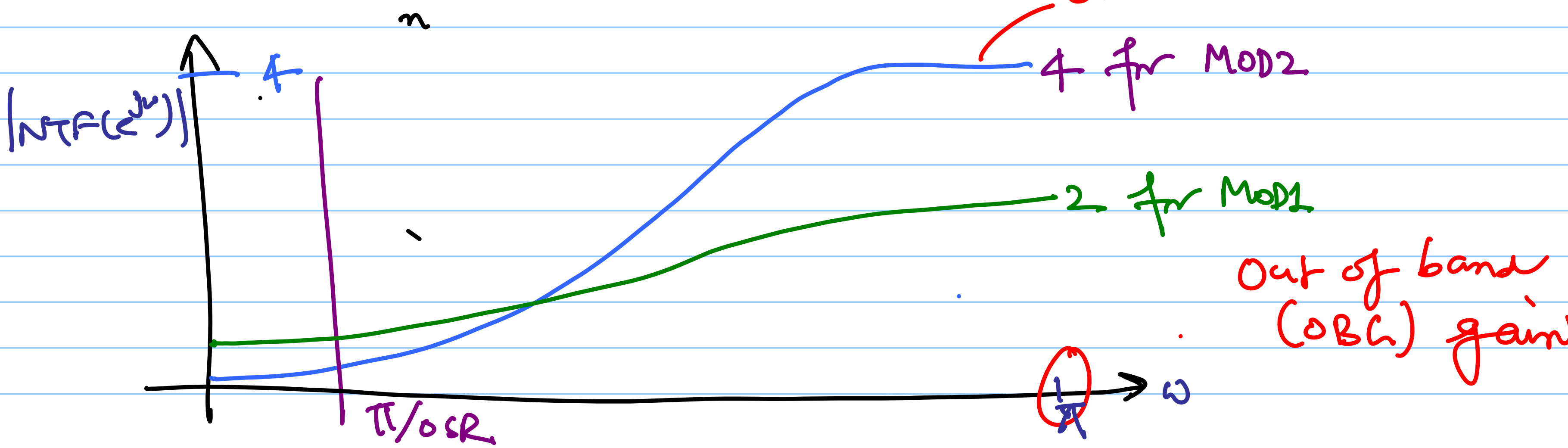
NTF gain at $\omega = \pi$

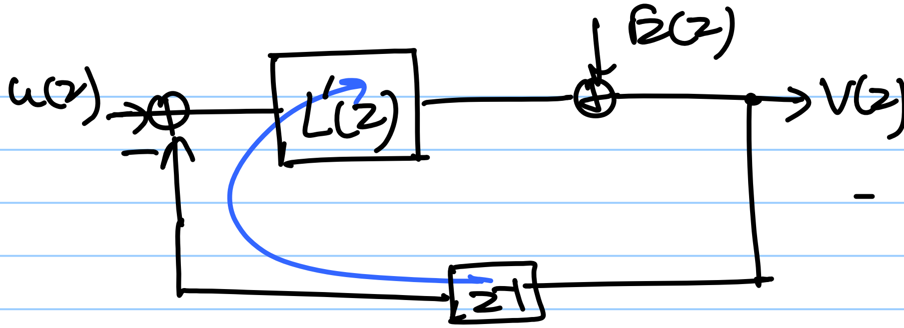
$$= \left| NTF(e^{j\omega}) \right|_{\omega=\pi}$$

$$= \sum_n (-1)^n h[n] = 4$$

$1 \quad -2 \quad 1$

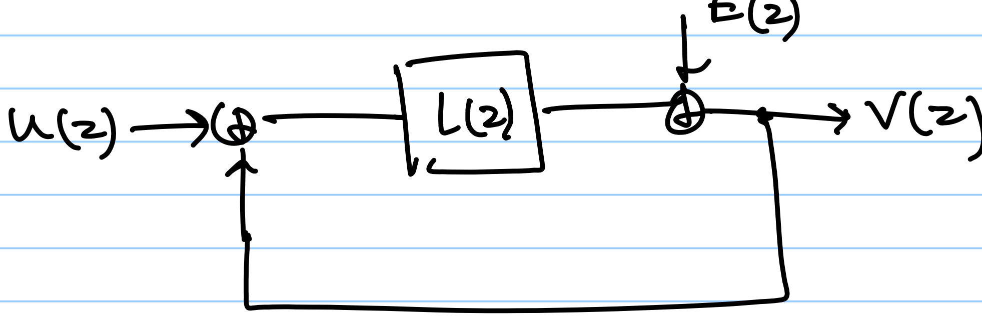
$1 \quad +2 \quad 1$



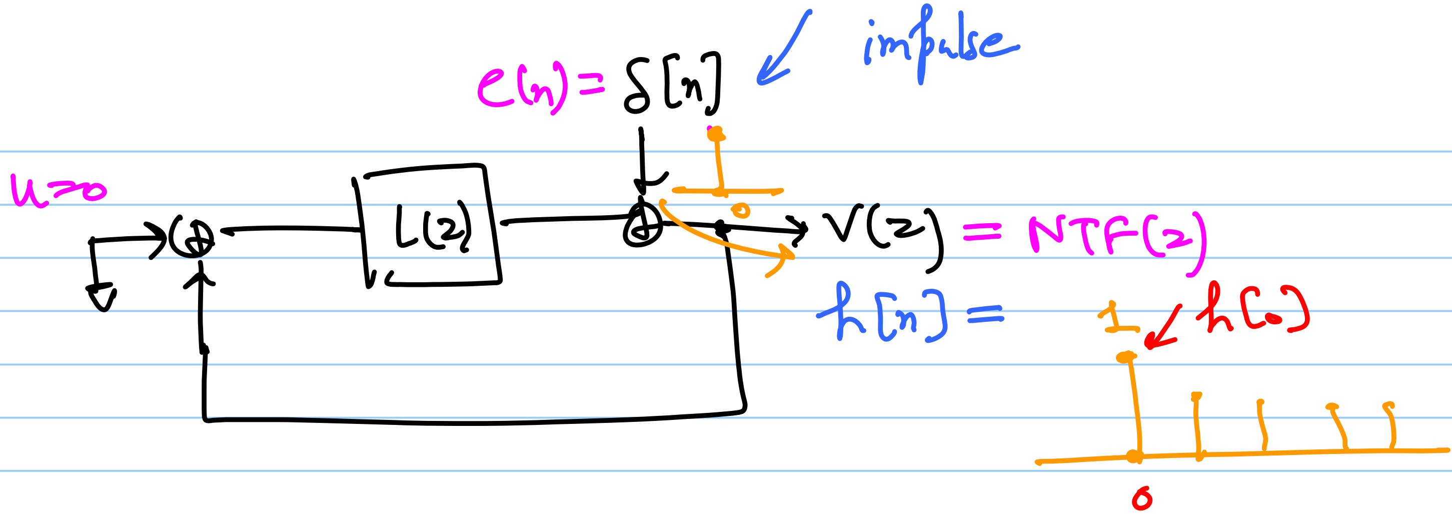


$$L'(z) = \frac{1}{1-z^{-1}}$$

$$L(z) = \frac{z^{-1}}{1-z^{-1}}$$



$\Rightarrow$ NO DELAY FREE LOOPS



first sample of the impulse response = 1

↳ no delay free loop

$$\boxed{h[0] = 1}$$

"Realizability condition"

$$\boxed{h[0] = 1 \quad \Rightarrow \quad NTF(z \to \infty) = 1}$$ fundamental result

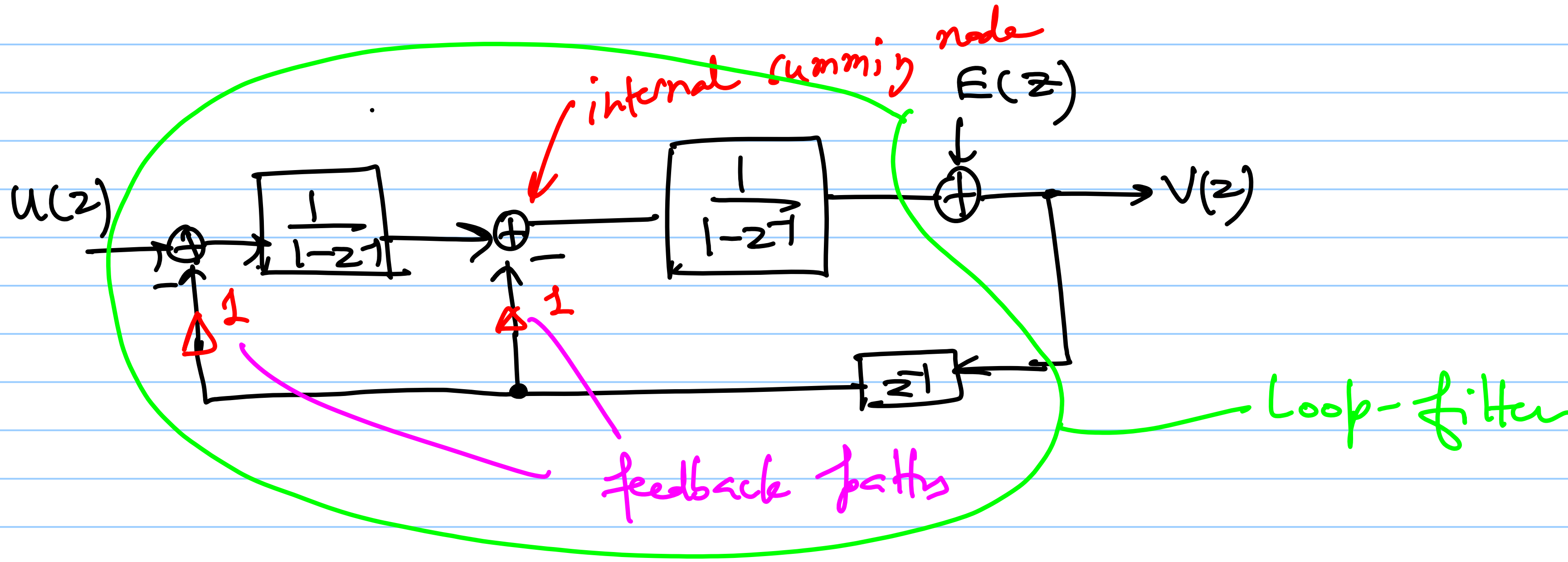
$$\underbrace{NTF(z)}_{z \to \infty \; = 1} = h[0] + \cancel{h[1] z^{-1}} + \cancel{h[2] z^{-2}} + \cdots$$

If $h[0] \neq 1 \Rightarrow$ NTF is not physically realizable

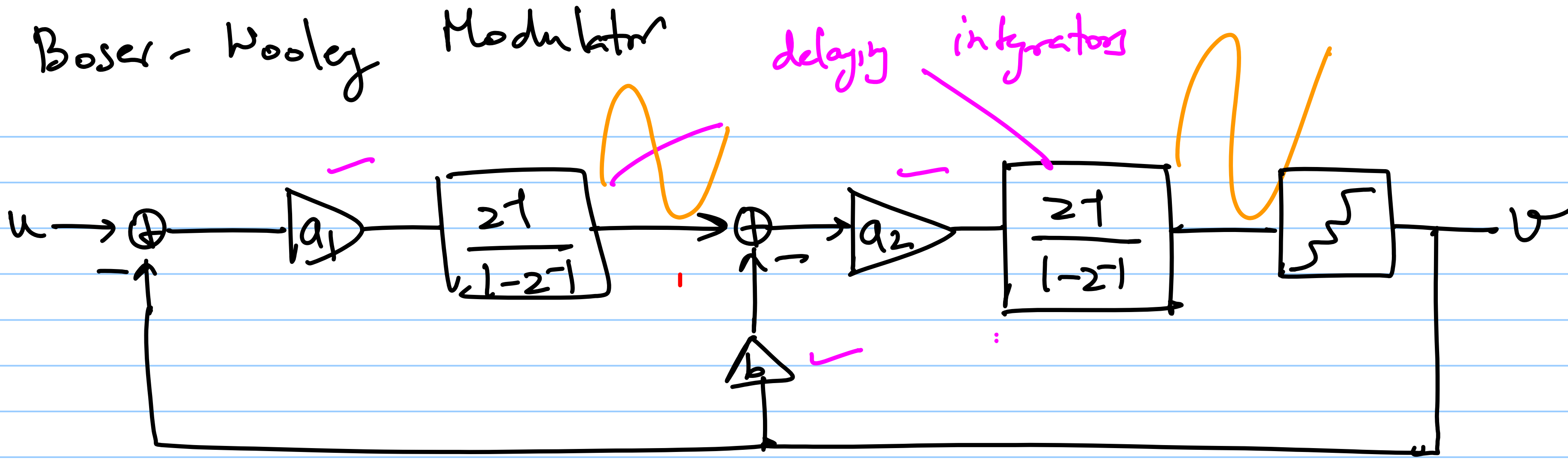
$$NTF(z) = 0.8 - \cancel{1.6 z^{-1}} + 0.8 z^{-2}$$

not realizable

Alternative 2nd-order modulators:



Boser - Wooley Modulator



$$NTF(z) = \frac{(1-z^{-1})^2}{D(z)}, \quad STF(z) = \frac{a_1 a_2 z^{-2}}{D(z)}$$

$$D(z) = (1-z^{-1})^2 + a_2 b z^{-1}(1-z^{-1}) + a_1 a_2 z^{-2}$$

for STF $= z^{-2}$ NTF $= (1-z^{-1})^2$

we can choose $a_1 a_2 = 1$ & $a_2 b = 2$

$\rightarrow D(z) = 1$

∞ solutions

$a_1 = a_2 = 1$ & $b = 2$

$a_1 = 1/2$, $a_2 = 2$, $b = 1$